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Finite time exergoeconomic performance optimization for an irreversible universal steady flow variable-temperature heat reservoir heat pump cycle model

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Abstract

An irreversible universal steady flow heat pump cycle model with variable-temperature heat reservoirs and the losses of heat-resistance and internal irreversibility is established by using the theory of finite time thermodynamics. The universal heat pump cycle model consists of two heat-absorbing branches, two heat-releasing branches and two adiabatic branches. Expressions of heating load, coefficient of performance (COP) and profit rate of the universal heat pump cycle model are derived, respectively. By means of numerical calculations, heat conductance distributions between hot- and cold-side heat exchangers are optimized by taking the maximum profit rate as objective. There exist an optimal heat conductance distribution and an optimal thermal capacity rate matching between the working fluid and heat reservoirs which lead to a double maximum profit rate. The effects of internal irreversibility, total heat exchanger inventory, thermal capacity rate of the working fluid and heat capacity ratio of the heat reservoirs on the optimal finite time exergoeconomic performance of the cycle are discussed in detail. The results obtained herein include the optimal finite time exergoeconomic performances of endoreversible and irreversible, constant- and variable-temperature heat reservoir Brayton, Otto, Diesel, Atkinson, Dual, Miller and Carnot heat pump cycles.

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Keywords: Finite time thermodynamics, Heating load, COP, Profit rate, Irreversible universal heat pump cycle, Internal irreversibility, Optimal heat capacity rate matching, Exergoeconomic performance.

1. Introduction

Finite time thermodynamics (FTT) [1-15] has been a powerful tool for the performance analyses and optimizations of various thermodynamic processes and cycles. The performance index in the analyses and optimizations are often pure thermodynamic parameters, which include power output, efficiency, entropy production rate, cooling load, heating load, coefficient of performance (COP), exergy loss, etc. Exergoeconomic (or thermoeconomic) analysis [16, 17] is a relatively new method that combines exergy with conventional concepts from long-run engineering economic optimization to evaluate and optimize the design and performance of energy systems. Salamon and Nitzan's work [18] combined the endoreversible model in finite time thermodynamics with exergoeconomic analysis. It was termed as finite time exergoeconomic analysis [19-36] to distinguish it from the endoreversible analysis with pure thermodynamic objectives and the exergoeconomic analysis with long-run economic optimization. This ideal has been extended to endoreversible [19-24] and generalized irreversible [25-27] Carnot heat engines, refrigerators and heat pumps, universal steady flow two-heat-reservoir heat engine, refrigerator

and heat pump cycles [28-31], three-heat-reservoir refrigerator and heat pump cycles [32, 33], endoreversible and irreversible four-heat-reservoir absorption refrigerator [34], as well as endoreversible closed-cycle simple and regenerative gas turbine heat and power cogeneration plants [35, 36]. In succession, a new thermoeconomic optimization criterion, thermodynamic output rates (power, cooling load or heating load for heat engine, refrigerator or heat pump) per unit total cost, was put forward by Sahin and Kodal [37-41]. It was used to analyze and optimize the performances of endoreversible [37, 38] and irreversible [39, 40] Carnot heat engines [37, 39], refrigerators and heat pumps [38, 40], and three-heat-reservoir absorption refrigerator and heat pump [41].

Generalization and unified description of thermodynamic cycle model is an important task of FTT research. Finite time exergoeconomic optimization for endoreversible [30] and irreversible [31] universal steady flow heat pump cycles with constant-temperature heat reservoirs have been studied, but practical heat pump cycles are always irreversible ones and with variable-temperature heat reservoirs. There are lacks of unified descriptions of exergoeconomic performances for various heat pump cycles with variable-temperature heat reservoir. On the basis of variable-temperature heat reservoir Carnot and Brayton heat pump cycle models [42-45], this paper will build an irreversible universal steady flow heat pump cycle model consisting of two heat-absorbing branches, two heat-releasing branches and two adiabatic branches with variable-temperature heat reservoirs and the losses of heat-resistance and internal irreversibility. The major work of this paper is to provide a unified description of the finite time exergoeconomic performance for various irreversible heat pump cycles with variable-temperature heat reservoirs. The results obtained herein include the optimal finite time exergoeconomic performance characteristics of end reversible and irreversible variable- and constant-temperature heat reservoir Brayton, Otto, Diesel, Atkinson, Dual, Miller and Carnot heat pump cycles.

2. Cycle model

An irreversible universal variable-temperature heat reservoir heat pump cycle model with heat-resistance and internal irreversibility is shown in Figure 1. The following assumptions are made for this model:

(1) The working fluid is an ideal gas and flows through the system in a quasi-steady fashion. The cycle consists of two heat-absorbing branches (1-2 and 2-3) with constant working fluid thermal capacity rates (mass flow rate of the working fluid and specific heat product) C_{wf1} and C_{wf2} , two heat-releasing branches (4-5 and 5-6) with constant working fluid thermal capacity rates C_{wf4} and C_{wf3} and two adiabatic branches (3-4 and 6-1). All six processes are irreversible.

(2) The hot- and cold-side heat exchangers are considered to be counter-flow heat exchangers, the working fluid temperatures are different from the heat reservoir temperatures owing to the heat transfer. The heat transfer rate (Q_H) released to the heat sink, i.e. the heating load of the cycle, and the heat transfer rate (Q_H) supplied by the heat source are:

$$Q_{H} = Q_{H1} + Q_{H2} \tag{1}$$

$$Q_L = Q_{L1} + Q_{L2} \tag{2}$$

where $Q_{H1} + Q_{H2}$ is due to the driving force of temperature differences between the high-temperature (hot-side) heat sink and working fluid, $Q_{L1} + Q_{L2}$ is due to the driving force of temperature differences between the low-temperature (cold-side) heat source and working fluid. The high-temperature heat sink is considered with thermal capacity rate C_H and the inlet and outlet temperatures of the heat-releasing fluid are T_{Hin} , T_{Hout1} and T_{Hout2} , respectively. The low-temperature heat source is considered with thermal capacity rate C_L and the inlet and outlet temperatures of the heat-releasing fluid are T_{Lin} , T_{Lout1} and T_{Lout2} , respectively.

(3) A constant coefficient ϕ is introduced to characterize the additional internal miscellaneous irreversibility effects: $\phi = (Q_{H1} + Q_{H2})/(Q_{H1} + Q_{H2}) \ge 1$, where $Q_{H1} + Q_{H2}$ is the rate of heat-flow from the warm working-fluid to the heat-sink for the irreversible cycle model, while $Q_{H1} + Q_{H2}$ is that for the endoreversible cycle model with the only loss of heat-resistance.

To summarize, the irreversible universal heat pump cycle model with variable-temperature heat reservoirs is characterized by the following three aspects:

(1) The different values of C_H and C_L . If $C_H \to \infty$ and $C_L \to \infty$, the cycle model is reduced to the irreversible universal heat pump cycle model with constant-temperature heat reservoirs [31].

