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Calculation of elastic deformation under the influence of high velocity impact on composite plate structures

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Abstract
Composite materials are widely used in many civil and military applications because of their excellent mechanical properties. In military applications, composite materials are used to protect vehicles and aircraft and to manufacture shields for their ability to absorb and disperse impact energy. In this paper, used high velocity impact with a bullet (it mass of $8\ g$, diameter $9\ mm$ and a semi-circular projectile head) with a specific velocity ranging from $210 - 365\ m/s$, to investigate the elastic deformation for composite plate structure with various composite laminated parameters effect. Where, studied the effect of the impact by using theoretically technique, in addition to, calculating the natural frequency for plates by using analytical and numerical techniques, and then, comparison the results obtained. In the theoretical part, problems were analysed into several categories, the first is the calculation of the elastic deformation in the composite materials samples which depends on several parameters such as the calculation of the natural frequency, the time and force contact, and elastic deformation. An equation for the calculation of elastic deformation was derived based on the classical theory of plates. Also, the natural frequency results calculated analytically by using classical theory of plates with drive the general equation of motion. Then, by using numerical technique calculating also the natural frequency of plates, and then, comparison the results obtained with analytical results obtained. When, the numerical technique include using for finite element technique to calculate the vibration behaviour for plate. Then, the comparison for results given a good agreement for analytical technique used with maximum error did not exceed about (1.23%).

Keywords: Elastic deformation; High velocity impact; Composite materials, Classical laminate plate theory; Contact force; Contact time.

1. Introduction
Composite materials are defined as, a substance consisting of two or more components, insoluble with each other, which are combined to form a useful new material that possesses properties that components do not, [1]. These materials consist of reinforcing materials that are either continuous or discontinuous embedded in a substance called a matrix, so that the matrix is reinforced with reinforcement materials. This work takes into account the improvement in the properties of the matrix when adding embedded reinforcing fibers, reinforcing materials whether they are long continuous in one direction or in more directions, or be short chopped or be particles. These materials offer more benefits than conventional materials such as aluminum, steel and other types and include high tensile stress, low density, corrosion

resistance ... etc. By changing the fiber arrangement, several designs can be designed according to the application requirements, [2]. The design of the compound includes composite materials of various structures. Conversely, for conventional materials whose properties are constant when manufacturing, these properties can be changed significantly with the freedom of control by the designer, [3]. Among the most preferment is the laminated fiber composite, which some of them passes in the higher specific moduli, such as Kevlar, glass and carbon fibers. The specific strength and stiffness are great importance than conventional materials such as steel and aluminium. This makes it attract for many applications that require light weight and high resistance to impact and corrosion applications, [4].
A theoretical solution is provided for the effect of the impact force at high velocity on the target of a composite material made of fiber woven in the matrix. Therefore, the analysis of the material properties of the plates is used in a composite material for woven fiber. The laminate layer consists of a number of layers of several materials arranged in a particular order and at certain angles to form a plate of compound material to give it the desired purpose to withstand impact and absorb energy, classical theory was based on the theoretical solution of this paper for elastic deformation. In addition the numerical solution used to calculate the natural frequency of plate structure, and then, comparison the results with analytical results obtained.

2. Analytical investigation

In most thin plates applications, the effect of stresses that affect the on the middle level is small. The plate is thin it means that the traction on any parallel surface is relatively small and as a rough application for its plane stress, x, y, z coordinate system standard, as shown in Figure 1, is used in deriving the equations, the displacements are denoted u, v, w, respectively. The following basic assumptions are made, [5],

1. The plate is made of a number of layers of panels linked together, and not necessarily of the same type, and,
2. The plate is thin and less thickness than other dimensions.
3. ε_x, ε_y and ε_{xy} are small in plane strain, and ε_{xz} and ε_{yz} are negligible.
4. The effects of force under nonlinear term conditions are negligible.
5. u and v are linear functions of the z direction.
6. Hook's law applies to each plate.
7. Constant thickness for plate.
8. Rotatory inertia parameters are negligible, no body forces.

The geometry of the layer laminate consists of N layers and along the Z coordinate as in Figure 2, h is the thickness of plate from the first plate of thickness j_1 and ending with the plate j_j. This plate is a thin plate and each plate is homogeneous, [6].

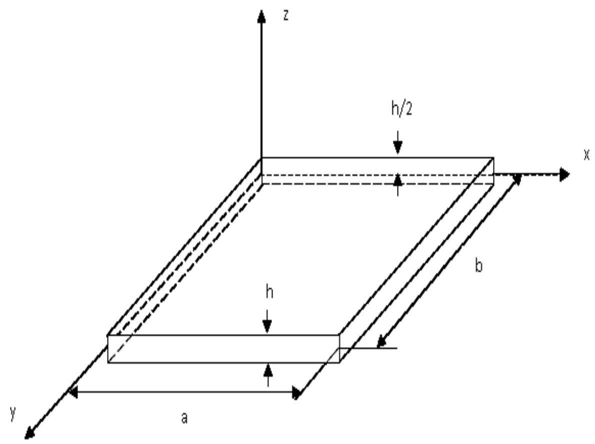

Figure 1. Coordinate System of Plate, [1].

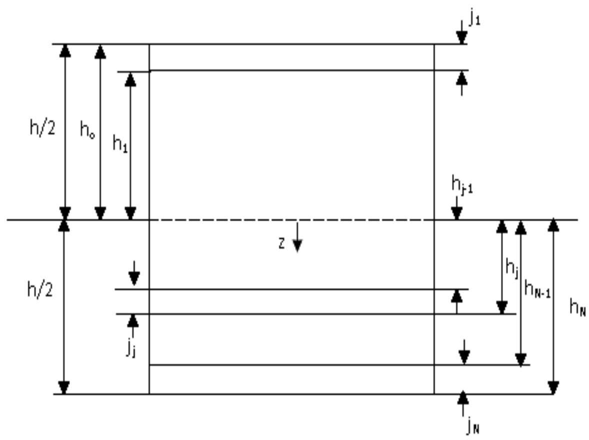

Figure 2. Laminate Geometry, [6].

