Smooth-operating point and theoretical analyses of unsmooth operation in pyrological processes

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Abstract
To emphasize the role of heat loss in energy conservation, this paper introduces smooth operation principle. By least square method, the smooth-operating point (SOP) formula of fuel gas with definite composition and temperature is deduced, which is then used to analyze changes in heat effect when injecting heat at unsmooth-operating point (UOP), educe allocation regulations of heat effect in furnace, and study the heat loss resulting from unsmooth operation. It indicates that for fuel gas with definite composition and temperature, the SOP is unique and there is no heat loss if injecting heat at SOP. With regard to unsmooth operation, low UOP is better than a high one.

Keywords: Furnace, Heat allocation, Heat loss, Smooth operation, Unsmooth operation.

1. Introduction
Analysis of energy utilization has attracted increasing attention during recent decades, especially since the first "oil crisis". And the correspondingly evaluating index, thermal efficiency, has already been studied by many researchers [1-3]. Lu [3] represented that it is reasonable for thermal efficiency to evaluate energy conversion processes but unreasonable to evaluate technological pyrological processes. Even for pyrological processes, it is just a black-box model which cannot clearly represent the contribution of each heat input or output item. However, thermal value theory combining energy conservation law and heat transfer theory solves this confusion [4]. And it has been extended for evaluating the energy effective utilization degree in pyrological processes and/or equipments [5-7].
To note that main feedstock and product in energy conversion processes are both energetic materials; e.g., feedstock is coal and product is electric in thermal power generation process. But neither material nor product is energy in pyrological processes, such as metallurgical production process in which feedstock is ore and product is metal and energy is just necessary for pushing the smelting process. In this work, heating process occurred in rolling furnaces is selected as an example of pyrological processes.
Thermal value theory, however, is still in terms of positive balance evaluation. With the advance of energy conservation, the counter balance method focused on heat losses has increasingly shown its importance [8,9]. Smooth operation principle, as will be shown later, is a method highly correlated with

thermal value analysis for evaluating energy effective utilization degree from the negative standpoint and next, its application in the pyrological process of furnaces will be represented.

2. Smooth operation principle

For the counter-flow heat transfer furnace modeled by Cai [6], smooth operation is an energy utilization condition at which steel obtains the most available heat or loses the least available heat from heat effect or by changing the motion of steel and/or original furnace gas (OG) and the resulting transferred heat attached on them. The "heat effect" here refers to chemical reaction [4].

It is smooth operation that the OG and steel which share the same heat transfer process move upon the order of heating and cooling, respectively. If OG at section A reverses its position with OG at section B, the heat attached on OG adjacent its inlet will be transferred to another position adjacent its outlet; if steel at section A reverses its position with steel at section B, the heat attached on steel adjacent its outlet will be transferred to another position adjacent its inlet. Both the two back-to-front movements of heat are unsmooth operation. According to thermal value equation [5], the thermal value of transferred heat is to be reduced whether the unsmooth operation is from OG or from steel.

When injecting fuel gas (FG) to a furnace, injection point reversing its position with adjacent OG by multi-step manner makes the injection point migration. If and only if injected FG has the same temperature with the originally smooth-operating OG at the injection section, the furnace is of smooth operation; and such injection point is smooth-operating point (SOP). It is unsmooth operation when injection point moves to the upstream or downstream of SOP. Moreover, injection point farther from SOP shows more character of unsmooth operation.

3. Determination of smooth-operating point (SOP)

Because it has different composition with OG, the injected FG reacts with and transfers heat to OG, even steel, after being injected into a furnace, which causes change in temperatures of FG as well as steel and OG at the injection section. And the resulting gas, which is called mixed gas (MG), derives from the reaction and mixing of injected FG with OG. To be in smooth operation, the temperatures of steel and OG after injecting FG must be the same as those before, i.e., the enthalpy change (H_{in}) caused by injected FG must be zero.