(2) The different values of C_{wf1} , C_{wf2} , C_{wf3} and C_{wf4} . If C_{wf1} , C_{wf2} , C_{wf3} and C_{wf4} have different values, the cycle model can be reduced to various special heat pump cycles.

(3) The different values of ϕ . If $\phi = 1$, the cycle model is reduced to the endoreversible universal heat pump cycle model with variable-temperature heat reservoirs. If $\phi = 1$, $C_H \rightarrow \infty$ and $C_L \rightarrow \infty$ further, the cycle model is reduced to the endoreversible universal heat pump cycle model with constant-temperature heat reservoirs [30].



Figure 1. Cycle model

According to the properties of heat transfer, heat reservoir, working fluid, and the theory of heat exchangers, the heat transfer rates $(Q_{H_1} \text{ and } Q_{H_2})$ released to the heat sink and the heat transfer rates $(Q_{L_1} \text{ and } Q_{L_2})$ supplied by heat source are, respectively, given by

$$Q_{H1} = U_{H1}[(T_5 - T_{Hout1}) - (T_6 - T_{Hin})] / \ln[(T_5 - T_{Hout1}) / (T_6 - T_{Hin})] = C_H (T_{Hout1} - T_{Hin}) = C_{wf3}(T_5 - T_6) = C_{H1\min} E_{H1}(T_5 - T_{Hin})$$
(3)

$$Q_{H_2} = U_{H_2}[(T_4 - T_{Hout_2}) - (T_5 - T_{Hout_1})] / \ln[(T_4 - T_{Hout_2}) / (T_5 - T_{Hout_1})] = C_H (T_{Hout_2} - T_{Hout_1}) = C_{wf_4} (T_4 - T_5) = C_{H_2 \min} E_{H_2} (T_4 - T_{Hout_1})$$
(4)

$$Q_{L1} = U_{L1}[(T_{Lout1} - T_2) - (T_{Lout2} - T_1)] / \ln[(T_{Lout1} - T_2) / (T_{Lout2} - T_1)] = C_L(T_{Lout1} - T_{Lout2})$$

= $C_{wf1}(T_2 - T_1) = C_{L1\min}E_{L1}(T_{Lout1} - T_1)$ (5)

$$Q_{L2} = U_{L2}[(T_{Lin} - T_3) - (T_{Lout1} - T_2)] / \ln[(T_{Lin} - T_3) / (T_{Lout1} - T_2)] = C_L(T_{Lin} - T_{Lout1})$$

= $C_{wf2}(T_3 - T_2) = C_{L2\min}E_{L2}(T_{Lin} - T_2)$ (6)

where E_{H_1} , E_{H_2} , E_{L_1} and E_{L_2} are the effectivenesses of the hot- and cold-side heat exchangers, and are defined as:

$$E_{H1} = \{1 - \exp[-N_{H1}(1 - C_{H1\min} / C_{H1\max})]\} / \{1 - (C_{H1\min} / C_{H1\max}) \exp[-N_{H1}(1 - C_{H1\min} / C_{H1\max})]\}$$
(7)

$$E_{H2} = \{1 - \exp[-N_{H2}(1 - C_{H2\min} / C_{H2\max})]\} / \{1 - (C_{H2\min} / C_{H2\max}) \exp[-N_{H2}(1 - C_{H2\min} / C_{H2\max})]\}$$
(8)

$$E_{L1} = \{1 - \exp[-N_{L1}(1 - C_{L1\min} / C_{L1\max})]\} / \{1 - (C_{L1\min} / C_{L1\max}) \exp[-N_{L1}(1 - C_{L1\min} / C_{L1\max})]\}$$
(9)

$$E_{L2} = \{1 - \exp[-N_{L2}(1 - C_{L2\min} / C_{L2\max})]\} / \{1 - (C_{L2\min} / C_{L2\max}) \exp[-N_{L2}(1 - C_{L2\min} / C_{L2\max})]\}$$
(10)

where $C_{H1\min}$ and $C_{H1\max}$ are the minimum and maximum of C_H and C_{wf3} , respectively; $C_{H2\min}$ and $C_{H2\max}$ are the minimum and maximum of C_H and C_{wf4} , respectively; $C_{L1\min}$ and $C_{L1\max}$ are the minimum and maximum of C_L and C_{wf1} , respectively; $C_{L2\min}$ and $C_{L2\max}$ are the minimum and maximum of C_L and C_{wf1} , respectively; $C_{L2\min}$ and $C_{L2\max}$ are the minimum and maximum of C_L and C_{wf1} , respectively; $C_{L2\min}$ and $C_{L2\max}$ are the minimum and maximum of C_L and C_{wf1} , respectively; N_{L1} and N_{L2} are the numbers of heat transfer units of the hot- and cold-side heat exchangers, respectively:

$$C_{H1\min} = \min\{C_H, C_{wf3}\}, C_{H1\max} = \max\{C_H, C_{wf3}\}$$
(11)

$$C_{H2\min} = \min\{C_H, C_{wf4}\}, C_{H2\max} = \max\{C_H, C_{wf4}\}$$
(12)

$$C_{L1\min} = \min\{C_L, C_{wf1}\}, C_{L1\max} = \max\{C_L, C_{wf1}\}$$
(13)

$$C_{L2\min} = \min\{C_L, C_{wf2}\}, C_{L2\max} = \max\{C_L, C_{wf2}\}$$
(14)

$$N_{H1} = U_{H1} / C_{H1\min}, N_{H2} = U_{H2} / C_{H2\min}, N_{L1} = U_{L1} / C_{L1\min}, N_{L2} = U_{L2} / C_{L2\min}$$
(15)

where U_{H1} , U_{H2} , U_{L1} and U_{L2} are the heat conductances, that is, the product of heat transfer coefficient α and heat transfer surface area F.

3. Finite time exergoeconomic performance analysis

Combining equations (3)-(6), one can obtain:

$$T_{5} = \left(C_{wf3}T_{6} - C_{H1\min}E_{H1}T_{Hin}\right) / \left(C_{wf3} - C_{H1\min}E_{H1}\right)$$
(16)

$$T_{4} = [C_{H1min}C_{H2min}C_{wf3}E_{H1}E_{H2}(-T_{6}+T_{Hin})/C_{H} + C_{H1min}E_{H1}T_{Hin}(-C_{wf4}+C_{H2min}E_{H2}) + C_{wf3}(C_{wf4}T_{6}-C_{H2min}E_{H2}T_{Hin})]/[(C_{wf3}-C_{H1min}E_{H1})(C_{wf4}-C_{H2min}E_{H2})]$$
(17)

$$T_{1} = [C_{L1min}C_{L2min}C_{wf2}E_{L1}E_{L2}(T_{Lin} - T_{3})/C_{L} + C_{L1min}E_{L1}T_{Lin}(C_{L2min}E_{L2} - C_{wf2}) + C_{wf1}(C_{wf2}T_{3} - C_{L2min}E_{L2}T_{Lin})]/[(C_{wf1} - C_{L1min}E_{L1})(C_{wf2} - C_{L2min}E_{L2})]$$
(18)

$$T_{2} = \left(C_{wf2}T_{3} - C_{L2\min}E_{L2}T_{Lin}\right) / \left(C_{wf2} - C_{L2\min}E_{L2}\right)$$
(19)