2.1 General equation of motion by using classical laminated plate theory

The simplest method for laminated theory is the "Classical Laminated Plate Theory (CLPT)", [7], in according to thin plate theory with in-plane forces. The displacement field is assumed to be,

$$u_1(x,y,t) = u(x,y,t) - z\frac{\partial w}{\partial x}$$
$$u_2(x,y,t) = v(x,y,t) - z\frac{\partial w}{\partial y}$$
$$u_3(x,y,t) = w(x,y,t) \tag{1}$$

Where u and v are the tangential displacement of the laminate. The strains are therefore given by,

$$\varepsilon_x = \frac{\partial u_1}{\partial x} = \frac{\partial u}{\partial x} - \frac{\partial^2 w}{\partial x^2}$$
$$\varepsilon_y = \frac{\partial u_2}{\partial y} = \frac{\partial v}{\partial y} - \frac{\partial^2 w}{\partial y^2}$$
$$\gamma_{xy} = \frac{\partial u_1}{\partial y} + \frac{\partial u_2}{\partial x} = \frac{\partial u}{\partial y} - z\frac{\partial^2 w}{\partial xy} + \frac{\partial v}{\partial x} - z\frac{\partial^2 w}{\partial xy}$$
$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} - 2z\,\frac{\partial^2 w}{\partial xy} \tag{2}$$

The corresponding stiffness matrix is obtained by inverting the compliance matrix given, [6],

$$\begin{bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{bmatrix} = \begin{bmatrix}\overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16}\\ \overline{Q}_{21} & \overline{Q}_{22} & \overline{Q}_{26}\\ \overline{Q}_{61} & \overline{Q}_{62} & \overline{Q}_{66}\end{bmatrix}\begin{bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{bmatrix} \tag{3}$$

Then, by substitution the results of Eq. (2) in Eq. (3), get,

$$\begin{bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{bmatrix} = \begin{bmatrix}\overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16}\\ \overline{Q}_{21} & \overline{Q}_{22} & \overline{Q}_{26}\\ \overline{Q}_{61} & \overline{Q}_{62} & \overline{Q}_{66}\end{bmatrix}\left[\begin{bmatrix}\frac{\partial u}{\partial x}\\ \frac{\partial v}{\partial y}\\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\end{bmatrix} - z\begin{bmatrix}\frac{\partial^2 w}{\partial x^2}\\ \frac{\partial^2 w}{\partial y^2}\\ 2\frac{\partial^2 w}{\partial x\,\partial y}\end{bmatrix}\right] \tag{4}$$

Or,

$$\begin{bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{bmatrix} = \begin{bmatrix}\overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16}\\ \overline{Q}_{21} & \overline{Q}_{22} & \overline{Q}_{26}\\ \overline{Q}_{61} & \overline{Q}_{62} & \overline{Q}_{66}\end{bmatrix}\begin{bmatrix}\frac{\partial u}{\partial x}\\ \frac{\partial v}{\partial y}\\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\end{bmatrix} - z\begin{bmatrix}\overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16}\\ \overline{Q}_{21} & \overline{Q}_{22} & \overline{Q}_{26}\\ \overline{Q}_{61} & \overline{Q}_{62} & \overline{Q}_{66}\end{bmatrix}\begin{bmatrix}\frac{\partial^2 w}{\partial x^2}\\ \frac{\partial^2 w}{\partial y^2}\\ 2\frac{\partial^2 w}{\partial x\,\partial y}\end{bmatrix} \tag{5}$$

The axial stress resultants are defined as, [6],

$$N_x = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_x dz = \sum_{j=1}^{n} \int_{h_{j-1}}^{h_j} \sigma_x^j dz$$
$$N_y = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_y dz = \sum_{j=1}^{n} \int_{h_{j-1}}^{h_j} \sigma_y^j dz$$
$$N_{xy} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \tau_{xy} dz = \sum_{j=1}^{n} \int_{h_{j-1}}^{h_j} \tau_{xy}^j dz \tag{6}$$

Where, N_x, N_y, N_{xy}, M_x, M_y and M_{xy} are a forces and moments per unit length respectively of the cross section of the laminate as shown in Figures 3 and 4.
Then, substitution the results of Eq. (5) in Eq. (6), get,

$$\begin{bmatrix}N_x\\ N_y\\ N_{xy}\end{bmatrix} = \sum_{j=1}^{n}\begin{bmatrix}\overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16}\\ \overline{Q}_{21} & \overline{Q}_{22} & \overline{Q}_{26}\\ \overline{Q}_{61} & \overline{Q}_{62} & \overline{Q}_{66}\end{bmatrix}\left[\int_{h_{j-1}}^{h_j}\begin{bmatrix}\frac{\partial u}{\partial x}\\ \frac{\partial v}{\partial y}\\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\end{bmatrix} dz - z\int_{-\frac{h}{2}}^{\frac{h}{2}}\begin{bmatrix}\frac{\partial^2 w}{\partial x^2}\\ \frac{\partial^2 w}{\partial y^2}\\ 2\frac{\partial^2 w}{\partial x\,\partial y}\end{bmatrix} dz\right] \tag{7}$$

Therefore, integrate the Eq. (7) to get,

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \sum_{j=1}^{n} \left[\begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{21} & A_{22} & A_{26} \\ A_{61} & A_{62} & A_{66} \end{bmatrix} \left(h_j - h_{j-1}\right) - \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{21} & B_{22} & B_{26} \\ B_{61} & B_{62} & B_{66} \end{bmatrix} (h_j^2 - h_{j-1}^2) \right] \tag{8}$$

Where,

$$A_{mn} = \sum_{j=1}^{n} [\overline{Q}_{mn}](h_j - h_{j-1})$$
$$B_{mn} = \frac{1}{2} \sum_{j=1}^{n} [\overline{Q}_{mn}](h_j^2 - h_{j-1}^2) \tag{9}$$

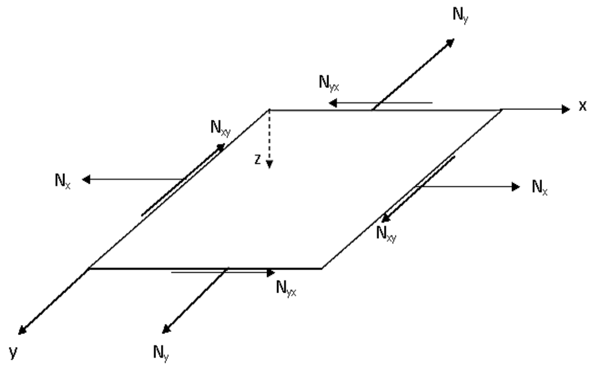

Figure 3. In Plane Forces on a Laminate, [7].