Assume that the chemical reaction attributed to injecting FG is completed at the injection section instantaneously. If the injected FG's composition is as listed in Table 1, the FG will alter its self-temperature and participate in the chemical reaction after being injected into the furnace, and the calculation formula of total heat effect (H_{in}) is

$$H_{in} = H_{CO} + H_{CO_2} + H_{CH_4} + H_{C_2H_6} + H_{C_3H_8} + H_{H_2} + H_{N_2} + H_{O_2} + H_{H_2O} \tag{1}$$

where H_{CO}, H_{CO_2}, etc. are the heat effects (J) caused by injecting corresponding substances contained in FG such as CO and CO_2, respectively.

Table 1. Wet ingredients of injected FG

item	CO	CO_2	CH_4	C_2H_6	C_3H_8	H_2	N_2	O_2	H_2O
percentage [%]	21.68	8.22	7.51	0.28	0.07	17.45	43.55	0.14	1.10

(1) H_{CO}

Assuming CO in the injected FG is completely burned, then

$$H_{CO} = n_{CO}H_1 \tag{2}$$

where n_{CO} is the amount of substance of CO (mol), and H_1 is the heat of reaction of $CO + 1/2 O_2 = CO_2$ considering change in temperature (J/mol).

The thermo-chemical equation of CO reacting with O_2 to produce CO_2 at the temperature of resulting MG is

$$\mathrm{CO}\big(T_{g,in}\big)+\frac{1}{2}\mathrm{O}_2\big(T_{g,in}\big)=\mathrm{CO}_2\big(T_{g,m}\big); \quad H_1 \tag{3}$$

where $T_{g,in}$ is the temperature of FG (K), and $T_{g,m}$ is the temperature of MG (K).
According to Hess law, H_1 is decomposed as

$$H_1 = -\big(H_{1-1}+H_{1-2}\big) \tag{4}$$

$$\mathrm{CO}\big(T_{g,in}\big)+\frac{1}{2}\mathrm{O}_2\big(T_{g,in}\big)=\mathrm{CO}_2\big(T_{g,in}\big); \quad H_{1-1} \tag{5}$$

$$\mathrm{CO}_2\big(T_{g,in}\big)\to \mathrm{CO}_2\big(T_{g,m}\big); \quad H_{1-2} \tag{6}$$

According to Kirchhoff's law, H_{1-1} is calculated as

$$H_{1-1} = \Delta H_{1-1,298,\mathrm{CO}} + \int_{298}^{T_{g,in}} \Delta c_{p,\mathrm{CO}}\,dT \tag{7}$$

$$\Delta c_{p,\mathrm{CO}} = c_{p,\mathrm{CO}_2} -\left(c_{p,\mathrm{CO}} +\frac{1}{2}c_{p,\mathrm{O}_2}\right) \tag{8}$$

where c_p is the heat capacity of each substance (J/(K·mol)), $\Delta c_{p,\mathrm{CO}}$ is the reaction's heat capacity difference (J/(K·mol)), and $\Delta H_{1-1,298,\mathrm{CO}}$ is the standard heat of formation of this reaction (J/mol).
In addition,

$$H_{1-2} = \int_{T_{g,in}}^{T_{g,m}} c_{p,\mathrm{CO}_2}\,dT \tag{9}$$

Combining Eqs. (2), (4), (7) and (9), H_{CO} can be expressed as

$$H_{\mathrm{CO}} = -n_{\mathrm{CO}}\left(\Delta H_{1-1,298,\mathrm{CO}} + \int_{298}^{T_{g,in}} \Delta c_{p,\mathrm{CO}}\,dT + \int_{T_{g,in}}^{T_{g,m}} c_{p,\mathrm{CO}_2}\,dT\right) \tag{10}$$