The second law of thermodynamics requires that:

$$\phi = (Q_{H1} + Q_{H2}) / (Q_{H1} + Q_{H2}) = (C_{wf3} \ln \frac{T_5}{T_6} + C_{wf4} \ln \frac{T_4}{T_5}) / (C_{wf1} \ln \frac{T_2}{T_1} + C_{wf2} \ln \frac{T_3}{T_2})$$
(20)

Thus:

$$T_2 = T_1 G \tag{21}$$

where:

$$G = x^{\frac{C_{wf3}}{\phi C_{wf1}}} y^{-\frac{C_{wf2}}{C_{wf1}}} \left\{ \frac{[C_{H1min}C_{H2min}C_{wf3}E_{H1}E_{H2}(-T_6 + T_{Hin})/C_H + C_{H1min}E_{H1}T_{Hin}(-C_{wf4} + C_{H2min}E_{H2}) + \frac{C_{wf4}}{\phi C_{wf1}}}{C_{wf3}(C_{wf4}T_6 - C_{H2min}E_{H2}T_{Hin})]/[(C_{wf3}T_6 - C_{H1min}E_{H1}T_{Hin})(C_{wf2} - C_{H2min}E_{H2})]} \right\}^{(22)}$$

where $x = T_5 / T_6$ and $y = T_3 / T_2$.

Combining equations (3)-(6) with equations (18)-(22) gives:

$$T_{1} = \frac{C_{L1min} \mathcal{E}_{L1} T_{Lin} [C_{L2min} C_{wf2} \mathcal{E}_{L2} + C_{L} (-C_{wf2} + C_{L2min} \mathcal{E}_{L2})]}{Gy C_{L1min} C_{L2min} C_{wf2} \mathcal{E}_{L1} \mathcal{E}_{L2} - C_{L} [C_{wf1} (G-1) + C_{L1min} \mathcal{E}_{L1}] (C_{wf2} - C_{L2min} \mathcal{E}_{L2})}$$
(23)

$$T_{2} = \frac{GC_{L1min} \mathcal{E}_{L1} T_{Lin} [C_{L2min} C_{wf2} \mathcal{E}_{L2} + C_{L} (-C_{wf2} + C_{L2min} \mathcal{E}_{L2})]}{GyC_{L1min} C_{L2min} C_{wf2} \mathcal{E}_{L1} \mathcal{E}_{L2} - C_{L} [C_{wf1} (G-1) + C_{L1min} \mathcal{E}_{L1}] (C_{wf2} - C_{L2min} \mathcal{E}_{L2})}$$
(24)

$$T_{3} = \frac{GyC_{L1min}E_{L1}T_{Lin}[C_{L2min}C_{wf2}E_{L2} + C_{L}(-C_{wf2} + C_{L2min}E_{L2})]}{GyC_{L1min}C_{L2min}C_{wf2}E_{L1}E_{L2} - C_{L}[C_{wf1}(G-1) + C_{L1min}E_{L1}](C_{wf2} - C_{L2min}E_{L2})}$$
(25)

$$T_{Hout2} = [C_{H}^{2}T_{Hin}(C_{wf3} - C_{H1min}E_{H1})(C_{wf4} - C_{H2min}E_{H2}) + C_{H1min}C_{H2min}C_{wf3}C_{wf3}E_{H1}E_{H2}(-T_{6} + T_{Hin}) + C_{H}C_{wf3}(T_{6} - T_{Hin})(C_{H2min}C_{wf4}E_{H2} + C_{H1min}E_{H1}C_{wf4} - C_{H1min}C_{H2min}E_{H1}E_{H2})]/$$

$$[C_{H}^{2}(C_{wf3} - C_{H1min}E_{H1})(C_{wf4} - C_{H2min}E_{H2})]$$
(26)

$$T_{Lout2} = \{C_{L1min}C_{L2min}C_{wf1}C_{wf2}E_{L1}E_{L2}T_{Lin}[(G-1)C_{wf1} + (1-Gy)C_{L1min}E_{L1}] + C_{L}^{2}T_{Lin}(C_{wf1} - C_{L1min}E_{L1})(C_{wf1}G + C_{L1min}E_{L1} - C_{wf1})(C_{wf2} - C_{L2min}E_{L2}) + C_{L}C_{wf2}T_{Lin}[(1-G)C_{L2min}C_{wf1}E_{L2} + C_{L1min}^{2}E_{L1}^{2}(C_{wf1}Gy + C_{L2min}E_{L2} - C_{wf1}) + (1-G)C_{wf1}C_{L1min}C_{wf1}E_{L1} + (G-2)C_{L2min}E_{L2}C_{L1min}C_{wf1}E_{L1}]\} / \{(C_{wf1}C_{L} - C_{L1min}C_{L}E_{L1})[-GyC_{L1min}C_{wf2}E_{L1}E_{L2} + C_{L}(C_{wf1}G + C_{L1min}E_{L1} - C_{wf1})(C_{wf2} - C_{L2min}E_{L2})]\} \}$$

$$(27)$$

Substituting equations (3), (4), (16) and (17) into equation (1) yields the heating load of the cycle:

$$Q_{H} = Q_{H1} + Q_{H2}$$

$$= [-C_{H1min}C_{H2min}C_{wf3}C_{wf4}E_{H1}E_{H2}(T_{6} - T_{Hin}) / C_{H} + C_{H2min}C_{wf3}C_{wf4}E_{H2}(T_{6} - T_{Hin}) + C_{H1min}E_{H1}C_{wf3}(T_{6} - T_{Hin})(C_{wf4} - C_{H2min}E_{H2})] / [(C_{wf3} - C_{H1min}E_{H1})(C_{wf4} - C_{H2min}E_{H2})]$$
(28)

Substituting equations (5), (6), (18) and (19) into equation (2) yields the heat transfer rate supplied by the heat source:

$$Q_{L} = Q_{L1} + Q_{L2}$$

$$= T_{Lin} \{C_{L2min}^{2} E_{L2}^{2} (1 - C_{L1min} E_{L1} / C_{L}) [-G(y - 1)C_{L1min} C_{wf2} E_{L1} / C_{L} - (G - 1)(C_{wf1} - C_{L1min} E_{L1})] + C_{L2min} E_{L2} [C_{L1min} C_{wf2} E_{L1} (C_{wf1} + C_{L1min} E_{L1} G - C_{L1min} E_{L1} G y - C_{wf1} G) / C_{L} + (G - 1)(C_{wf1} C_{wf2} - C_{wf1} C_{L1min} E_{L1} - C_{wf2} C_{L1min} E_{L1})] + (G - 1)C_{L1min} C_{wf1} C_{wf2} E_{L1} \}/ [(C_{wf1} G + C_{L1min} E_{L1} - C_{wf1})(C_{wf2} - C_{L2min} E_{L2}) - GyC_{L1min} C_{wf2} E_{L1} E_{L2} / C_{L}]$$
(29)