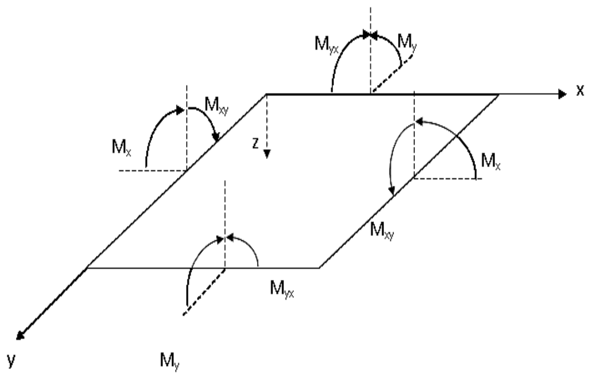

Figure 4. Moments on a Laminate, [7].

Also, bending and twisting moment resultants are defined as [6],

$$M_x = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_x z dz = \sum_{j=1}^{n} \int_{h_{j-1}}^{h_j} \sigma_x^j z dz$$
$$M_y = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_y z dz = \sum_{j=1}^{n} \int_{h_{j-1}}^{h_j} \sigma_y^j z dz$$
$$M_{xy} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \tau_{xy} z dz = \sum_{j=1}^{n} \int_{h_{j-1}}^{h_j} \tau_{xy}^j z dz \tag{10}$$

Or,

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} z\, dz = \sum_{j=1}^{n} \int_{h_{j-1}}^{h_j} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} z\, dz \tag{11}$$

Then, substitution the results of Eq. (5) in Eq. (10), get,

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \sum_{j=1}^{n} \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{21} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{61} & \overline{Q}_{62} & \overline{Q}_{66} \end{bmatrix} \left[\int_{h_{j-1}}^{h_j} \begin{bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \end{bmatrix} z.\, dz - \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{bmatrix} \frac{\partial^2 w}{\partial x^2} \\ \frac{\partial^2 w}{\partial y^2} \\ 2\frac{\partial^2 w}{\partial x \partial y} \end{bmatrix} z^2.\, dz \right] \tag{12}$$

Integrate the Eq. (11) to get,

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \sum_{j=1}^{n} \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{21} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{61} & \overline{Q}_{62} & \overline{Q}_{66} \end{bmatrix} \left[\begin{bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \end{bmatrix} \frac{1}{2}\left(h_j^2 - h_{j-1}^2\right) - \frac{1}{2} \begin{bmatrix} \frac{\partial^2 w}{\partial x^2} \\ \frac{\partial^2 w}{\partial y^2} \\ 2\frac{\partial^2 w}{\partial x \partial y} \end{bmatrix} (h_j^3 - h_{j-1}^3) \right] \tag{13}$$

So, can be know the matrices A, B as follows, $[A]=\begin{bmatrix}A_{11} & A_{12} & A_{16}\\ A_{21} & A_{22} & A_{26}\\ A_{61} & A_{62} & A_{66}\end{bmatrix}$, $[B]=\begin{bmatrix}B_{11} & B_{12} & B_{16}\\ B_{21} & B_{22} & B_{26}\\ B_{61} & B_{62} & B_{66}\end{bmatrix}$

In addition, can be defined D matrix as, $[D]=\begin{bmatrix}D_{11} & D_{12} & D_{16}\\ D_{21} & D_{22} & D_{26}\\ D_{61} & D_{62} & D_{66}\end{bmatrix}$

Where,

$$D_{mn}=\frac{1}{3}\sum_{j=1}^{n}[\overline{Q}_{mn}](h_j^3-h_{j-1}^3) \tag{14}$$

By using Eqs. (7) to (9) and substituting Eqs. (12) to (14) gets, [8-10],

$$\begin{bmatrix}N_x\\ N_y\\ N_{xy}\\ M_x\\ M_y\\ M_{xy}\end{bmatrix}=\begin{bmatrix}A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16}\\ A_{21} & A_{22} & A_{26} & B_{21} & B_{22} & B_{26}\\ A_{61} & A_{62} & A_{66} & B_{61} & B_{62} & B_{66}\\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16}\\ B_{21} & B_{22} & B_{26} & D_{21} & D_{22} & D_{26}\\ B_{61} & B_{62} & B_{66} & D_{61} & D_{62} & D_{66}\end{bmatrix}\begin{bmatrix}\frac{\partial u}{\partial x}\\ \frac{\partial v}{\partial y}\\ \frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\\ \frac{\partial^2 w}{\partial x^2}\\ \frac{\partial^2 w}{\partial y^2}\\ 2\frac{\partial^2 w}{\partial x\,\partial y}\end{bmatrix} \tag{15}$$