(2) H_{CH_4}, $H_{\mathrm{C}_2\mathrm{H}_6}$, $H_{\mathrm{C}_3\mathrm{H}_8}$ and H_{H_2}
Similarly, the expressions of H_{CH_4}, $H_{\mathrm{C}_2\mathrm{H}_6}$, $H_{\mathrm{C}_3\mathrm{H}_8}$ and H_{H_2} are

$$H_{\mathrm{CH}_4} = -n_{\mathrm{CH}_4}\left[\Delta H_{1-1,298,\mathrm{CH}_4} + \int_{298}^{T_{g,in}} \Delta c_{p,\mathrm{CH}_4}\,dT + \int_{T_{g,in}}^{T_{g,m}}\big(c_{p,\mathrm{CO}_2}+2c_{p,\mathrm{H}_2\mathrm{O}}\big)\,dT\right] \tag{11}$$

$$H_{\mathrm{C}_2\mathrm{H}_6} = -n_{\mathrm{C}_2\mathrm{H}_6}\left[\Delta H_{1-1,298,\mathrm{C}_2\mathrm{H}_6} + \int_{298}^{T_{g,in}} \Delta c_{p,\mathrm{C}_2\mathrm{H}_6}\,dT + \int_{T_{g,in}}^{T_{g,m}}\big(2c_{p,\mathrm{CO}_2}+3c_{p,\mathrm{H}_2\mathrm{O}}\big)\,dT\right] \tag{12}$$

$$H_{\mathrm{C}_3\mathrm{H}_8} = -n_{\mathrm{C}_3\mathrm{H}_8}\left[\Delta H_{1-1,298,\mathrm{C}_3\mathrm{H}_8} + \int_{298}^{T_{g,in}} \Delta c_{p,\mathrm{C}_3\mathrm{H}_8}\,dT + \int_{T_{g,in}}^{T_{g,m}}\big(3c_{p,\mathrm{CO}_2}+4c_{p,\mathrm{H}_2\mathrm{O}}\big)\,dT\right] \tag{13}$$

$$H_{\mathrm{H}_2} = -n_{\mathrm{H}_2}\left(\Delta H_{1-1,298,\mathrm{H}_2} + \int_{298}^{T_{g,in}} \Delta c_{p,\mathrm{H}_2}\,dT + \int_{T_{g,in}}^{T_{g,m}} c_{p,\mathrm{H}_2\mathrm{O}}\,dT\right) \tag{14}$$

(3) H_{CO_2}

CO_2 in the FG cannot react with any other substance after being injected into the furnace. When calculating its heat effect, only that caused by temperature change is considered, i.e.,

$$H_{CO_2} = -n_{CO_2} \int_{T_{g,in}}^{T_{g,m}} c_{p,CO_2} \, dT \tag{15}$$

(4) H_{N_2}, H_{O_2} and H_{H_2O}

Like CO_2, N_2 and H_2O in the FG also react with no substance after being injected into the furnace; while O_2 in the FG has the same function with that in OG besides the heat effect caused by temperature change. By the same method as calculating H_{CO_2}, H_{N_2}, H_{O_2} and H_{H_2O} can be calculated by

$$H_{N_2} = -n_{N_2} \int_{T_{g,in}}^{T_{g,m}} c_{p,N_2} \, dT \tag{16}$$

$$H_{O_2} = -n_{O_2} \int_{T_{g,in}}^{T_{g,m}} c_{p,O_2} \, dT \tag{17}$$

$$H_{H_2O} = -n_{H_2O} \int_{T_{g,in}}^{T_{g,m}} c_{p,H_2O} \, dT \tag{18}$$

Assuming X is any a substance (CO, for example) in FG,

$$n_X = \frac{V_X}{V_m} = \frac{1}{V_m} \cdot \frac{V_X}{V} \cdot V = \frac{1}{V_m} \cdot X' \cdot V \tag{19}$$

where V_X is the volume of X (m^3), V is the volume of FG (m^3) and hence, X' is the percent of X by volume (vol.%), and V_m is molar volume (m^3/mol) with the value of 0.0224.

Substituting Eqs. (10) to (18) into Eq. (1) and arranging them using Eq. (19), the format of heat effect per volume of FG signed as h_{in} (kJ/m^3) can be achieved. Set $h_{in} = 0$, then the temperature of SOP ($T_{g,m}$) can be solved.