Combining equations (28) with (29) gives the COP of the cycle:

$$\beta = \frac{Q_{H}}{P} = \frac{Q_{H}}{Q_{H} - Q_{L}} \\ = \frac{[-C_{H1min}C_{H2min}C_{wf3}C_{wf4}E_{H1}E_{H2}(T_{6} - T_{Hin})/C_{H} + C_{H2min}C_{wf3}C_{wf4}E_{H2}(T_{6} - T_{Hin}) + C_{H1min}E_{H1}C_{wf3}(T_{6} - T_{Hin})(C_{wf4} - C_{H2min}E_{H2})]/[(C_{wf3} - C_{H1min}E_{H1})(C_{wf4} - C_{H2min}E_{H2})]}{[-C_{H1min}C_{H2min}C_{wf3}C_{wf4}E_{H1}E_{H2}(T_{6} - T_{Hin})/C_{H} + C_{H2min}C_{wf3}C_{wf4}E_{H2}(T_{6} - T_{Hin}) + C_{H1min}E_{H1}C_{wf3}(T_{6} - T_{Hin})(C_{wf4} - C_{H2min}E_{H2})]/[(C_{wf3} - C_{H1min}E_{H1})(C_{wf4} - C_{H2min}E_{H2})] - C_{H1min}E_{H1}C_{wf3}(T_{6} - T_{Hin})(C_{wf4} - C_{H2min}E_{H2})]/[(C_{wf3} - C_{H1min}E_{H1})(C_{wf4} - C_{H2min}E_{H2})] - T_{Lin}\{C_{L2min}^{2}E_{L2}^{2}(1 - C_{L1min}E_{L1}/C_{L})[-G(y - 1)C_{L1min}C_{wf2}E_{L1}/C_{L} - (G - 1)(C_{wf1} - C_{L1min}E_{L1})] + C_{L2min}E_{L2}[C_{L1min}C_{wf2}E_{L1}(C_{wf1} + C_{L1min}E_{L1}G - C_{L1min}E_{L1}Gy - C_{wf1}G)/C_{L} + (G - 1)(C_{wf1}C_{wf2} - C_{wf1}C_{L1min}E_{L1} - C_{wf2}C_{L1min}E_{L1})] + (G - 1)C_{L1min}C_{wf2}E_{L1}\}/[(C_{wf1}G + C_{L1min}E_{L1} - C_{wf1})(C_{wf2} - C_{L2min}E_{L2}) - GyC_{L1min}C_{wf2}E_{L1}E_{L2}/C_{L}]$$

where T_{Hout2} and T_{Lout2} are calculated by equations (26) and (27).

Assuming that the environmental temperature is T_0 , the exergy output rate of the cycle is:

$$A = \int_{T_{Hin}}^{T_{Hour2}} C_H (1 - T_0/T) dT - \int_{T_{Lin}}^{T_{Lour2}} C_L (T_0/T - 1) dT$$

$$= Q_H - Q_L - T_0 [C_H \ln(T_{Hour2}/T_{Hin}) + C_L \ln(T_{Lour2}/T_{Lin})] = Q_H \eta_1 - Q_L \eta_2$$
(31)

where $\eta_1 = 1 - T_0 / [(T_{Hout2} - T_{Hin}) / \ln(T_{Hout2} / T_{Hin})]$, and $\eta_2 = 1 - T_0 / [(T_{Lin} - T_{Lout2}) / \ln(T_{Lin} / T_{Lout2})]$. Assuming that the prices of exergy output rate and power input are ψ_1 and ψ_2 , the profit rate of the cycle

is:

$$\Pi = \psi_1 A - \psi_2 P = (\psi_1 \eta_1 - \psi_2) Q_H + (\psi_2 - \psi_1 \eta_2) Q_L$$
(32)

Substituting equations (28) and (29) into equation (32) yields the profit rate of the cycle:

$$\Pi = (\psi_{1}\eta_{1} - \psi_{2})[-C_{H1min}C_{H2min}C_{wf3}C_{wf4}E_{H1}E_{H2}(T_{6} - T_{Hin})/C_{H} + C_{H2min}C_{wf3}C_{wf4}E_{H2}(T_{6} - T_{Hin}) + C_{H1min}E_{H1}C_{wf3}(T_{6} - T_{Hin})(C_{wf4} - C_{H2min}E_{H2})]/[(C_{wf3} - C_{H1min}E_{H1})(C_{wf4} - C_{H2min}E_{H2})] + T_{Lin}(\psi_{2} - \psi_{1}\eta_{2})\{C_{L2min}^{2}E_{L2}^{2}(1 - C_{L1min}E_{L1}/C_{L})[-G(y-1)C_{L1min}C_{wf2}E_{L1}/C_{L} - (G-1)(C_{wf1} - C_{L1min}E_{L1})] + C_{L2min}E_{L2}[C_{L1min}C_{wf2}E_{L1}(-C_{wf1}G + C_{wf1} + C_{L1min}E_{L1}G - C_{L1min}E_{L1}Gy)/C_{L} + (G-1)(C_{wf1}C_{wf2} - C_{wf1}C_{L1min}E_{L1} - C_{wf2}C_{L1min}E_{L1})] + (G-1)C_{L1min}C_{wf1}C_{wf2}E_{L1}\}/ [(C_{wf1}G + C_{L1min}E_{L1} - C_{wf1})(C_{wf2} - C_{L2min}E_{L2}) - GyC_{L1min}C_{wf2}E_{L1}E_{L2}/C_{L}]]$$

$$(33)$$

In order to make the cycle operate normally, state point 2 must be between state points 1 and 3, and state point 5 must be between state points 4 and 6. Therefore, the ranges of x and y are:

$$1 \le x \le [C_{H1min}C_{H2min}C_{wf3}E_{H1}E_{H2}(-T_6 + T_{Hin})/C_H + C_{H1min}E_{H1}T_{Hin}(-C_{wf4} + C_{H2min}E_{H2}) + C_{wf3}(C_{wf4}T_6 - C_{H2min}E_{H2}T_{Hin})]/[T_6(C_{wf3} - C_{H1min}E_{H1})(C_{wf4} - C_{H2min}E_{H2})]$$

