INTERNATIONAL JOURNAL OF
ENERGY AND ENVIRONMENT

Volume 1, Issue 3, 2010 pp.467-478

Journal homepage: www.IJEE.IEEFoundation.org

Hydromagnetic convective flow past a vertical porous plate through a porous medium with suction and heat source

S.S.Das[1], U.K.Tripathy[2], J.K.Das[3]

[1] Department of Physics, KBDAV College, Nirakarpur, Khurda-752 019 (Orissa), India.
[2] Department of Physics, B S College, Daspalla, Nayagarh-752 078 (Orissa), India.
[3] Department of Physics, Stewart Science College, Mission Road, Cuttack-753 001 (Orissa), India.

Abstract

This paper theoretically analyzes the unsteady hydromagnetic free convective flow of a viscous incompressible electrically conducting fluid past an infinite vertical porous plate through a porous medium in presence of constant suction and heat source. Approximate solutions are obtained for velocity field, temperature field, skin friction and rate of heat transfer using multi-parameter perturbation technique. The effects of the flow parameters on the flow field are analyzed with the aid of figures and tables. The problem has some relevance in the geophysical and astrophysical studies.

Keywords: Free convection, Heat source, Hydromagnetic flow, Porous medium, Suction.

1. Introduction

The problem of convective hydromagnetic flow with heat transfer has been a subject of interest of many researchers because of its possible applications in the field of geophysical studies, astrophysical sciences, engineering sciences and also in industry. In view of its wide range of applications, Hasimoto [1] estimated the boundary layer growth on a flat plate with suction or injection. Gersten and Gross [2] studied the flow and heat transfer along a plane wall with periodic suction. Soundalgekar [3] analyzed the effect of free convection on steady MHD flow of an electrically conducting fluid past a vertical plate. Raptis and Singh [4] discussed free convection flow past an accelerated vertical plate in presence of a transverse magnetic field. Singh and Sacheti [5] reported the unsteady hydromagnetic free convection flow with constant heat flux employing finite difference scheme. Mansutti et *al*. [6] investigated the steady flow of a non-Newtonian fluid past a porous plate with suction or injection. Jha [7] analyzed the effect of applied magnetic field on transient free convective flow in a vertical channel. Kim [8] studied the unsteady free convective MHD flow with heat transfer past a semi-infinite vertical porous moving plate with variable suction. Choudhury and Das [9] explained the magnetohydrodynamic boundary layer flows of non-Newtonian fluid past a flat plate.

The behaviour of steady free convective MHD flow past a vertical porous moving surface was presented by Sharma and Pareek [10]. Singh and his associates [11] discussed the effect of heat and mass transfer in MHD flow of a viscous fluid past a vertical plate under oscillatory suction velocity. Makinde *et al* [12] analyzed the unsteady free convective flow with suction on an accelerating porous plate. Sahoo *et al.* [13] studied the unsteady free convective MHD flow past an infinite vertical plate with constant suction and heat sink. Sarangi and Jose [14] investigated the unsteady free convective MHD flow and mass transfer past a vertical porous plate with variable temperature. Ogulu and Prakash [15] discussed the heat transfer to unsteady magneto-hydrodynamic flow past an infinite vertical moving plate with variable suction. Das and his co-workers [16] estimated the mass transfer effects on unsteady flow past an accelerated vertical porous plate with suction employing finite difference analysis. Recently, Das *et al.* [17] investigated numerically the unsteady free convective MHD flow past an accelerated vertical plate with suction and heat flux.

The study reported herein analyzes the unsteady free convective flow of a viscous incompressible electrically conducting fluid past an infinite vertical porous plate with constant suction and heat flux in presence of a transverse magnetic field. Approximate solutions are obtained for velocity field, temperature field, skin friction and rate of heat transfer using multi-parameter perturbation technique. The effects of the flow parameters on the flow field are analyzed with the help of figures and tables. The problem has some relevance in the geophysical and astrophysical studies.

2. Formulation of the problem

Consider the unsteady free convective flow of a viscous incompressible electrically conducting fluid past an infinite vertical porous plate in presence of constant suction and heat flux and transverse magnetic field. Let the x′-axis be taken in vertically upward direction along the plate and y′-axis normal to it. Neglecting the induced magnetic field and the Joulean heat dissipation and applying Boussinesq's approximation the governing equations of the flow field are given by:

Continuity equation:

$$\frac{\partial v'}{\partial y'} = 0 \quad \Rightarrow \quad v' = v_0' \ \text{(Constant)} \tag{1}$$

Momentum equation:

$$\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y'} = g\beta\left(T' - T'_\infty\right) + \nu\frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_0^2}{\rho}u' - \frac{\nu}{K'}u' \tag{2}$$

Energy equation:

$$\frac{\partial T'}{\partial t'} + v'\frac{\partial T'}{\partial y'} = k\frac{\partial^2 T'}{\partial y'^2} + \frac{\nu}{C_p}\left(\frac{\partial u'}{\partial y'}\right)^2 + S'\left(T' - T'_\infty\right) \tag{3}$$

The boundary conditions of the problem are:

$$u' = 0, v' = -v_0', T' = T_w' + \varepsilon\left(T_w' - T'_\infty\right)e^{i\omega' t'} \ \text{at} \ y' = 0,$$

$$u' \to 0, \quad T' \to T'_\infty \quad \text{as} \quad y' \to \infty. \tag{4}$$