Then, the equation of the motion of the plate is taken from the balance of forces from Figures 3 and 4, including the effect of the plate rotating inertia according to classical laminated plate theory as, [6],

$$\begin{aligned}
&\frac{\partial N_x}{\partial x}+\frac{\partial N_{xy}}{\partial y}=I_1\frac{\partial^2 u}{\partial t^2}-I_2\frac{\partial^3 w}{\partial x\,\partial t^2}\\
&\frac{\partial N_{xy}}{\partial x}+\frac{\partial N_x}{\partial y}=I_1\frac{\partial^2 v}{\partial t^2}-I_2\frac{\partial^3 w}{\partial y\,\partial t^2}\\
&\frac{\partial^2 M_x}{\partial x^2}+2\frac{\partial^2 M_{xy}}{\partial x\,\partial y}+\frac{\partial^2 M_y}{\partial y^2}=I_1\frac{\partial^2 w}{\partial t^2}+I_2\left(\frac{\partial^3 u}{\partial x\,\partial t^2}+\frac{\partial^3 u}{\partial x\,\partial t^2}\right)-I_3\left(\frac{\partial^4 u}{\partial x^2\,\partial t^2}+\frac{\partial^4 u}{\partial y^2\,\partial t^2}\right)-q(x,y,t)
\end{aligned} \tag{16}$$

Where,

$$(I_1,I_2,I_3)=\sum_{k=1}^{n}\int_{k-1}^{k}\rho^{(k)}(1,z,z^2)dz \tag{17}$$

$\rho^{(k)}$ material density of k^{th}, and $q(x,y,t)$ dynamic force subjected on a system dynamic. The final result for the equation of motion is below, which represents the sum of the effects in z coordinate,

$$\left(\left(\begin{gathered}B_{11}\frac{\partial^3 u}{\partial x^3}+B_{12}\frac{\partial^3 v}{\partial x^2\,\partial y}+B_{16}\left(\frac{\partial^3 u}{\partial x^2\,\partial y}+\frac{\partial^3 v}{\partial x^3}\right)+D_{11}\frac{\partial^4 w}{\partial x^4}+D_{12}\frac{\partial^4 w}{\partial x^2\,\partial y^2}+D_{16}\frac{\partial^4 w}{\partial x^3\,\partial y}+\\ 2\left(B_{61}\frac{\partial^3 u}{\partial x^2\,\partial y}+B_{62}\frac{\partial^3 v}{\partial x\,\partial y^2}+B_{66}\left(\frac{\partial^3 u}{\partial x\,\partial y^2}+\frac{\partial^3 v}{\partial x^2\,\partial y}\right)+D_{61}\frac{\partial^4 w}{\partial x^3\,\partial y}+D_{62}\frac{\partial^4 w}{\partial x\,\partial y^3}+D_{66}\frac{\partial^4 w}{\partial x^2\,\partial y^2}\right)+\\ B_{21}\frac{\partial^3 u}{\partial x\,\partial y^2}+B_{22}\frac{\partial^3 v}{\partial y^3}+B_{26}\left(\frac{\partial^3 u}{\partial y^3}+\frac{\partial^3 v}{\partial x\,\partial y^2}\right)+D_{21}\frac{\partial^4 w}{\partial x^2\,\partial y^2}+D_{22}\frac{\partial^4 w}{\partial y^4}+D_{26}\frac{\partial^4 w}{\partial x\,\partial y^3}\end{gathered}\right)= \\ I_1\frac{\partial^2 w}{\partial t^2}+I_2\left(\frac{\partial^3 u}{\partial x\,\partial t^2}+\frac{\partial^3 u}{\partial x\,\partial t^2}\right)-I_3\left(\frac{\partial^4 u}{\partial x^2\,\partial t^2}+\frac{\partial^4 u}{\partial y^2\,\partial t^2}\right)-q(x,y,t)\right) \tag{18}$$

The effect of the impact force for the projectile on the plate towards the z coordinate. Therefore, use Eq. (18) to analyze the effect of the impact, and because the plate is symmetric laminates, [B] = 0, reduces the equation to get as, [11, 12],

$$\left(\begin{gathered}D_{11}\frac{\partial^4 w}{\partial x^4}+D_{12}\frac{\partial^4 w}{\partial x^2\,\partial y^2}+D_{16}\frac{\partial^4 w}{\partial x^3\,\partial y}+D_{21}\frac{\partial^4 w}{\partial x^2\,\partial y^2}+D_{22}\frac{\partial^4 w}{\partial y^4}+D_{26}\frac{\partial^4 w}{\partial x\,\partial y^3}\\ +2\left(D_{61}\frac{\partial^4 w}{\partial x^3\,\partial y}+D_{62}\frac{\partial^4 w}{\partial x\,\partial y^3}+D_{66}\frac{\partial^4 w}{\partial x^2\,\partial y^2}\right)+\end{gathered}\right)=I_1\frac{\partial^2 w}{\partial t^2}-q(x,y,t) \tag{19}$$

Then, calculating the general actual displacement for simply cross-ply laminated plate as, [13-15],

$$w(x,y,t) = \sin\alpha x.\sin\beta y.w(t) \quad (20)$$

Therefore, by substitution the Eq. (20) in Eq. (19) and multiply the two sides of the equation by $\sin\alpha x.\sin\beta y$, [16, 17], to get as,

$$\left(\left(\begin{array}{c} D_{11}\alpha^4.\sin^2\alpha x.\sin^2\beta y.w(t) + 2D_{12}\alpha^2\beta^2.\sin^2\alpha x.\sin^2\beta y.w(t) - \\ 3D_{16}\alpha^3\beta.\cos\alpha x.\cos\beta y.\sin\alpha x.\sin\beta y.w(t) - 3D_{62}\alpha\beta^3.\cos\alpha x.\cos\beta y.\sin\alpha x.\sin\beta y.w(t) \\ +D_{22}\beta^4.\sin^2\alpha x.\sin^2\beta y.w(t) + D_{66}\alpha^2\beta^2.\sin^2\alpha x.\sin^2\beta y.w(t) + kw(t) \end{array}\right) = \begin{array}{c} \\ \rho h.\sin\alpha x.\sin\beta y.\frac{\partial^2 w}{\partial t^2}(t) - \int_0^b\int_0^a q(x,y,t).\sin\alpha x.\sin\beta y.dxdy \end{array}\right) \quad (21)$$