$$(34)$$

$$1 \le y \le x^{\frac{C_{wf3}}{\phi C_{wf2}}} \left\{ \begin{bmatrix} C_{H1min} C_{H2min} C_{wf3} E_{H1} E_{H2} (-T_6 + T_{Hin}) / C_H + C_{H1min} E_{H1} T_{Hin} (-C_{wf4} + C_{H2min} E_{H2}) + \\ C_{wf3} (C_{wf4} T_6 - C_{H2min} E_{H2} T_{Hin}) \end{bmatrix} / \begin{bmatrix} C_{wf3} T_6 - C_{H1min} E_{H1} T_{Hin} (-C_{wf4} + C_{H2min} E_{H2}) + \\ C_{wf3} (C_{wf4} T_6 - C_{H2min} E_{H2} T_{Hin}) \end{bmatrix} / \begin{bmatrix} C_{wf3} T_6 - C_{H1min} E_{H1} T_{Hin} (-C_{wf4} + C_{H2min} E_{H2}) + \\ C_{wf3} (C_{wf4} T_6 - C_{H2min} E_{H2} T_{Hin}) \end{bmatrix} / \begin{bmatrix} C_{wf3} T_6 - C_{H1min} E_{H1} T_{Hin} (-C_{wf4} + C_{H2min} E_{H2}) + \\ C_{wf3} (C_{wf4} T_6 - C_{H2min} E_{H2} T_{Hin}) \end{bmatrix} / \begin{bmatrix} C_{wf3} T_6 - C_{H1min} E_{H1} T_{Hin} (-C_{wf4} + C_{H2min} E_{H2}) + \\ C_{wf3} (C_{wf4} T_6 - C_{H2min} E_{H2} T_{Hin}) \end{bmatrix} / \begin{bmatrix} C_{wf4} T_6 - C_{H2min} E_{H2} T_{Hin} \\ C_{wf3} (C_{wf4} T_6 - C_{H2min} E_{H2} T_{Hin}) \end{bmatrix} / \begin{bmatrix} C_{wf4} T_6 - C_{H2min} E_{H2} T_{Hin} \\ C_{wf3} (C_{wf4} T_6 - C_{H2min} E_{H2} T_{Hin}) \end{bmatrix} / \begin{bmatrix} C_{wf4} T_6 - C_{H2min} E_{H2} T_{Hin} \\ C_{wf3} (C_{wf4} T_6 - C_{H2min} E_{H2} T_{Hin}) \end{bmatrix} / \begin{bmatrix} C_{wf4} T_6 - C_{H2min} E_{H2} T_{Hin} \\ C_{wf3} (C_{wf4} T_6 - C_{H2min} E_{H2} T_{Hin}) \end{bmatrix} / \begin{bmatrix} C_{wf4} T_6 - C_{H2min} E_{H2} T_{Hin} \\ C_{wf3} (C_{wf4} T_6 - C_{H2min} E_{H2} T_{Hin}) \end{bmatrix} / \begin{bmatrix} C_{wf4} T_6 - C_{H2min} E_{H2} T_{Hin} \\ C_{wf4} T_6 - C_{H2min} E_{H2} T_{Hin} \\ C_{wf4} T_6 - C_{H2min} E_{H2} T_{Hin} \\ C_{wf4} T_6 - C_{H2min} E_{H2} T_{Hin} \end{bmatrix} + \begin{bmatrix} C_{wf4} T_6 - C_{H2min} E_{H2} T_{Hin} \\ C_{wf4} T_6 T_{H1} T_{Hin} \\ C_{wf4} T_6 - C_{H2min} E_{H2} T_{Hin} \\ C_{wf4} T_6 - C_{H2min} E_{H2} T_{Hin} \\ C_{wf4} T_6 - C_{H2min} E_{H2} T_{Hin} \\ C_{wf4} T_6 T_{H1} \\ C_{wf4} T_6 T_{H1} \\ C_{wf4} T_{H1} \\$$

Note that for the process to be potential profitable, the following relationship must exist: $0 < \psi_2/\psi_1 < 1$, because one unit of work input must give rise to at least one unit of exergy output.

When the price of exergy output rate becomes very large compared with the price of the power input, i.e. $\psi_2/\psi_1 \rightarrow 0$, equation (32) becomes:

 $\Pi = \psi_1 A$

where A is the exergy output rate of the irreversible universal heat pump cycle. That is, the profit rate maximization approaches the exergy output rate maximization.

When the price of exergy output rate approaches the price of the power input, i.e. $\psi_2/\psi_1 \rightarrow 1$, equation (32) becomes

$$\Pi = -\psi_1 T_0 [C_H \ln(T_{Hout2}/T_{Hin}) + C_L \ln(T_{Lout2}/T_{Lin})] = -\psi_1 T_0 \sigma$$
(37)

where $\sigma = C_H \ln(T_{Hout2}/T_{Hin}) + C_L \ln(T_{Lout2}/T_{Lin})$ is the entropy production rate of the irreversible universal heat pump cycle. That is, the profit rate maximization approaches the entropy production rate minimization, i.e., the minimum exergy loss.

4. Discussion

Equations (30) and (33) are generalized. If C_H , C_L and ϕ have different values, equations (30) and (33) can be simplified into the corresponding analytical formulae for various endoreversible and irreversible, constant- and variable-temperature heat reservoir heat pump cycles.

Figure 2 shows the finite time exergoeconomic performance characteristics of the irreversible universal heat pump cycle with variable-temperature heat reservoirs. Heat conductances of the hot- and cold-side heat exchangers are set as $U_{H1} = 0$, $U_{L2} = 0$ and $U_{H2} = U_{L1} = 3 kW/K$ for Brayton, Otto, Diesel and Atkinson heat pump cycles; $U_{H1} = U_{H2} = U_{L1} = 2 kW/K$ and $U_{L2} = 0$ for Dual heat pump cycle; $U_{H1} = 0$ and $U_{H2} = U_{L1} = 0$ for Miller heat pump cycle, respectively. Internal irreversibility and price ratio are set as $\phi = 1.1$ and $\psi_1/\psi_2 = 5$, respectively. One can continue to discuss the special cases of the universal heat pump cycle for different thermal capacity rates of the working fluid (C_{wf1} , C_{wf2} , C_{wf3} and C_{wf4}) in detail, whose dimensionless profit rate versus COP curves are also shown in Figure 2.



Figure 2. $\overline{\Pi}$ vs. β characteristics of irreversible universal heat pump cycle with variable-temperature heat reservoirs

(1) When $C_{wf1} = C_{wf2} = \dot{m}C_p$ (mass flow rate \dot{m} of the working fluid and constant pressure specific heat C_p product) and $C_{wf3} = C_{wf4} = \dot{m}C_p$, $U_{H1} = 0$, $U_{L2} = 0$ and x = y = 1, equations (30) and (33) become the COP and finite time exergoeconomic performance characteristics of an irreversible variable-temperature heat reservoir steady flow Brayton heat pump cycle with the losses of heat-resistance and internal irreversibility.