Introducing the following non-dimensional variables and parameters,

$$y = \frac{y' v_0'}{\nu}, t = \frac{t' v_0'^2}{4\nu}, \omega = \frac{4\nu\omega'}{v_0'^2}, u = \frac{u'}{v_0'}, \nu = \frac{\eta_0}{\rho}, M = \left(\frac{\sigma B_0^2}{\rho}\right)\frac{\nu}{v_0'^2}, K_p = \frac{v_0^2 K'}{\nu^2},$$

$$T = \frac{T' - T'_\infty}{T'_w - T'_\infty}, P_r = \frac{\nu}{k}, G_r = \frac{\nu g\beta\left(T'_w - T'_\infty\right)}{v_0'^3}, S = \frac{4S'\nu}{v_0'^2}, E_c = \frac{v_0'^2}{C_p\left(T'_w - T'_\infty\right)}, \tag{5}$$

where g is the acceleration due to gravity, ρ is the density, σ is the electrical conductivity, ν is the coefficient of kinematic viscosity, β is the volumetric coefficient of expansion for heat transfer, ω is the angular frequency, η_0 is the coefficient of viscosity, k is the thermal diffusivity, T is the temperature, T_w is the temperature at the plate, T_∞ is the temperature at infinity, C_p is the specific heat at constant pressure, P_r is the Prandtl number, G_r is the Grashof number for heat transfer, S is the heat source parameter, K_p is the permeability parameter, E_c is the Eckert number and M is the magnetic parameter in equations (2) and (3) under boundary conditions (4), we get:

$$\frac{1}{4}\frac{\partial u}{\partial t} - \frac{\partial u}{\partial y} = G_r T + \frac{\partial^2 u}{\partial y^2} - Mu - \frac{u}{K_p}, \tag{6}$$

$$\frac{1}{4}\frac{\partial T}{\partial t} - \frac{\partial T}{\partial y} = \frac{1}{P_r}\frac{\partial^2 T}{\partial y^2} + \frac{1}{4}ST + E_c\left(\frac{\partial u}{\partial y}\right)^2. \tag{7}$$

The corresponding boundary conditions are:

$$u = 0, T = 1 + \varepsilon e^{i\omega t} \text{ at } y = 0,$$
$$u \to 0, T \to 0 \quad \text{as} \quad y \to \infty. \tag{8}$$

3. Method of solution

To solve equations (6) and (7), we assume ε to be very small and the velocity and temperature in the neighbourhood of the plate as

$$u(y,t) = u_0(y) + \varepsilon e^{i\omega t} u_1(y), \tag{9}$$

$$T(y,t) = T_0(y) + \varepsilon e^{i\omega t} T_1(y). \tag{10}$$

Substituting equations (9) and (10) in equations (6) and (7) respectively, equating the harmonic and non harmonic terms and neglecting the coefficients of ε^2, we get

Zeroth order:

$$u_0'' + u_0' - \left(M + \frac{1}{K_p}\right)u_0 = -G_r T_0, \tag{11}$$

$$T_0'' + P_r T_0' + \frac{P_r S}{4} T_0 = -P_r E_c \left(\frac{\partial u_0}{\partial y}\right)^2. \tag{12}$$

First order:

$$u_1'' + u_1' - \frac{i\omega}{4} u_1 - \left(M + \frac{1}{K_p}\right)u_1 = -G_r T_1, \tag{13}$$

$$T_1'' + P_r T_1' - \frac{P_r}{4}(i\omega - S)T_1 = -2P_r E_c \left(\frac{\partial u_0}{\partial y}\right)\left(\frac{\partial u_1}{\partial y}\right). \tag{14}$$

Using multi-parameter perturbation technique and taking E_c <<1, we assume

$$u_0 = u_{00} + E_c u_{01}, \tag{15}$$

$$T_0 = T_{00} + E_c T_{01}, \tag{16}$$

$$u_1 = u_{10} + E_c u_{11}, \tag{17}$$

$$T_1 = T_{10} + E_c T_{11}. \tag{18}$$

Now using equations (15)-(18) in equations (11)-(14) and equating the coefficients of like powers of E_c, we get the following set of differential equations

Zeroth order:

$$u_{00}'' + u_{00}' - \left(M + \frac{1}{K_p}\right)u_{00} = -G_r T_{00}, \tag{19}$$

$$u_{10}'' + u_{10}' - \frac{i\omega}{4} u_{10} - \left(M + \frac{1}{K_p}\right)u_{10} = -G_r T_{10}, \tag{20}$$

$$T_{00}'' + P_r T_{00}' + \frac{P_r S}{4} T_{00} = 0, \tag{21}$$

$$T_{10}'' + P_r T_{10}' - \frac{P_r}{4}(i\omega - S)T_{10} = 0. \tag{22}$$

The corresponding boundary conditions are,

$$y = 0 : u_{00} = 0, T_{00} = 1, u_{10} = 0, T_{10} = 1, \qquad y \to \infty : u_{00} = 0, T_{00} = 0, u_{10} = 0, T_{10} = 0. \tag{23}$$