Thermodynamic data used in the following calculation are cited from Borgnakke [10]. By substituting data in Table 1 into Eqs. (10) to (19), it can be arrived at

$$h_{in} = 5.41 \times 10^{-9} T_{g,in}^4 + 3.51 \times 10^{-6} T_{g,in}^3 + 0.24 T_{g,in}^2 + 1771.81 T_{g,in} + 1.22 \times 10^6 T_{g,in}^{-1} - \\ 0.19 T_{g,m}^2 - 1761.19 T_{g,m} - 1.41 \times 10^7 T_{g,m}^{-1} + 7591.60 \tag{20}$$

Making h_{in} equal to zero and arranging, the relationship of $T_{g,m}$ and $T_{g,in}$ can be written as

$$T_{g,m}^3 + 9629 T_{g,m}^2 - \left(2.85 \times 10^{-8} T_{g,in}^4 + 1.85 \times 10^{-5} T_{g,in}^3 + 1.26 T_{g,in}^2 + 9325 T_{g,in} + 6.42 \times 10^6 T_{g,in}^{-1} + \\ 39956 \right) T_{g,m} + 7.42 \times 10^7 = 0. \tag{21}$$

If $T_{g,in}$ is given, $T_{g,m}$ can be worked out by roots formula about three power equation; the results are given in terms of Celsius scale as listed in Table 2. In the range of calculated temperatures, $t_{g,m}$ shows a linear relationship with $t_{g,in}$. Define $t_{g,in} = x$ and $t_{g,m} = y$, then the linear equation $t_{g,m} = a + b t_{g,in}$ is rewritten as $y = a + bx$ (a, b are undetermined coefficients); the results based upon least square method are listed in Table 3.

Table 2. $t_{g,m}$ corresponding to different values of $t_{g,in}$

$t_{g,in}$ [°C]	400	500	600	700	800	900	1000	1100
$t_{g,m}$ [°C]	411	517	623	730	837	945	1053	1162

Table 3. Calculation table of least square method

i	x_i [°C]	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	y_i [°C]	$y_i - \bar{y}$	$(y_i - \bar{y})^2$	$(x_i - \bar{x})(y_i - \bar{y})$
1	400	-350	122500	411	-373.75	139689.0625	130812.5
2	500	-250	62500	517	-267.75	71690.0625	66937.5
3	600	-150	22500	623	-161.75	26163.0625	24262.5
4	700	-50	2500	730	-54.75	2997.5625	2737.5
5	800	50	2500	837	52.25	2730.0625	2612.5
6	900	150	22500	945	160.25	25680.0625	24037.5
7	1000	250	62500	1053	268.25	71958.0625	67062.5
8	1100	350	122500	1162	377.25	142317.5625	132037.5
Σ	6000	NA	420000	6278	NA	483225.5	450500

where NA is not available, $\bar{x} = 750$, $\bar{y} = 784.75$

Based on Table 3, the correlation coefficient (r) of variables x and y is

$$r = \frac{\sum_{i=1}^{8}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{8}(x_i - \bar{x})^2 \sum_{i=1}^{8}(y_i - \bar{y})^2}} = 0.999989 \tag{22}$$

And the undetermined coefficients $b = \sum_{i=1}^{8}(x_i - \bar{x})(y_i - \bar{y}) \Big/ \sum_{i=1}^{8}(x_i - \bar{x})^2 = 1.0726$, $a = \bar{y} - b\bar{x} = -19.7143$.

So the linear regression equation is

$$t_{g,m} = 1.0726 t_{g,in} - 19.7143 \quad (r = 0.999989) \tag{23}$$

Equation (23) is the formula for determining SOP and is plotted in Figure 1.

4. Analyses of unsmooth operation
4.1 Change in heat effect
If FG is injected into at unsmooth-operating point (UOP) with the composition referred to above, the heat effect will change depending upon different temperatures of UOP, for details see Figure 2 originated from Eq. (20).