(36)

(2) When $C_{wf1} = C_{wf2} = \dot{m}C_v$ (mass flow rate \dot{m} of the working fluid and constant volume specific heat C_v product) and $C_{wf3} = C_{wf4} = \dot{m}C_v$, $U_{H1} = 0$, $U_{L2} = 0$ and x = y = 1, equations (30) and (33) become the COP and finite time exergoeconomic performance of an irreversible variable-temperature heat reservoir steady flow Otto heat pump cycle with the losses of heat-resistance and internal irreversibility.

(3) When $C_{wf1} = C_{wf2} = \dot{m}C_v$ and $C_{wf3} = C_{wf4} = \dot{m}C_p$, $U_{H1} = 0$, $U_{L2} = 0$ and x = y = 1, equations (30) and (33) become the COP and finite time exergoeconomic performance characteristics of an irreversible variable-temperature heat reservoir steady flow Diesel heat pump cycle with the losses of heat-resistance and internal irreversibility.

(4) When $C_{wf1} = C_{wf2} = \dot{m}C_p$ and $C_{wf3} = C_{wf4} = \dot{m}C_v$, $U_{H1} = 0$, $U_{L2} = 0$ and x = y = 1, equations (30) and (33) become the COP and finite time exergoeconomic performance characteristics of an irreversible variable-temperature heat reservoir steady flow Atkinson heat pump cycle with the losses of heat-resistance and internal irreversibility.

(5) When $C_{wf1} = C_{wf2} = \dot{m}C_v$, $C_{wf3} = \dot{m}C_v$ and $C_{wf4} = \dot{m}C_p$, $U_{H1} \neq 0$, $U_{H2} \neq 0$, $U_{L2} = 0$ and y = 1, equations (30) and (33) become the COP and finite time exergoeconomic performance characteristics of an irreversible variable-temperature heat reservoir steady flow Dual heat pump cycle with the losses of heat-resistance and internal irreversibility. If $U_{H1} \rightarrow 0$, $U_{L2} = 0$ and x = y = 1 further, the Dual heat pump cycle is close to the Diesel heat pump cycle. If $U_{H2} \rightarrow 0$, $U_{L2} = 0$ and y = 1 further, the Dual heat pump cycle is close to the Otto heat pump cycle.

In this case, the range of x becomes:

$$1 \le x \le [C_{H1min}C_{H2min}C_{wf3}E_{H1}E_{H2}(-T_6 + T_{Hin})/C_H + C_{H1min}E_{H1}T_{Hin}(-C_{wf4} + C_{H2min}E_{H2}) + C_{wf3}(C_{wf4}T_6 - C_{H2min}E_{H2}T_{Hin})]/[T_6(C_{wf3} - C_{H1min}E_{H1})(C_{wf4} - C_{H2min}E_{H2})]$$
(38)

and the value of x is given by:

$$x = T_5 / T_6 = [C_{H1min} C_{H2min} C_{wf3} E_{H1} E_{H2} (-T_6 + T_{Hin}) / C_H + C_{H1min} E_{H1} T_{Hin} (-C_{wf4} + C_{H2min} E_{H2}) + C_{wf3} (C_{wf4} T_6 - C_{H2min} E_{H2} T_{Hin})] / [(C_{wf3} T_6 - C_{H1min} E_{H1} T_{Hin}) (C_{wf2} - C_{H2min} E_{H2})]$$
(39)

(6) When $C_{wf1} = \dot{m}C_p$, $C_{wf2} = \dot{m}C_v$ and $C_{wf3} = C_{wf4} = \dot{m}C_v$, $U_{H1} = 0$, $U_{L1} \neq 0$, $U_{L2} \neq 0$ and x = 1, equations (30) and (33) become the COP and finite time exergoeconomic performance characteristics of an irreversible variable-temperature heat reservoir steady flow Miller heat pump cycle with the losses of heat-resistance and internal irreversibility. If $U_{H1} = 0$, $U_{L2} \rightarrow 0$ and x = y = 1 further, the Miller heat pump cycle is close to the Atkinson heat pump cycle. If $U_{H1} = 0$, $U_{L1} \rightarrow 0$ and x = 1 further, the Miller heat pump cycle is close to the Otto heat pump cycle. In this case, the range of y is:

$$1 \le y \le \begin{cases} [C_{H1min}C_{H2min}C_{wf3}E_{H1}E_{H2}(-T_6 + T_{Hin})/C_H + C_{H1min}E_{H1}T_{Hin}(-C_{wf4} + C_{H2min}E_{H2}) +]^{\frac{1}{\phi}} \\ C_{wf3}(C_{wf4}T_6 - C_{H2min}E_{H2}T_{Hin})]/[(C_{wf3}T_6 - C_{H1min}E_{H1}T_{Hin})(C_{wf2} - C_{H2min}E_{H2})] \end{cases}$$

$$(40)$$

Combining equations (18), (19) and (22) give the following equation that the working fluid temperature T_3 should satisfy:

$$(C_{wf1} - C_{L1min} E_{L1}) (C_{wf2} T_3 - C_{L2min} E_{L2} T_{Lin}) [T_3 (C_{wf2} - C_{L2min} E_{L2}) / (C_{wf2} T_3 - C_{L2min} E_{L2} T_{Lin})]^{\frac{1}{k}} / \\ [C_{L1min} C_{L2min} C_{wf2} E_{L1} E_{L2} (T_{Lin} - T_3) / C_L + C_{L1min} E_{L1} T_{Lin} (C_{L2min} E_{L2} - C_{wf2}) + C_{wf1} (C_{wf2} T_3 - C_{L2min} E_{L2} T_{Lin})] =$$

$$\left\{ [C_{H1min} C_{H2min} C_{wf3} E_{H1} E_{H2} (-T_6 + T_{Hin}) / C_H + C_{H1min} E_{H1} T_{Hin} (-C_{wf4} + C_{H2min} E_{H2}) + \right\}^{\frac{1}{k}} \\ C_{wf3} (C_{wf4} T_6 - C_{H2min} E_{H2} T_{Hin})] / [(C_{wf3} T_6 - C_{H1min} E_{H1} T_{Hin}) (C_{wf2} - C_{H2min} E_{H2})] \right\}^{\frac{1}{k}}$$

where k is the ratio of the specific heats. Moreover, combining equations (18), (19), (21) with equation (41) gives G and y.

(7) When $C_{wf1} = C_{wf2} = C_{wf3} = C_{wf4} \rightarrow \infty$, equations (30) and (33) become the COP and finite time exergoeconomic performance characteristics of an irreversible variable-temperature heat reservoir steady flow Carnot heat pump cycle with the losses of heat-resistance and internal irreversibility. Specially, if $C_H \rightarrow \infty$ and $C_L \rightarrow \infty$ further, the finite time exergoeconomic performance characteristic of an irreversible Carnot heat pump cycle with variable-temperature heat reservoirs become the finite time exergoeconomic performance characteristics of endoreversible ($\phi = 1$) [21, 24] and irreversible ($\phi > 1$) [28] Carnot heat pump cycle with constant-temperature heat reservoirs, respectively.