First order:

$$u_{01}'' + u_{01}' - \left(M + \frac{1}{K_p}\right)u_{01} = -G_r T_{01}, \tag{24}$$

$$u_{11}'' + u_{11}' - \frac{i\omega}{4} u_{11} - \left(M + \frac{1}{K_p} \right) u_{11} = -G_r T_{11}, \tag{25}$$

$$T_{01}'' + P_r T_{01}' + \frac{P_r S}{4} T_{01} = -P_r \left(u_{00}' \right)^2, \tag{26}$$

$$T_{11}'' + P_r T_{11}' - \frac{P_r}{4} (i\omega - S) T_{11} = -2 P_r \left(\frac{\partial u_{00}}{\partial y} \right) \left(\frac{\partial u_{10}}{\partial y} \right). \tag{27}$$

The corresponding boundary conditions are,

$$y = 0 : u_{01} = 0, T_{01} = 0, u_{11} = 0, T_{11} = 0,$$

$$y \to \infty : u_{01} = 0, T_{01} = 0, u_{11} = 0, T_{11} = 0. \tag{28}$$

Solving equations (19)-(22) subject to boundary condition (23), we get

$$u_{00} = A_1 \left(e^{-m_1 y} - e^{-m_5 y} \right), \tag{29}$$

$$T_{00} = e^{-m_1 y}, \tag{30}$$

$$T_{10} = e^{-m_3 y}, \tag{31}$$

$$u_{10} = A_2 \left(e^{-m_3 y} - e^{-m_7 y} \right). \tag{32}$$

Solving equations (24)-(27) subject to boundary condition (28), we get

$$T_{01} = P_r A_1^2 \left(A_3 e^{-2m_5 y} + A_4 e^{-2m_1 y} - A_5 e^{-(m_1 + m_5) y} - A_6 e^{-m_1 y} \right), \tag{33}$$

$$T_{11} = 2 P_r \left(A_7 e^{-m_5 y} - A_8 e^{-m_1 y} + A_9 e^{-m_7 y} - A_{10} e^{-m_3 y} \right), \tag{34}$$

$$u_{01} = B_1 e^{-2m_5 y} + B_2 e^{-2m_1 y} + B_3 e^{-(m_1 + m_5) y} + B_4 e^{-m_3 y} - B_5 e^{-m_5 y}, \tag{35}$$

$$u_{11} = B_6 e^{-m_5 y} + B_7 e^{-m_1 y} + B_8 e^{-m_3 y} - B_9 e^{-m_7 y}. \tag{36}$$

Using equations (15), (17), (29), (32), (35) and (36) in equation (9) and equations (16), (18), (30), (31), (33) and (34) in equation (10), the solutions for velocity and temperature of the flow field are given by

$$\begin{aligned} u &= u_{00} + E_c u_{01} + \varepsilon e^{i\omega t} \left\{ u_{10} + E_c u_{11} \right\} \\ &= A_1 \left(e^{-m_1 y} - e^{-m_5 y} \right) + E_c \left(B_1 e^{-2m_5 y} + B_2 e^{-2m_1 y} + B_3 e^{-(m_1 + m_5) y} + B_4 e^{-m_3 y} - B_5 e^{-m_5 y} \right) + \varepsilon e^{i\omega t} \left\{ A_2 \left(e^{-m_3 y} - e^{-m_7 y} \right) \right. \\ &\quad \left. + E_c \left(B_6 e^{-m_5 y} + B_7 e^{-m_1 y} + B_8 e^{-m_3 y} - B_9 e^{-m_7 y} \right) \right\}, \end{aligned} \tag{37}$$

$$\begin{aligned} T &= T_{00} + E_c T_{01} + \varepsilon e^{i\omega t} \left\{ T_{10} + E_c T_{11} \right\} \\ &= e^{-m_1 y} + E_c P_r A_1^2 \left(A_3 e^{-2m_5 y} + A_4 e^{-2m_1 y} - A_5 e^{-(m_1 + m_5) y} - A_6 e^{-m_1 y} \right) + \varepsilon e^{i\omega t} \left\{ e^{-m_3 y} \right. \\ &\quad \left. + 2 E_c P_r \left(A_7 e^{-m_5 y} - A_8 e^{-m_1 y} + A_9 e^{-m_7 y} - A_{10} e^{-m_3 y} \right) \right\} \end{aligned} \tag{38}$$

3.1. Skin Friction

The skin friction at the wall is given by

$$\tau_w = \left(\frac{\partial u}{\partial y} \right)_{y=0} \tag{39}$$

Using equations (37) in equation (39), the skin friction at the wall becomes

$$\begin{aligned} \tau_w &= A_1 (m_5 - m_1) - E_c \left\{ 2B_1 m_5 + 2B_2 m_1 + B_3 (m_1 + m_5) + B_4 m_1 - B_5 m_5 \right\} \\ &\quad + \varepsilon e^{i\omega t} \left\{ A_2 (m_7 - m_3) - E_c \left(B_6 m_5 + B_7 m_1 + B_8 m_3 - B_9 m_7 \right) \right\}. \end{aligned} \tag{40}$$