It can be found from Figure 2 that plot of h_{in} vs. $T_{g,m}$ presents a near-linear relationship. The linear regression equations are

$$T_{g,in} = 673\text{K}: \quad h_{in} = -2.1839 T_{g,m} + 1521.4526 \quad (r = -0.99969) \tag{24}$$

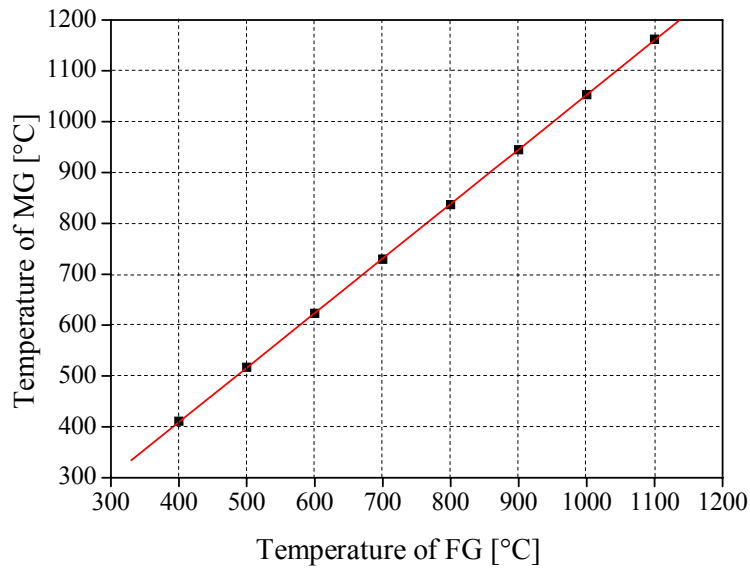$$T_{g,in} = 773\text{K}: \quad h_{in} = -2.1839 T_{g,m} + 1734.2468 \quad (r = -0.99969) \tag{25}$$

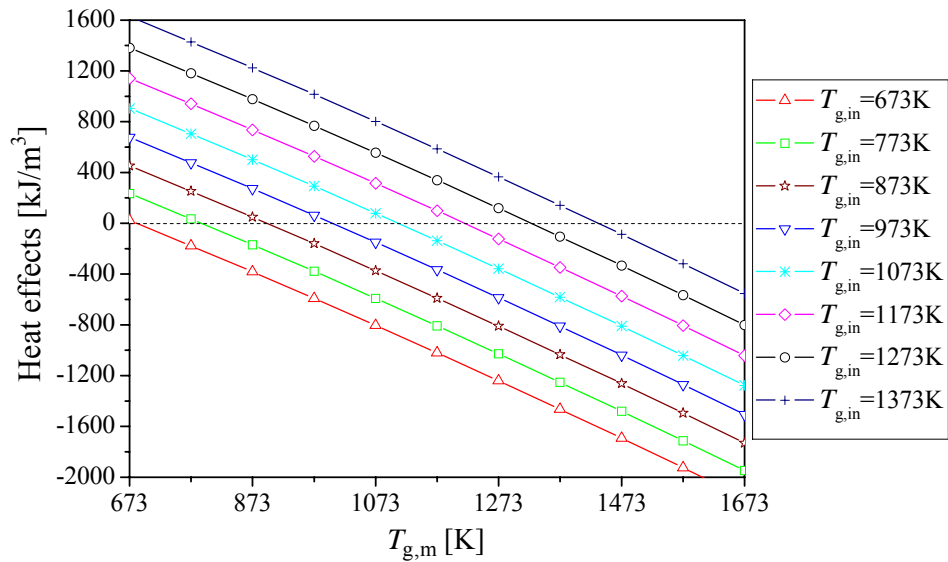Figure 1. Relationship between temperatures of MG (SOP) and FG

Figure 2. Heat effects (h_{in}) corresponding to different values of $T_{g,in}$ and $T_{g,m}$

$$T_{g,in} = 873K: \quad h_{in} = -2.1839 T_{g,m} + 1952.4149 \quad (r = -0.99969) \tag{26}$$

$$T_{g,in} = 973K: \quad h_{in} = -2.1839 T_{g,m} + 2176.0683 \quad (r = -0.99969) \tag{27}$$

$$T_{g,in} = 1073K: \quad h_{in} = -2.1839 T_{g,m} + 2405.3376 \quad (r = -0.99969) \tag{28}$$

$$T_{g,in} = 1173K: \quad h_{in} = -2.1839 T_{g,m} + 2640.3399 \quad (r = -0.99969) \tag{29}$$

$$T_{g,in} = 1273K: \quad h_{in} = -2.1839 T_{g,m} + 2881.3276 \quad (r = -0.99969) \tag{30}$$

$$T_{g,in} = 1373K: \quad h_{in} = -2.1839 T_{g,m} + 3128.3872 \quad (r = -0.99969) \tag{31}$$