Where, D_{ij} are the flexural rigidity coefficients of the laminated plate, w(t) is the deflection along the z direction and q(x,y,t) is the intensity of transverse distributed load per unit area acting on the thin plate, [18]. Then, for cross ply laminates, the material directions are oriented at 0 or 90 degrees, so have, [6], $(D_{16})_0 = (D_{16})_{90}$, $(D_{26})_0 = (D_{26})_{90}$
Then,

$$\begin{pmatrix} (D_{11}\alpha^4 + D_{22}\beta^4) + \\ 2\alpha^2\beta^2(D_{12} + 2D_{66}) \end{pmatrix} w(t) + kw(t) + \rho h\frac{\partial^2 w}{\partial t^2} = -\frac{4}{ab}\int_0^b\int_0^a q(x,y).\sin\alpha x.\sin\beta y.dxdy.Q(t) \quad (22)$$

Therefore, the equation of motion is, [19],

$$Kw(t) + M\ddot{w}(t) = q(t) \quad (23)$$

Where, the response to the strength of the impact affecting the plate becomes, [20-22],

$$w(t) = \frac{4}{abM\omega_n}\int_0^t q(\tau).\sin\omega_n(t-\tau)d\tau \quad (24)$$

And natural frequency for the plate is,

$$\omega_n = \sqrt{\frac{[(D_{11}\alpha^4 + D_{22}\beta^4) + 2\alpha^2\beta^2(D_{12} + 2D_{66})]}{\sum_{j=1}^{n}\rho h}} \quad (25)$$

And the $q(\tau)$ is the forcing function with time can represent it as a sine pulse, [23],

$$q(\tau) = q_o \sin\left(\frac{\pi\tau}{t_1}\right) \qquad 0 \le t \le t_1$$
$$q(\tau) = 0 \qquad t > t_1 \quad (26)$$

2.2 The projectile and the target
The investigation of the effect of the deferent parameters of the projectile mass of m and the velocity of v against a flexible for sample by a stiffness spring k. The kinetic energy for the bullet is,

$$K.E = \frac{1}{2}mv^2 \quad (27)$$

As it is clearly dependent on the mass and velocity of the bullet, this energy is absorbed by deflections of the sample, internal damage and local deformations in the contact zone. Express the contact time and the value of the contact force by using Eq. (26). It is possible to calculate contact time in case of impact of relationship, [24],

$$T_c = 3.2145\left(\frac{m^2}{v.k_c{}^2}\right)^{\frac{1}{5}} \quad (28)$$

And the contact force is,

$$q_o = \left(\frac{5}{4}\right)^{\frac{3}{5}}.(m)^{\frac{3}{5}}.(v)^{\frac{6}{5}}.(k_c)^{\frac{2}{5}} \quad (29)$$

Where, m is mass of the bullet, v is velocity of the bullet, and, k_c contact stiffness defining as,

$$k_c = \frac{4}{3} E^* R^{\frac{1}{2}} \quad (30)$$

For, R is the equivalent radius, can be calculating from,

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \quad (31)$$

And E^* is the effective modulus calculating as,

$$\frac{1}{E^*} = \frac{(1-\nu_1^2)}{E_1} + \frac{(1-\nu_2^2)}{E_2} \quad (32)$$

Subscripts 1 and 2 refer to the radius, elastic moduli and Poisson ratios of the projectile and the target.

3. Numerical technique

The numerical technique included used finite element method to evaluate the natural frequency of composite laminated plate structure with different number of layers and reinforcement fiber effect. There, by using Ansys program can be calculating the natural frequency for plates. Then, firstly must be selecting the element types required used to calculate the dynamic behavior of plate, [25, 26], then, selecting the best element number required to given the best results for natural frequency, by using mesh generation technique, [27, 28], finally, after applied boundary condition for structure can be calculating the parameters required by solving the problem with Ansys program, [29, 30]. Therefore, the element type can be used to calculate the natural frequency of laminate plate was (SHELL91 16-Layer), which may be used for layered applications of a structural shell model or for modeling thick sandwich structures. Up to 16 different layers are permitted for applications with the sandwich option turned off. Another element, allowing more layers, but no nonlinear materials. The element has six degrees of freedom at each node: translations in the nodal x,y, and z directions and rotations about the nodal x,y, and z axes. The eight-node quadratic quadrilateral element is basically a higher order version of two-dimensional four-node quadrilateral element. This types of element is better suited for modeling problems with curved boundaries, [7]. Where, the geometry, node locations, and the coordinate system for this element are shown in Figure 5. The element is defined by eight nodes, layer thicknesses, layer material direction angles, and orthotropic material properties. Mid side nodes may not be removed (with a zero node number) from this element, and the impartment assuming for its element are, [10],

1. Normals to the center-plane are assumed to remain straight after deformation, but not necessarily normal to the center-plane.
2. Each pair of integration points (in the direction) is assumed to have the same element (material) orientation.
3. There is no significant stiffness associated with rotation about the element r axis. A nominal value of stiffness is assigned using the approach of Zienkiewicz, however, to prevent free rotation at the node.
4. This element dose not generate a consistent mass matrix; only the lumped mass matrix is available.

Where, the parameters investigation for effect on the natural frequency were the number of layers for laminated plate and the fiber reinforcement of simply supported plate structure. Then, after calculated the natural frequency of plate by numerical technique, comparison the results obtained with analytical results calculated by drive of general equation of motion, to given the agreement for techniques used.

4. Results and discussion

The results of the theoretical model and numerical technique are listed and discussed in this paper. Elastic deformation values were calculated for each type of samples subject to high-velocity impact, and the natural frequency for various plate sample were calculated analytically and numerically. Note that the samples used are reinforcing materials such as Kevlar and carbon fibers and glass with a matrix of polyester and a volumetric friction of 30% and samples of Kevlar and glass by 40% and a hybrid sample of glass and carbon fibers. The impact test was carried out on samples of composite materials at a limited velocity about from 210 m/s to 365 m/s, the mass of the projectile head (8 g) and the type of the semi-circular projectile head. Mechanical properties were extracted such as, modulus of elasticity, modulus of rigidity, Poisson's ratio, ultimate strength and strain to determine the ranges and limits that the sample can withstand the impact and applied the values of these properties on the equations as in the Tables 1and 2. In addition, the number of layer used to manufacture the laminated plate samples were presenting in Table 3, and, the

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dimensions for plate manufactured are, plate length $l = 20\ cm$, plate width $w = 20\ cm$, and plate thickness used shown in Table 3 for each samples used.