3.2. Heat Flux

The heat flux at the wall in terms of Nusselt number is given by

$$N_u = \left(\frac{\partial T}{\partial y} \right)_{y=0} \tag{41}$$

Using equation (38) in equation (41), the heat flux at the wall becomes

$$N_u = -m_1 - E_c P_r A_1^2 \left\{ 2A_3 m_5 + 2A_4 m_1 - A_5 (m_1 + m_5) - A_6 m_1 \right\}$$

$$-\varepsilon e^{i\omega t}\left\{m_3 + 2E_c P_r\left(A_7 m_5 - A_8 m_1 + A_9 m_7 - A_{10} m_3\right)\right\}, \tag{42}$$

where

$$m_1 = \frac{1}{2}\left[P_r + \sqrt{P_r^2 - SP_r}\right],\ m_2 = \frac{1}{2}\left[-P_r + \sqrt{P_r^2 - SP_r}\right],\ m_3 = \frac{1}{2}\left[P_r + \sqrt{P_r^2 - P_r\left(S - i\omega\right)}\right],$$

$$m_4 = \frac{1}{2}\left[-P_r + \sqrt{P_r^2 - P_r\left(S - i\omega\right)}\right],\ m_5 = \frac{1}{2}\left[1 + \sqrt{1 + 4\left(M + \frac{1}{K_p}\right)}\right],\ m_6 = \frac{1}{2}\left[-1 + \sqrt{1 + 4\left(M + \frac{1}{K_p}\right)}\right],$$

$$m_7 = \frac{1}{2}\left[1 + \sqrt{1 + i\omega + 4\left(M + \frac{1}{K_p}\right)}\right],\ m_8 = \frac{1}{2}\left[-1 + \sqrt{1 + i\omega + 4\left(M + \frac{1}{K_p}\right)}\right],\ A_1 = \frac{G_r}{\left(m_5 - m_1\right)\left(m_6 + m_1\right)},$$

$$A_2 = \frac{G_r}{\left(m_7 - m_3\right)\left(m_8 + m_3\right)},\ A_3 = \frac{m_5^2}{\left(m_1 - 2m_5\right)\left(m_2 + 2m_5\right)},\ A_4 = \frac{-m_1}{\left(m_2 + 2m_1\right)},\ A_5 = \frac{-2m_5}{\left(m_5 + m_2 + m_1\right)},$$

$$A_6 = -A_5 + A_4 + A_3,\ A_7 = \frac{A_1 m_5}{\left(m_3 - m_5\right)\left(m_4 + m_5\right)},\ A_8 = \frac{A_1 m_1}{\left(m_3 - m_1\right)\left(m_4 + m_1\right)},\ A_9 = \frac{A_2 m_7}{\left(m_3 - m_7\right)\left(m_4 + m_7\right)},$$

$$A_{10} = A_7 - A_8 + A_9,\ B_1 = \frac{A_1^2 A_3 P_r G_r}{m_5\left(2m_5 + m_6\right)},\ B_2 = \frac{-A_1^2 A_4 P_r G_r}{\left(m_5 - 2m_1\right)\left(m_6 + 2m_1\right)},\ B_3 = \frac{A_1^2 A_5 P_r G_r}{m_1\left(m_6 + m_5 + m_1\right)},$$

$$B_4 = \frac{A_1^2 P_r G_r\left(A_3 + A_4 - A_5\right)}{\left(m_5 - m_1\right)\left(m_6 + m_1\right)},\ B_5 = B_1 + B_2 + B_3 + B_4,\ B_6 = \frac{-2P_r G_r A_7}{\left(m_7 - m_5\right)\left(m_8 + m_5\right)},\ B_7 = \frac{2P_r G_r A_8}{\left(m_7 - m_1\right)\left(m_8 + m_1\right)},$$

$$B_8 = \frac{2P_r G_r\left(A_7 - A_8 + A_9\right)}{\left(m_7 - m_3\right)\left(m_3 + m_8\right)},\ B_9 = B_6 + B_7 + B_8.$$

4. Discussions and results

The effect of magnetic field and permeability of the medium on unsteady free convective flow of a viscous incompressible electrically conducting fluid past an infinite vertical porous plate with constant suction and heat source in presence of a transverse magnetic field has been studied. The governing equations of the flow field are solved employing multi-parameter perturbation technique and approximate solutions are obtained for velocity field, temperature field, skin friction and rate of heat transfer. The effects of the pertinent parameters on the flow field are analyzed and discussed with the help of velocity profiles (Figures 1-4); temperature profiles (Figures 5-8) and Tables 1-4.

4.1. Velocity field

The velocity of the flow field suffers a change in magnitude with the variation of the flow parameters. The factors affecting the velocity of the flow field are magnetic parameter M, permeability parameter K_p, Grashof number for heat transfer G_r and heat source parameter S. The effects of these parameters on the velocity field have been analyzed with the help of Figures 1-4.

Figure 1 depicts the effect of magnetic parameter on transient velocity of the flow field. Comparing the curves of the figure, it is observed that a growing magnetic parameter decelerates the transient velocity of the flow field at all points due to the magnetic pull of the Lorentz force acting on the flow field. The effect of permeability parameter on the transient velocity of the flow field is shown in Figure 2. For lower values of permeability parameter K_p, the transient velocity is found to increase at all points of the flow field while for higher values the effect reverses. Figure 3 presents the effect of Grashof number for heat transfer on the transient velocity. The Grashof number for heat transfer has an accelerating effect on the transient velocity of the flow field at all points due to the action of free convection current in the flow field. Figure 4 analyzes the effect of heat source parameter on the transient velocity of the flow field. A growing heat source parameter is found to enhance the transient velocity of the flow field at all points.

4.2. Temperature field

The temperature of the flow field suffers a change in magnitude with the variation of the flow parameters such as Prandtl number P_r, magnetic parameter M, permeability parameter K_p and heat source parameter S. The variations in the temperature of the flow field are shown in Figures. 5-8. Figure 5 shows the effect of Prandtl number against y on the temperature field keeping other parameters of the flow field constant.