Regressing the constant term in Eqs. (24) to (31),

$$Const = -2.2949T_{g,in} - 42.7152 \quad (r = -0.99969) \tag{32}$$

Then heat effect can be expressed in Celsius scale uniformly as

$$h_{in} = -2.1839t_{g,m} + 2.2949t_{g,in} - 12.3956 \tag{33}$$

To note that $t_{g,m}$ used above is the temperature of MG which is the mixture after the heat transfer process and chemical reaction of FG but not the temperature of OG. At a certain FG's temperature of $t_{g,in}$, to get the change in heat effect when injection occurs at where the temperature of OG is $t_{g,0}$, the relationship between $t_{g,m}$ and $t_{g,0}$ should be found out, which will be studied later.

4.2 Allocation of heat effect
Heat effect caused by injecting FG is partially absorbed (or released, depending upon the sign of H_{in}) by OG, expressed as $H_{in,g}$ and partially absorbed (or released) by steel, expressed as $H_{in,s}$; that is,

$$H_{in} = H_{in,g} + H_{in,s} \tag{34}$$

According to heat transfer theory,

$$W_g\left(t_{g,m} - t_{g,0}\right) = H_{in,g} \tag{35}$$

$$W_s\left(t_{s,m} - t_{s,0}\right) = H_{in,s} \tag{36}$$

where $t_{s,m}$ is steel's temperature after heat transfer (°C), $t_{s,0}$ is steel's temperature before injecting FG (°C), and W_g, W_s are water equivalents of OG and steel (kJ/(h·°C)), respectively.
Assume that

$$t_{s,m}/t_{g,m} = \kappa \tag{37}$$

$$t_{s,0}/t_{g,0} = \kappa \tag{38}$$

Transforming and arranging Eqs. (35) to (38), it can be achieved

$$t_{g,0} = t_{g,m} - \frac{H_{in}}{W_g + \kappa W_s} \tag{39}$$

That is, to reach $t_{g,m}$, the injection point of which temperature is $t_{g,0}$ can be calculated by using Eq. (39) when heat effect caused by injected FG is H_{in}. Because W_g and W_s are both positive, it is obvious that $t_{g,0} < t_{g,m}$ when $H_{in} > 0$ (i.e., temperature of MG will increase after mixing of FG and OG) and $t_{g,0} > t_{g,m}$ when $H_{in} < 0$ (i.e., temperature of MG will decrease after mixing of FG and OG), while $t_{g,0} = t_{g,m}$ when FG is injected at SOP where $H_{in} = 0$.
Combining Eqs. (35) and (36),

$$\frac{W_g \left(t_{g,m} - t_{g,0} \right)}{W_s \left(t_{s,m} - t_{s,0} \right)} = \frac{H_{in,g}}{H_{in,s}} \tag{40}$$

into which κ is substituted, it can be got

$$\frac{W_g}{\kappa W_s} = \frac{H_{in,g}}{H_{in,s}} \tag{41}$$

Combining Eqs. (34) and (41), the heat allocation regulation is obeyed upon

$$H_{in,g} = \frac{H_{in}}{1 + \frac{\kappa W_s}{W_g}} \tag{42}$$

$$H_{in,s} = \frac{H_{in}}{1 + \frac{W_g}{\kappa W_s}} \tag{43}$$

4.3 Heat loss of unsmooth operation

When injecting FG at SOP, there is no heat loss because of zero H_{in}. Where the temperature is lower than that at SOP is called low UOP. If FG is injected at low UOP, $H_{in} > 0$ and the temperature of MG is higher than that of OG; i.e., injected FG carries heat of H_{in} more than OG does. The heat is attached on both OG and steel. $H_{in,s}$, the heat attached on steel, is absorbed by steel and contributes to target heat [7] while the other part, $H_{in,g}$, is attached on MG. And part of the heat attached on MG transfers to steel in MG's moving process, which is expressed as $H_{in,g\text{-}s}$; while other part, $H_{in,g\text{-}g}$, becomes heat loss with the off-gas. The heat loss caused by injecting FG at low UOP is expressed as $H_{loss,1}$. The heat contained in discharging steel is from MG by heat transfer, with the exception of that contained in charging steel, then