5. Finite time exergoeconomic performance optimization

5.1 Optimal distributions of heat conductance

If heat conductances of hot- and cold-side heat exchangers are changeable, the profit rate of the irreversible universal heat pump cycle may be optimized by searching the optimal heat conductance distributions for the fixed total heat exchanger inventory. For the fixed heat exchanger inventory U_T , that is, for the constraint of $U_{H1}+U_{H2}+U_{L1}+U_{L2}=U_T$, defining the distributions of heat conductance $u_{H1}=U_{H1}/U_T$, $u_{H2}=U_{H2}/U_T$, $u_{L1}=U_{L1}/U_T$ and $u_{L2}=U_{L2}/U_T$ leads to:

$$U_{H1} = u_{H1}U_T, U_{H2} = u_{H2}U_T, U_{L1} = u_{L1}U_T, U_{L1} = u_{L1}U_T, U_{L2} = u_{L2}U_T$$
(42)

The following conditions should be satisfied: $0 \le u_{H1} \le 1$, $0 \le u_{H2} \le 1$, $0 \le u_{L1} \le 1$, $0 \le u_{L2} \le 1$, and $u_{H1} + u_{H2} + u_{L1} + u_{L2} = 1$. Moreover, heat conductance distributions are set as $u_{H1} = 0$ and $u_{L2} = 0$ for Brayton, Otto, Diesel and Atkinson heat pump cycles; $u_{L2} = 0$ for Dual heat pump cycle; $u_{H1} = 0$ for Miller heat pump cycle, respectively.

To illustrate the preceding analyses, one can take the irreversible Brayton heat pump cycle with variabletemperature heat reservoirs (air as the working fluid) as a numerical example. In the calculations, it is set that $T_{Hin} = 290.0K$, $T_{Lin} = 268.0K$, $C_H = C_L = 1.2 \ kW / K$, $C_v = 0.7165 \ kJ / (kg \cdot K)$, $C_p = 1.0031 \ kJ / (kg \cdot K)$, k = 1.4, $\phi = 1.1$, $U_T = 5 \ kW / K$, $\dot{m} = 1.1165 \ kg / s$ and $\psi_1 / \psi_2 = 5$. If there are no special explanations, the parameters are set as above. The working fluid temperature T_6 is a variable and its reasonable value is greater than T_{Hin} . The calculations illustrate that the values of x and y are always in their ranges. The dimensionless profit rate is defined as $\overline{\Pi} = \Pi / (0.9 \dot{m} T_L C_V \psi_2)$.

Figure 3 shows the effect of the price ratio (ψ_1/ψ_2) on the dimensionless profit rate $(\overline{\Pi})$ versus COP (β) for irreversible variable-temperature heat reservoir Brayton heat pump cycle. From Figure 3, one can see that $\overline{\Pi}$ increases with the increase in ψ_1/ψ_2 for the fixed β . Moreover, when $\psi_1/\psi_2 = 1$, the maximum profit rate is not greater than zero, i.e., the heat pump is not profitable regardless of any working condition.

The dimensionless profit rate ($\overline{\Pi}$) versus COP (β) and the hot-side heat conductance distribution (u_{H2}) of an irreversible variable-temperature heat reservoir Brayton heat pump cycle with $\psi_1/\psi_2 = 5$ and $\phi = 1.1$ is shown in Figure 4. It indicates that the curve of dimensionless profit rate versus hot-side heat conductance distribution is a parabolic-like one for the fixed COP. There exists an optimal heat conductance distribution ($u_{H2,opt,\Pi}$) which leads to the optimal dimensionless profit rate ($\overline{\Pi}_{opt,u}$). For Otto, Diesel and Atkinson heat pump cycles, the three-dimensional diagram characteristics among dimensionless profit rate versus COP and heat conductance distribution are similar with those shown in Figure 4.

The three-dimensional diagram among the dimensionless profit rate $(\overline{\Pi})$ and heat conductance distributions $(u_{H1} \text{ and } u_{H2})$ of an irreversible variable-temperature heat reservoir Dual heat pump cycle with $\beta = 3$, $\psi_1/\psi_2 = 5$ and $\phi = 1.1$ is shown in Figure 5. It indicates that there exists a pair of $u_{H1,opt,\Pi}$ near zero and $u_{H2,opt,\Pi}$ near 0.5, which lead to the optimal dimensionless profit rate. In this case, Dual heat pump cycle becomes Diesel heat pump cycle. The three-dimensional diagram among the dimensionless

profit rate ($\overline{\Pi}$) and heat conductance distributions (u_{L1} and u_{L2}) of an irreversible variable-temperature heat reservoir Miller heat pump cycle with $\beta = 3$, $\psi_1/\psi_2 = 5$ and $\phi = 1.1$ is shown in Figure 6. It indicates that there exists a pair of $u_{L1,opt,\Pi}$ near 0.5 and $u_{L2,opt,\Pi}$ near zero, which lead to the optimal dimensionless profit rate. In this case, Miller heat pump cycle becomes Atkinson heat pump cycle. Figure 7 show the optimal heat conductance distribution ($u_{H2,opt,\Pi}$) versus COP (β) for Brayton, Otto, Diesel and Atkinson heat pump cycles. It indicates that $u_{H2,opt,\Pi}$ is a little greater than 0.5 for Brayton, Otto, Diesel and Atkinson heat pump cycles, and the COP has little effects on $u_{H2,opt,\Pi}$. Moreover, when carrying out heat conductance optimizations, $u_{H2,opt,\Pi}$ for Dual heat pump cycle and $u_{L2,opt,\Pi}$ for Miller heat pump cycle are close to the corresponding optimal heat conductance distributions of Diesel and Atkinson heat pump cycles as shown in Figures 5 and 6, respectively.



Figure 3. Effect of ψ_1/ψ_2 on $\overline{\Pi}$ vs. β characteristic for irreversible variable-temperature heat reservoir Brayton heat pump cycle



Figure 4. $\overline{\Pi}$ vs. β and u_{H2} for irreversible variable-temperature heat reservoir Brayton heat pump cycle



Figure 5. $\overline{\Pi}$ vs. u_{H_1} and u_{H_2} for irreversible variable-temperature heat reservoir Dual heat pump cycle



Figure 6. $\overline{\Pi}$ vs. u_{L1} and u_{L2} for irreversible variable-temperature heat reservoir Miller heat pump cycle



Figure 7. $u_{H2opt,\Pi}$ vs. β for irreversible variable-temperature heat reservoir Brayton, Otto, Diesel and Atkinson heat pump cycles

5.2 Optimal finite time exergoeconomic performance

Figure 8 shows the optimal dimensionless profit rate $(\overline{\Pi}_{opt,u})$ versus COP (β) characteristic for irreversible variable-temperature heat reservoir Brayton, Otto, Diesel, Atkinson, Dual and Miller heat pump cycles with $\psi_1/\psi_2 = 5$ and $\phi = 1.1$. It indicates that $\overline{\Pi}_{opt,u}$ decreases with the increase in β . Dual and Diesel, Miller and Atkinson heat pump cycles have the same optimal dimensionless profit rate versus COP characteristics, respectively. For the fixed β , Brayton heat pump cycle has the maximum $\overline{\Pi}_{opt,u}$ among the six heat pump cycles, and Otto heat pump cycle has the minimum.