The Prandtl number reduces the temperature of the flow field at all points. Figure 6 depicts the effect of magnetic parameter on the temperature of the flow field. The effect of magnetic parameter is to decrease the temperature of the flow field at all points. Curve with M=0 corresponds to the non-MHD flow. It is observed that in absence of magnetic field the temperature first rises near the plate and thereafter, it falls. In other curves there is a decrease in temperature at all points. This shows the dominating effect of the magnetic field due to the action of the Lorentz force acting on the flow field. In Figure 7, we analyze the effect of permeability parameter on the temperature of the flow field. A growing permeability parameter is found to increase the temperature of the flow field at all points. For higher values of K_p, the temperature first increases near the plate and thereafter it decreases at al points. Figure 8 shows the effect of heat source parameter on the temperature field. The heat source parameter is found to enhance the temperature of the flow field at all points.

4.3. Skin friction

The variations in the values of skin friction at the wall against K_p for different values of magnetic parameter M and heat source parameter S are entered in Tables 1 and 2 respectively. From Table 1, we observe that a growing magnetic parameter M reduces the skin friction at the wall for a given value of the permeability parameter due to the action of Lorentz force in the flow field. On the other hand, for a given value of magnetic parameter the permeability parameter reverses the effect. It is further noted from Table 2 that both heat source parameter S and permeability parameter enhance the skin friction at the wall.

4.4. Rate of heat transfer

The variations in the values of rate of heat transfer at the wall in terms of Nusselt number against P_r for different values of magnetic parameter M and heat source parameter S are entered in Tables 3-4 respectively. From Table 3, it is observed that a growing Prandtl number P_r or magnetic parameter M increases the magnitude of the rate of heat transfer at the wall. Further, it is observed from Table 4 that an increase in heat source parameter reduces its value for a given value of Prandtl number, while for a given heat source parameter the Prandtl number enhances the magnitude of rate of heat transfer at the wall.

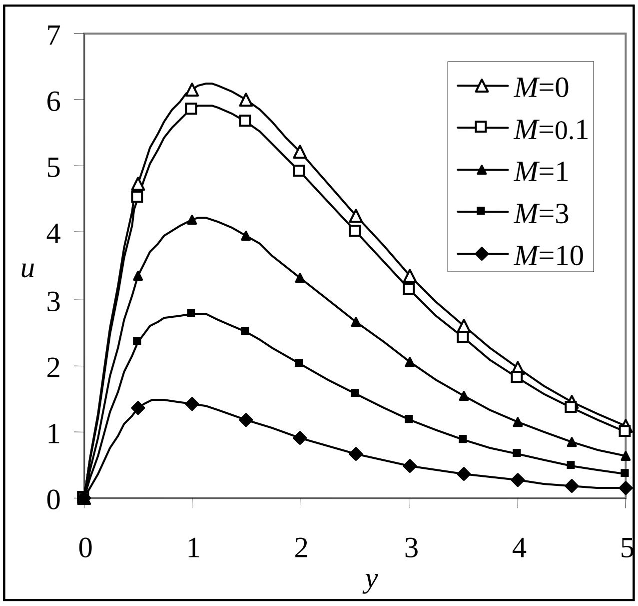

Figure 1. Transient velocity profiles against y for different values of M with G_r=5, K_p=1, S=0.1, P_r=0.71, E_c=0.002, ω=5.0, ε=0.2, ωt=π/2

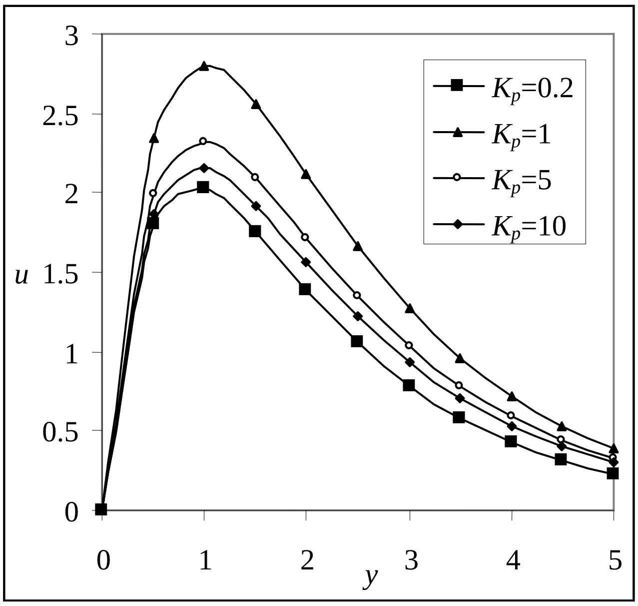

Figure 2. Transient velocity profiles against y for different values of K_p with G_r=5, S=0.1, P_r=0.71, M=1, E_c=0.002, ω=5.0, ε=0.2, ωt=π/2