$$\frac{H_{in,g\text{-}s}}{H_s} = \frac{H_{in,g}}{H_g} \tag{44}$$

where H_s and H_g are heat obtained by discharging steel and contained in MG (kJ/h), respectively, and

$$H_s = W_s t_{s,m} - W_s' t_s' \tag{45}$$

$$H_g = W_{g,m} t_{g,m} \tag{46}$$

where t_s' is temperature of charging steel (°C), W_s' and $W_{g,m}$ are water equivalents of charging steel and MG (kJ/(h·°C)), respectively.

According to the analyses above,

$$H_{loss,1} = H_{in,g} - H_{in,g\text{-}s} \tag{47}$$

Substituting κ into Eqs. (45) to (47) lead to

$$H_{\text{loss,l}} = H_{\text{in}} \left(\dfrac{\dfrac{W_{\text{g,m}}}{W_{\text{s}}} - \kappa + \dfrac{W_{\text{s}}' t_{\text{s}}'}{W_{\text{s}} t_{\text{g,m}}}}{\dfrac{W_{\text{g,m}}}{W_{\text{s}}} + \kappa} \right) \tag{48}$$

Similarly, if FG is injected at high UOP where the temperature is higher than that at SOP, $H_{\text{in}} < 0$ and the temperature of MG is lower than that of OG; i.e., to reach $t_{\text{g,m}}$, injected FG should absorb heat of H_{in} transferred from both OG and steel. $H_{\text{in,s}}$ from steel transfers to MG and then, part of the heat transfers to steel in MG's moving process, which is expressed as $H_{\text{in,s-s}}$; while other part, $H_{\text{in,s-g}}$, contributes to heat loss with the off-gas. The heat loss caused by injecting FG at high UOP is expressed as $H_{\text{loss,h}}$. Because steel transfers heat of $H_{\text{in,s}}$ to MG, the discharging temperature of steel decreases, of course. To ensure that steel's discharging temperature meets the target temperature, heat compensation is of necessity at UOP or other point. To note that heat compensation is also transferred from MG to steel and there is existing heat loss in the heat transfer process, so heat compensation must be greater than $H_{\text{in,s}}$. Assuming the heat loss without heat compensation is $H_{\text{loss,h0}}$, then it can be got according to the analysis above that

$$H_{\text{loss,h0}} = H_{\text{in,s}} - H_{\text{in,s-s}} \tag{49}$$

$$\frac{H_{\text{in,s-s}}}{H_{\text{s}}} = \frac{H_{\text{in,s}}}{H_{\text{g}}} \tag{50}$$

By the same method as calculating $H_{\text{loss,l}}$, it can be arrived at

$$H_{\text{loss,h0}} = H_{\text{in}} \left[\dfrac{\dfrac{W_{\text{g,m}}}{W_{\text{s}}} - \kappa + \dfrac{W_{\text{s}}' t_{\text{s}}'}{W_{\text{s}} t_{\text{g,m}}}}{\dfrac{W_{\text{g,m}}}{W_{\text{s}}} \left(1 + \dfrac{W_{\text{g,m}}}{W_{\text{s}} \kappa} \right)} \right] \tag{51}$$

Part of heat compensation (H_{com}) releases with the off-gas. In fact, only heat of $H_{\text{loss,h0}}$ is compensated to steel; that is,

$$\frac{H_{\text{loss,h0}}}{H_{\text{s,com}}} = \frac{H_{\text{com}}}{H_{\text{g,com}}} \tag{52}$$

where $H_{\text{s,com}}$ and $H_{\text{g,com}}$ are heats attached on steel and MG, respectively, at the injecting section of heat compensation (kJ/h) with

$$H_{\text{g,com}} = W_{\text{g,com}} t_{\text{g,com}} \tag{53}$$

$$H_{\text{s,com}} = W_{\text{s,com}} t_{\text{s,com}} - W_{\text{s}}' t_{\text{s}}' \tag{54}$$

where $W_{\text{s,com}}$ and $W_{\text{g,com}}$ are water equivalents of steel and MG at the injecting section of heat compensation (kJ/(h·°C)), respectively; $t_{\text{s,com}}$ and $t_{\text{g,com}}$ are temperatures of steel and MG at the injecting section of heat compensation (°C), respectively.