Figure 8. $\overline{\Pi}_{opt, u}$ vs. β characteristics for six heat pump cycles

Figures 9-11 show the effect of internal irreversibility (ϕ), total heat exchanger inventory (U_T) and thermal capacity rate of the working fluid (C_{wf4}) on the optimal dimensionless profit rate ($\overline{\Pi}_{opt,u}$) versus COP (β) characteristics of an irreversible variable-temperature heat reservoir Brayton heat pump cycle with $\psi_1/\psi_2 = 5$ and $C_H = C_L = 1.2 \, kW/K$, respectively. From Figure 9, one can see that for the fixed COP, $\overline{\Pi}_{opt,u}$ decreases with the increase in ϕ . Moreover, when $\phi = 1$, $\overline{\Pi}_{opt,u}$ versus β characteristic of an irreversible Brayton heat pump cycle with variable-temperature heat reservoirs becomes that of an endoreversible one. From Figure 10, one can see that $\overline{\Pi}_{opt,u}$ increases with the increase gradually. From Figure 11, one can see that $\overline{\Pi}_{opt,u}$ increases with the increase of C_{wf4} is lower than C_H and C_L ; $\overline{\Pi}_{opt,u}$ decreases with the increase of C_{wf4} when C_{wf4} is greater than C_H and C_L . Moreover, the effects of internal irreversibility, total heat exchanger inventory and thermal capacity rate of the working fluid on the optimal finite time exergoeconomic performances of Otto, Diesel, Atkinson, Dual and Miller heat pump cycles are similar with those shown in Figures 9-11.



Figure 9. Effect of ϕ on $\overline{\Pi}_{opt,u}$ vs. β characteristic for irreversible variable-temperature heat reservoir Brayton heat pump cycle



Figure 10. Effect of U_T on $\overline{\Pi}_{opt,u}$ vs. β characteristic for irreversible variable-temperature heat reservoir Brayton heat pump cycle



Figure 11. Effect of C_{wf4} on $\overline{\Pi}_{opt,u}$ vs. β characteristic for irreversible variable-temperature heat reservoir Brayton heat pump cycle

5.3 Optimal thermal capacity rate matching between the working fluid and heat reservoirs

Figure 12 shows a three-dimensional diagram among the dimensionless profit rate ($\overline{\Pi}$), thermal capacity rate matching ($c = C_{wf4}/C_L$) between the working fluid and heat reservoirs and heat conductance distribution (u_{H2}) of an irreversible variable-temperature heat reservoir Brayton heat pump cycle with $\beta = 3$, $\psi_1/\psi_2 = 5$ and $\phi = 1.1$. From Figure 12, one can see that the curve of $\overline{\Pi}$ versus *c* is a parabolic-like one for the fixed u_{H2} . There exist an optimal thermal capacity rate matching (c_{opt}) between the working fluid and heat reservoirs and an optimal heat conductance distribution ($u_{H2,opt}$) which lead to the double maximum dimensionless profit rate.

Figures 13-15 show the effect of internal irreversibility (ϕ), total heat exchanger inventory (U_T) and heat capacity ratio of the heat reservoirs (C_H / C_L) on the optimal dimensionless profit rate ($\overline{\Pi}_{opt,u}$) versus thermal capacity rate matching (c) between the working fluid and heat reservoirs characteristics of an irreversible variable-temperature heat reservoir Brayton heat pump cycle with $\psi_1/\psi_2 = 5$, respectively. From Figure 13, for the fixed c, $\overline{\Pi}_{opt,u}$ decreases with the increase in ϕ . Moreover, when $\phi = 1$, $\overline{\Pi}_{opt,u}$ versus c characteristic of an irreversible Brayton heat pump cycle with variable-temperature heat reservoirs becomes that of an endoreversible one. From Figure 14, one can see that $\overline{\Pi}_{opt,u}$ increases with the increase in U_T for the fixed c, but the increment decreases gradually. From Figure 15, one can see that when $C_H / C_L = 1$, the optimal thermal capacity rate matching between the working fluid and heat reservoirs is $c_{opt} = 1$, which leads to the double maximum dimensionless profit rate. Meanwhile, c_{opt} increases with the increase in C_H / C_L . Moreover, the effects of internal irreversibility, total heat exchanger inventory and heat capacity ratio of the heat reservoirs on the optimal finite time exergoeconomic performances of Otto, Diesel, Atkinson, Dual and Miller heat pump cycles are similar with those shown in Figures 13-15.



Figure 12. $\overline{\Pi}$ vs. c and $u_{\mu\nu}$ for irreversible variable-temperature heat reservoir Brayton heat pump cycle



Figure 13. Effect of ϕ on $\overline{\Pi}_{opt,u}$ vs. *c* characteristic for irreversible variable-temperature heat reservoir Brayton heat pump cycle



Figure 14. Effect of U_T on $\overline{\Pi}_{opt,u}$ vs. *c* characteristic for irreversible variable-temperature heat reservoir Brayton heat pump cycle



Figure 15. Effect of C_H / C_L on $\overline{\Pi}_{opt,u}$ vs. *c* characteristic for irreversible variable-temperature heat reservoir Brayton heat pump cycle

6. Conclusion

Finite time exergoeconomic performance of an irreversible universal steady flow heat pump cycle model with variable-temperature heat reservoirs, and the losses of heat transfer and internal irreversibility is analyzed and optimized by using the theory of finite time thermodynamics. Expressions for COP and profit rate are derived and are used to discuss the optimal finite time exergoeconomic performance of the universal heat pump cycle. Numerical examples show that the optimal hot-side heat conductance distributions are a little greater than 0.5 for Brayton, Otto, Diesel and Atkinson heat pump cycles; optimal performances of Dual and Miller heat pump cycles are close to those of Diesel and Atkinson heat pump cycles, respectively. There exist an optimal heat conductance distribution and an optimal thermal capacity rate matching between the working fluid and heat reservoirs which lead to the double maximum profit rate. Moreover, the effects of internal irreversibility, total heat exchanger inventory, thermal capacity rate of the working fluid and heat capacity ratio of the heat reservoirs on the optimal finite time exergoeconomic performance and optimal thermal capacity rate matching between the working fluid and heat reservoirs are discussed. The results obtained herein include the optimal finite time exergoeconomic performance of endoreversible and irreversible, constant- and variable- temperature heat reservoir Brayton, Otto, Diesel, Atkinson, Dual, Miller and Carnot heat pump cycles, and can provide some theoretical guidelines for parameter designs and performance optimizations of various practical heat pumps.

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