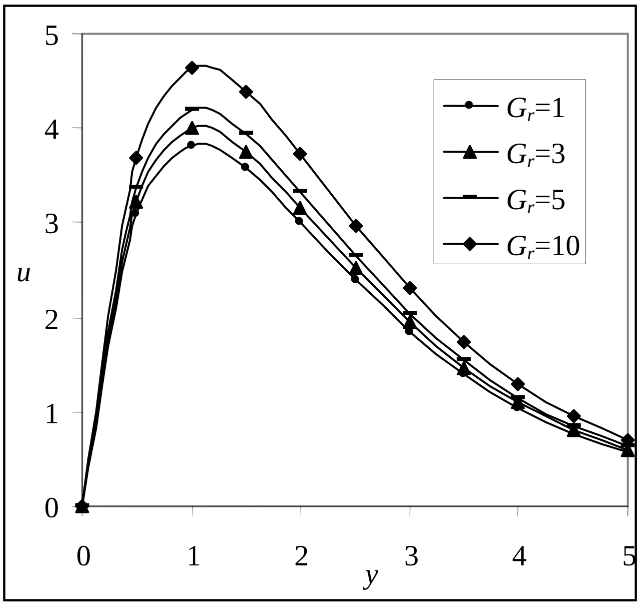

Figure 3. Transient velocity profiles against y for different values of G_r with M=1, K_p=1, S=0.1, P_r=0.71, E_c=0.002, ω=5.0, ε=0.2, ωt=π/2

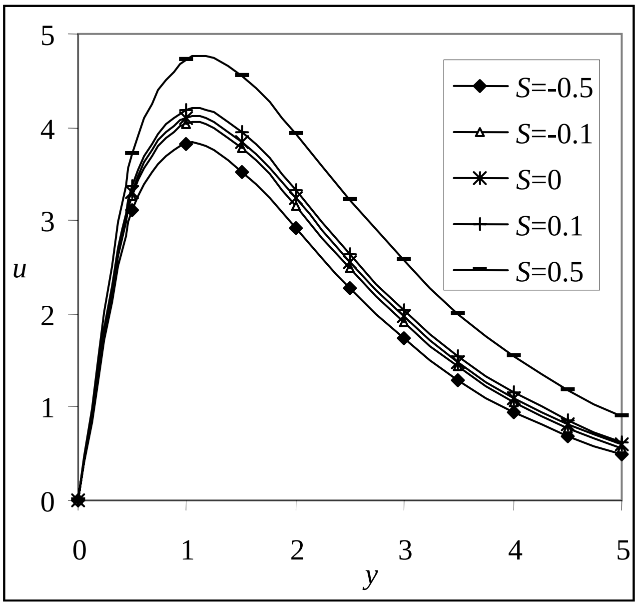

Figure 4. Transient velocity profiles against *y* for different values of *S* with G_r=5, *M*=1, K_p=1, P_r=0.71, E_c=0.002, ω=5.0, ε=0.2, ωt=π/2

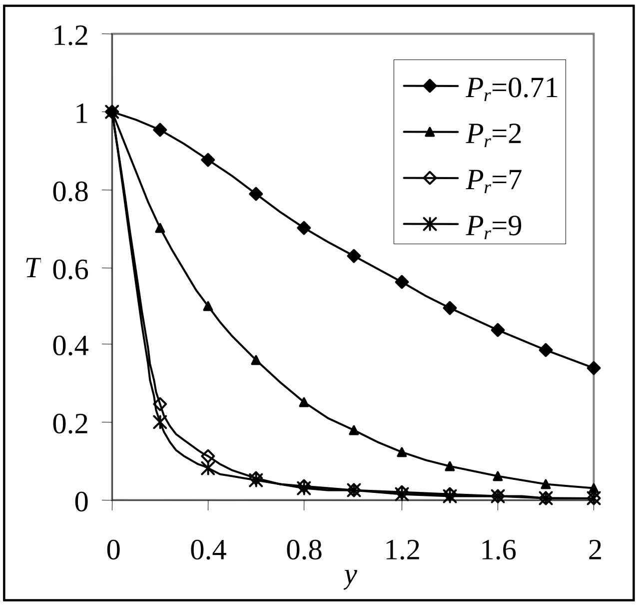

Figure 5. Transient temperature profiles against *y* for different values of P_r with G_r=5, *M*=1, K_p=1, *S*=0.1, E_c=0.002, ω=5.0, ε=0.2, ωt=π/2

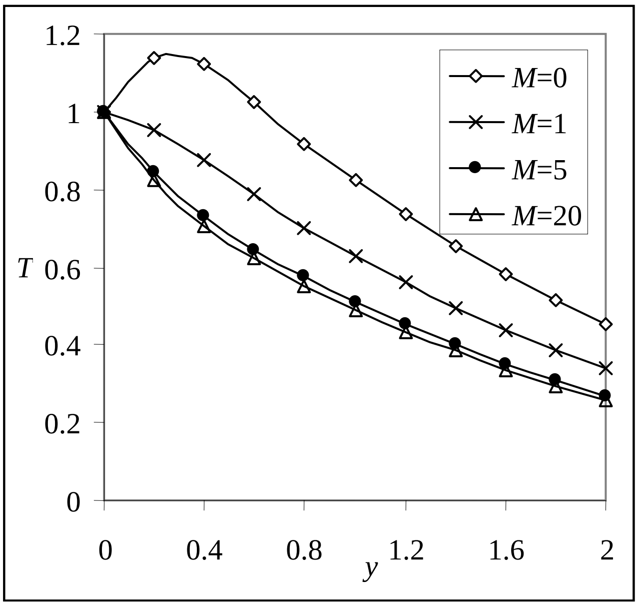

Figure 6. Transient temperature profiles against y for different values of M with G_r=5, K_p=1, S=0.1, P_r=0.71, E_c=0.002, ω=5.0, ε=0.2, ωt=π/2