Combining Eqs. (52) to (54),

$$H_{\text{com}} = H_{\text{loss,h0}} \frac{W_{g,\text{com}} t_{g,\text{com}}}{W_{s,\text{com}} t_{s,\text{com}} - W'_s t'_s} \tag{55}$$

And substituting Eq. (51) into Eq. (55), then

$$H_{\text{loss,h}} = H_{\text{com}} = H_{\text{in}} \frac{\dfrac{W_{g,m}}{W_s} - \kappa + \dfrac{W'_s t'_s}{W_s t_{g,m}}}{\dfrac{W_{g,m}}{W_s}\left(1 + \dfrac{W_{g,m}}{W_s \kappa}\right)} \cdot \frac{W_{g,\text{com}} t_{g,\text{com}}}{W_{s,\text{com}} t_{s,\text{com}} - W'_s t'_s} \tag{56}$$

4.4 Comments on unsmooth operation
Heat transfer in furnaces is always from OG to steel, thus,

$$H_g > H_s \tag{57}$$

Rewrite Eq. (57) in form of water equivalent as

$$W_g t_g > W_s t_s \tag{58}$$

i.e.,

$$W_g / W_s > t_s / t_g = \kappa \tag{59}$$

It can be found from Eq. (56) that denominator diminishes when UOP is closer to furnace head due to the increasing W_s. $\left(W'_s t'_s / W_s t_{g,m}\right)$ in numerator is determined by heat of charging steel and heat of steel at the injection section and its value is small, especially for cold charging. If the difference of $\left(W_{g,m} / W_s\right)$ and κ keeps constant, $H_{\text{loss,h}}$ will increase with the increasing W_s. This represents that, for high UOP, the more adjacent to furnace head, the larger heat loss. And when injecting FG at high UOP, the higher $t_{g,m}$, the larger absolute value of H_{in}, and thus the more heat loss.

It is shown from Eq. (48) that when UOP is closer to furnace end the difference of $\left(W_g / W_s\right)$ and κ increases due to the increasing $\left(W_g / W_s\right)$ and decreasing κ. One term in denominator increases and the other decreases and thus, the sum of them changes little. So, $H_{\text{loss,l}}$ increases. That is, for low UOP, the more adjacent to point with low temperature, the more heat loss.

Comparing Eqs. (48) with (56), it can be seen that numerator in Eq. (48) is the same as that in Eq. (56) when whose factor greater than 1, $\left(W_{g,\text{com}} t_{g,\text{com}} / \left(W_{s,\text{com}} t_{s,\text{com}} - W'_s t'\right)\right)$, is negligible. As for high UOP and low UOP causing the same H_{in}, $H_{\text{loss,l}}$ is less than $H_{\text{loss,h}}$ because $\left(W_{g,m} / W_s\right)$ in Eq. (48) is greater than that in Eq. (56). In other words, injecting FG at low UOP is relatively better than at high UOP though they are both unsmooth operation. Moreover, injecting at low UOP necessitates no heat compensation because of zero reduction in steel's discharging temperature.

5. Conclusion
This work is a further study on thermal value theory which aims at analyzing each heat input and/or output item's influence on energy effective utilization degree for pyrological processes. Different from conventional analysis methods, this paper focuses on reducing heat loss, instead of increasing available heat, using smooth operation principle. Based on least square method, FG's SOP formula is proposed and unsmooth operation is analyzed. The results show that SOP at which no heat loss is generated when

injecting FG is unique. As for unsmooth operation, low UOP is better than high UOP due to the zero heat compensation.

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