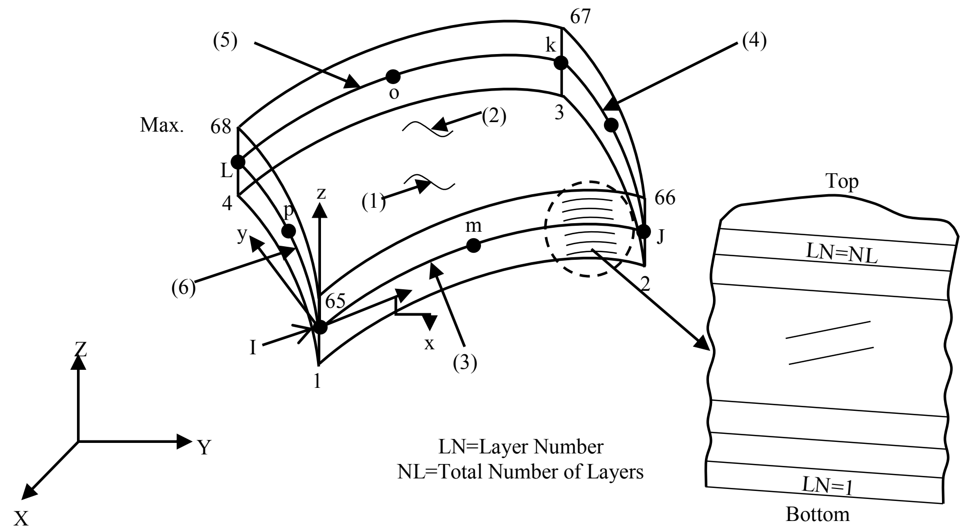


Figure 5. SHELL91 16-layer element type.

Table 1. Mechanical Properties for Composite Materials (30-70%).

Samples	P (kg/m³)	υ	Tension				Compressive	
			E (Gpa)	σ (Mpa)	ε	G (Gpa)	σ (Mpa)	ε
K-P (30-70)%	1200	0.4	9.44	286	0.0305	3.4	43	0.038
C-P (30-70)%	1320	0.385	12.77	238	0.0231	4.61	58	0.043
G-P (30-70)%	1500	0.355	8.8	249	0.0283	3.24	66	0.048

Table 2. Mechanical Properties for Composite Materials (40-60)%.

Samples	P (kg/m³)	υ	Tension				Compressive	
			E (Gpa)	σ (Mpa)	ε	G (Gpa)	σ (Mpa)	ε
K-P (40-60)%	1230	0.4	9.75	341	0.035	3.48	32	0.035
C-P (40-60)%	1390	0.38	13.2	243	0.0236	4.78	42	0.037
G-P (40-60)%	1620	0.34	9.13	256	0.0285	3.4	60	0.044

Table 3. Number of Layers and Thickness for Composite Laminated Plate.

Composite	Number of Layers	Plate Thickness (cm)
K-P (30-70)%	35	11
G-P (30-70)%	30	11.2
C-P (30-70)%	40	10.3
K-P (40-60)%	28	10
G-P (40-60)%	26	11.5
Hybrid	10-C(40-60%)+20-G(30-70%)+10-C(40-60%)	11.5

4.1 Natural frequency results

The results for natural frequency for different laminated plate, shown in Table 3, calculated by using analytical investigation, by using general equation of motion, and numerical technique, by using finite element technique, and then, comparison the results obtained. Then, as shown in Table 4, the natural frequency for laminated increase with increasing the mechanical properties for laminated. Also, as shown in Figure 6, can be shown that the comparison for analytical and numerical results given a good agreement for results with maximum error about (1.23%).

Table 4. Analytical Results for Natural Frequency.

Composite	Natural Frequency (rad/sec)
K-P (30-70)%	3936
G-P (30-70)%	3396
C-P (30-70)%	4045
K-P (40-60)%	3969
G-P (40-60)%	3490
Hybrid	4752