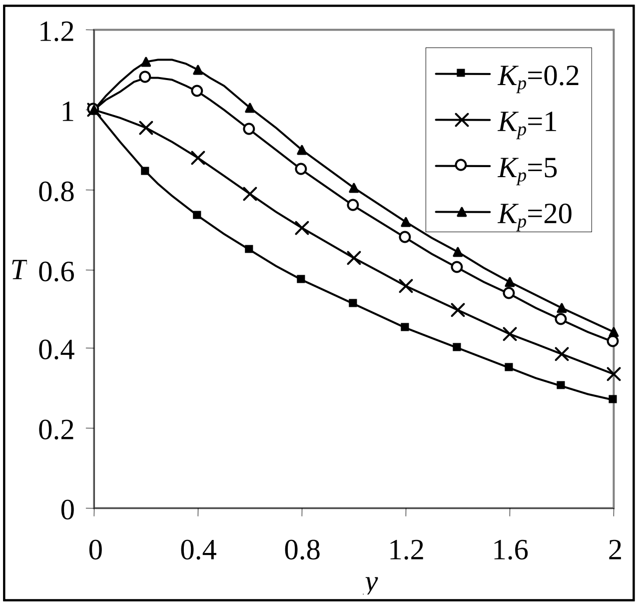

Figure 7. Transient temperature profiles against y for different values of K_p with G_r=5, M=1, S=0.1, P_r=0.71, E_c=0.002, ω=5.0, ε=0.2, ωt=π/2

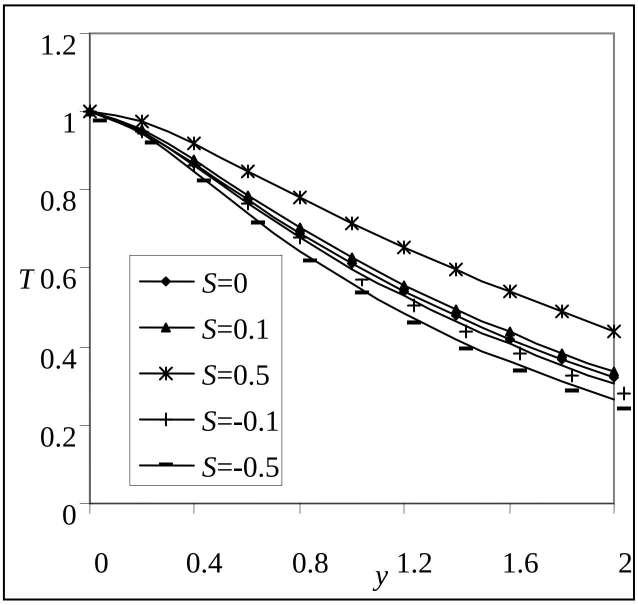

Figure 8. Transient temperature profiles against y for different values of S with G_r=5, M=1, K_p=1, P_r=0.71, E_c=0.002, ω=5.0, ε=0.2, ωt=π/2

Table 1. Variation in the value of skin friction (τ) at the wall against K_p for different values of M with G_r =5, S=0.1, E_c=0.002, ω=5.0, ε=0.2, ωt=π/2

K_p	τ			
	M=0	M=0.5	M=1	M=10
0.5	2.973518	2.716441	2.518142	1.357226
1	3.862856	3.327076	2.973518	1.413200
5	6.211482	4.363239	3.616990	1.463444
10	7.164187	4.585850	3.733386	1.470116

Table 2. Variation in the value of skin friction (τ) at the wall against K_p for different values of S with G_r=5, E_c=0.002, ω=5.0, ε=0.2, ωt=π/2

K_p	τ			
	S= -0.5	S= -0.1	S= 0.1	S= 0.5
0.5	2.358984	2.456320	2.518142	2.702705
1	2.754887	2.887993	2.973518	3.233433
5	3.301969	3.492482	3.616990	4.005289
10	3.399721	3.601249	3.733386	4.147470

Table 3. Variation in the value of heat flux (N_u) at the wall against P_r for different values of M with G_r=5, K_p =1, S=0.1, E_c=0.002, ω=5.0, ε=0.2, ωt=π/2

P_r	N_u			
	M=0	M=0.5	M=1	M=10
0.71	-0.893847	-0.894761	-0.895256	-0.8964530
2	-2.442254	-2.442407	-2.442867	-2.4437120
7	-8.409239	-8.409261	-8.409277	-8.4093510
9	-10.80198	-10.80199	-10.80200	-10.802062

Table 4. Variation in the value of heat flux (N_u) at the wall against P_r for different values of S with M=1, G_r=5, E_c=0.002, ω=5.0, ε=0.2, ωt=π/2

P_r	N_u			
	S= -0.5	S= -0.1	S= 0.1	S= 0.5
0.71	-1.0385520	-0.9483380	-0.8952560	-0.7532970
2	-2.6011110	-2.4979410	-2.4415670	-2.3232590
7	-8.5828100	-8.4678740	-8.4092770	-8.2896700
9	-10.977352	-10.861061	-10.802006	-10.681982

5. Conclusion

The above study brings out the following results of physical interest on the velocity and temperature of the flow field and also on the wall shear stress and rate of heat transfer at the wall.

1. The effect of increasing magnetic parameter M is to retard the transient velocity of the flow field at all points, while a growing Grashof number for heat transfer G_r or heat source/sink parameter S accelerates the transient velocity of the flow field at all points.
2. For smaller values of permeability parameter K_p (≤1), the transient velocity increases at all points of the flow field with increasing K_p, whereas for higher values of K_p the effect reverses.
3. A growing magnetic parameter M or Prandtl number P_r decelerates the transient temperature of the flow field at all points while a growing permeability parameter K_p or heat source parameter S reverses the effect.
4. The effect of increasing magnetic parameter M is to reduce the skin friction at the wall while a growing permeability parameter K_p or heat source parameter S reverses the effect.
5. A growing Prandtl number P_r or magnetic parameter M increases the magnitude of the rate of heat transfer at the wall. On the other hand, a growing heat source S parameter reverses the effect.

References

[1] Hasimoto H. Boundary layer growth on a flat plate with suction or injection. J. Phys. Soc. Japan. 1957, 12, 68-72.

[2] Gersten K., Gross J.F. Flow and heat transfer along a plane wall with periodic suction. Z. Angew. Math. Phys.1974, 25(3), 399-408.

[3] Soundalgekar V. M. Free convection effects on steady MHD flow past a vertical porous plate. J. Fluid Mech.1974, 66, 541-551.

[4] Raptis A., Singh A.K. MHD free convection flow past an accelerated vertical plate. Int. Comm. Heat Mass Transfer. 1983, 10, 313-321.

[5] Singh A.K., Sacheti N.C. Finite difference analysis of unsteady hydromagnetic free convection flow with constant heat flux. Astrophys. Space Sci. 1988,150, 303-308.

[6] Mansutti D., Pontrelli G., Rajagopal K. R. Steady flows of non-Newtonian fluids past a porous plate with suction or injection. Int. J. Num. Methods Fluids. 1993, 17, 927-941.

[7] Jha B. K. Effects of applied magnetic field on transient free convective flow in a vertical channel. Ind. J.Pure Appl. Math. 1998, 29(4), 441-445.

[8] Kim Y. J. Unsteady MHD convective heat transfer past a semi-infinite vertical porous moving plate with variable suction. Int. J. Engng. Sci. 2000, 38, 833-845.

[9] Choudhury R., Das A. Magnetohydrodynamic boundary layer flows of non-Newtonian fluid past a flat plate. Ind. J. Pure Appl. Math. 2000, 31(11), 1429-1441.

[10] Sharma P. R., Pareek D. Steady free convection MHD flow past a vertical porous moving surface. Ind. J. Theo. Phys. 2002, 50, 5-13.

[11] Singh A. K., Singh A. K., Singh N. P. Heat and mass transfer in MHD flow of a viscous fluid past a vertical plate under oscillatory suction velocity. Ind. J. Pure Appl. Math. 2003, 34(3), pp. 429-442.

[12] Makinde O. D., Mango J. M., Theuri D. M. Unsteady free convection flow with suction on an accelerating porous plate, AMSE J. Mod. Meas. Cont., B. 2003, 72(3), 39-46.

[13] Sahoo P. K., Datta N., Biswal S. Magnetohydrodynamic unsteady free convention flow past an infinite vertical plate with constant suction and heat sink. Ind. J. Pure Appl. Math. 2003, 34(1), 145-155.
[14] Sarangi K. C., Jose C. B. Unsteady free convective MHD flow and mass transfer past a vertical porous plate with variable temperature. Bull. Cal. Math. Soc.2005, 97 (2), 137-146.
[15] Ogulu A., Prakash J. Heat transfer to unsteady magneto-hydrodynamic flow past an infinite vertical moving plate with variable suction, Physica Scripta. 2006, 74(2), 232-238.
[16] Das S. S., Sahoo S. K., Dash G. C. Numerical solution of mass transfer effects on unsteady flow past an accelerated vertical porous plate with suction. Bull. Malays. Math. Sci., Soc. 2006, 29(1), 33-42.
[17] Das S. S., Satapathy A., Das J. K., Sahoo S. K. Numerical solution of unsteady free convective MHD flow past an accelerated vertical plate with suction and heat flux. J. Ultra Sci. Phys. Sci. 2007, 19(1), 105-112.

S. S. Das did his M. Sc. degree in Physics from Utkal University, Orissa (India) in 1982 and obtained his Ph. D degree in Physics from the same University in 2002. He served as a Faculty of Physics in Nayagarh (Autonomous) College, Orissa (India) from 1982-2004 and presently working as the Head of the faculty of Physics in KBDAV College, Nirakarpur, Orissa (India) since 2004. He has 27 years of teaching experience and 10 years of research experience. He has produced 2 Ph. D scholars and presently guiding 15 Ph. D scholars. Now he is carrying on his Post Doc. Research in MHD flow through Porous Media.
His major field of study is MHD flow, Heat and Mass Transfer Flow through Porous Media, Polar fluid, Stratified flow etc. He has 48 papers in the related area, 34 of which are published in Journals of International repute. Also he has reviewed a good number of research papers of some International Journals. Dr. Das is currently acting as the honourary member of editorial board of Indian Journal of Science and Technology and as Referee of AMSE Journal, France; Central European Journal of Physics; International Journal of Medicine and Medical Sciences etc.
E-mail address: drssd2@yahoo.com