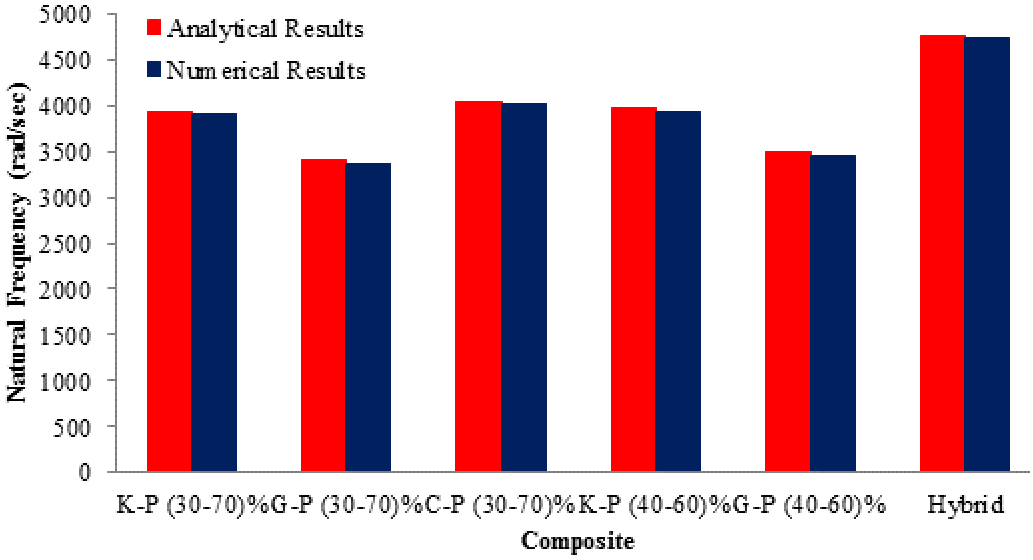
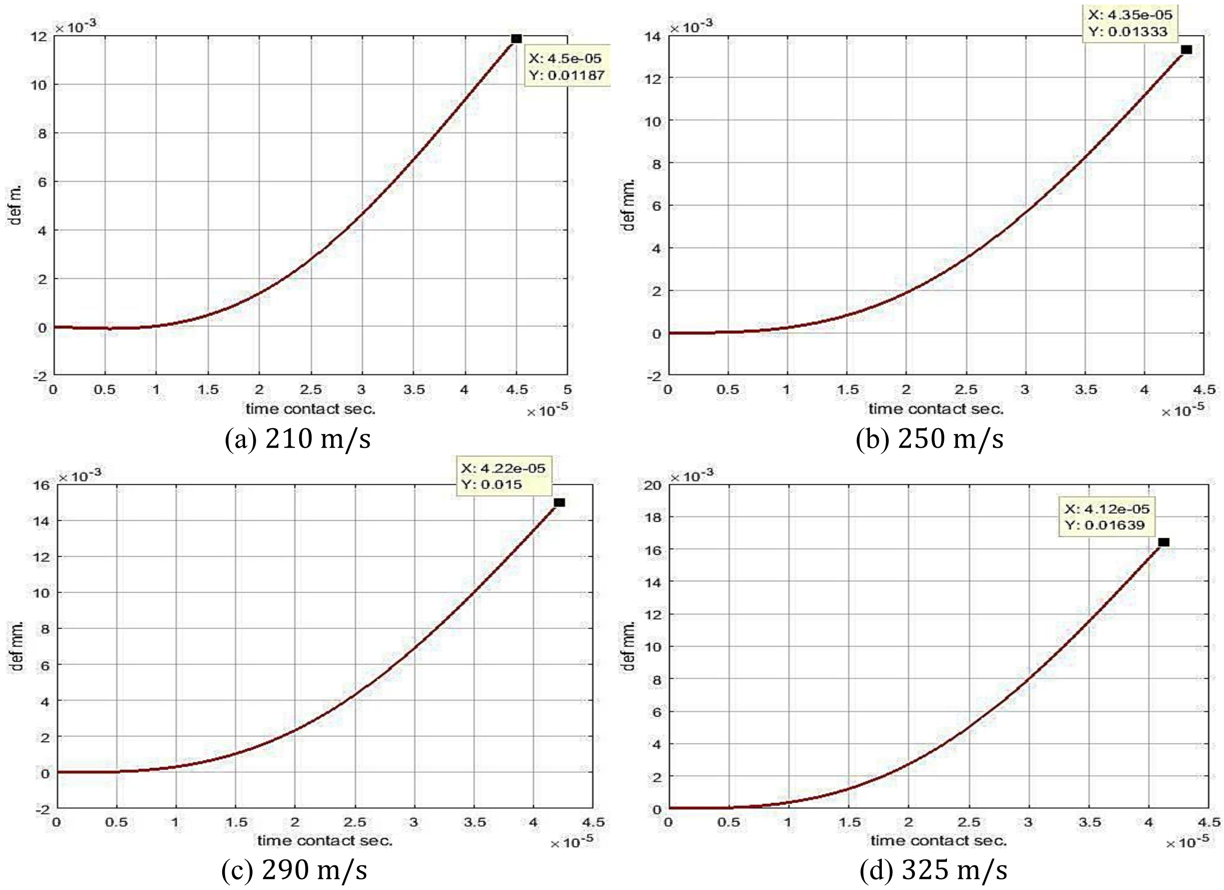


Figure 6. Comparison for Analytical and Numerical Natural Frequency for Various Composite Plate.

4.2 Elastic deformation results

To calculate the elastic deformation in the plate due to the effect of the impact chose the classical theory of the plate, in the theoretical part analyzed the sample and derived an equation that was able to calculate the elastic deformation relative to time Eq. (24), then, notice from this equation there are several variables whose value must be found are the force and time of the impact and the natural frequency of the sample, by using the Hertz's Contact Law to calculate the contact time and contact force of Eqs. (28) and (29), respectively, and the natural frequency calculation was theoretically part from the Eq. (26) with a comparison of its value in numerical technique.

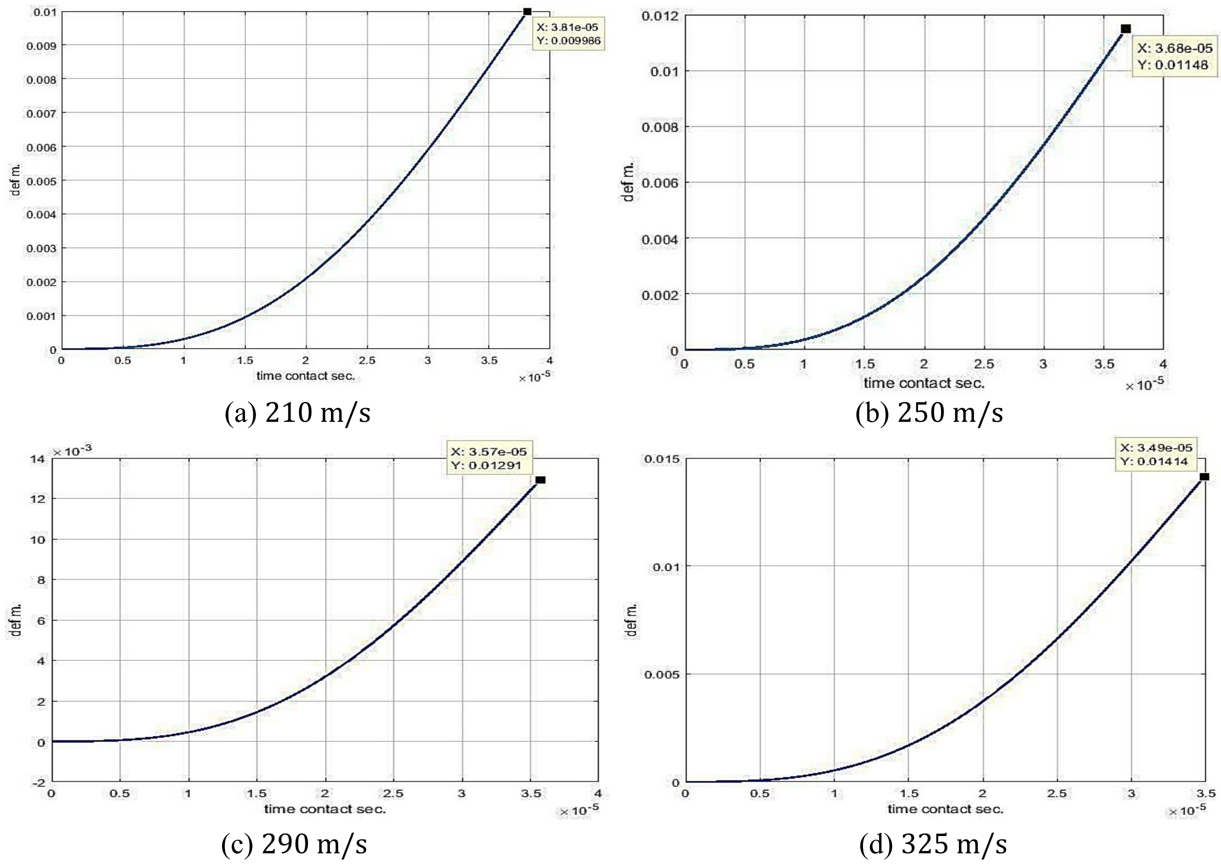
Therefore, in the Figures 7 to 12, note the variation in the amount of elastic deformation shows an increase in the value of the elastic deformation with the increase of the velocity of the projectile and decreasing in the amount of contact time, with increasing velocity the kinetic energy is increase to lead to an increase in the strain rate of reduce contact time. The increasing the projectile velocity, the contact force increases significantly as it is directly proportional to contact stiffness (kc), which is a property of the material based on the modulus of elasticity of the target and the geometry of the projectile. Then, when comparing the Figures 7.d, 8.d and 9.b, at velocity 325m/s for samples (30-70%) note that the elastic deformation of the glass-polyester is lower than carbon-polyester and finally K-P, while the contact force of the C-P was higher than the K-P and finally the G-P. Also the Figures 10 and 11 shown the deformation for Kevlar samples higher than for glass samples of (40-60%) samples, but this does not mean that the depth of penetration more. In addition from Figure 12, can be shown that the deformation for hyper composite were reduce with various velocity impact for (40-60%) samples.



(a) 210 m/s (b) 250 m/s

(c) 290 m/s (d) 325 m/s

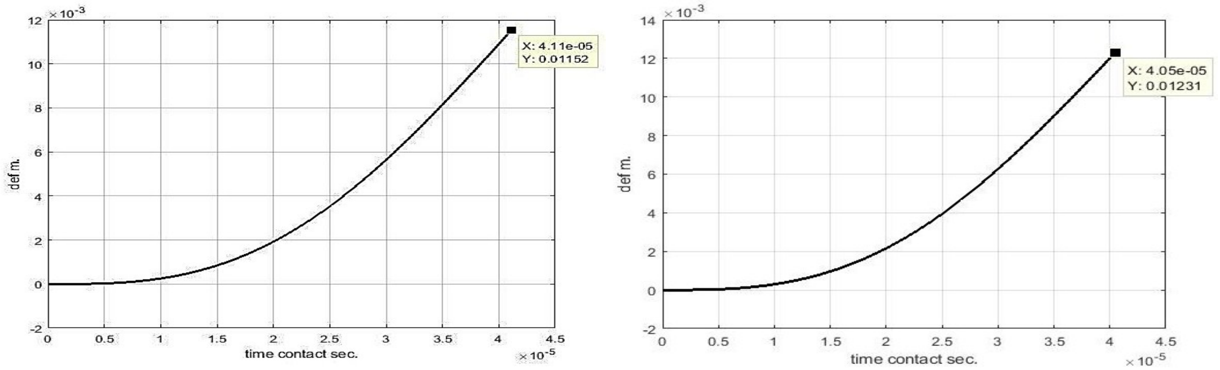
Figure 7. Deformation for Kevlar Fiber-Polyester $(30-70)\%$ at Different Velocity Impact.



(a) 210 m/s (b) 250 m/s

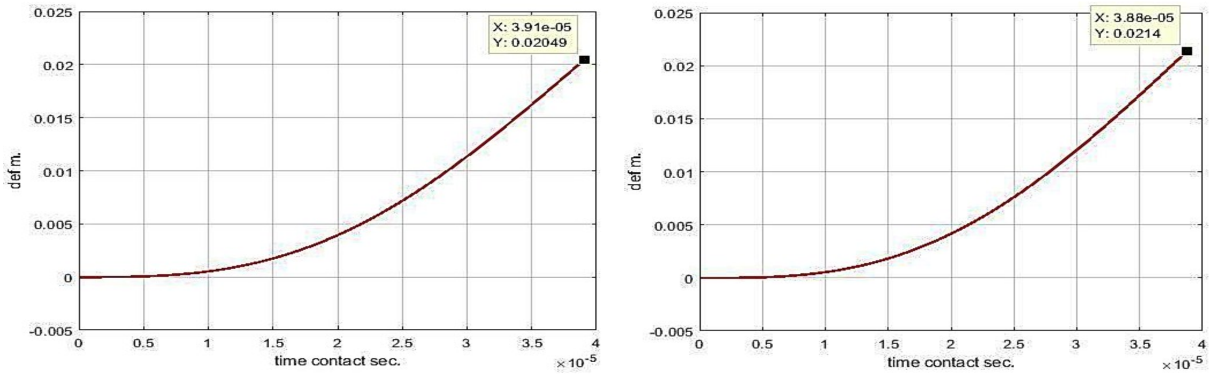
(c) 290 m/s (d) 325 m/s

Figure 8. Deformation for carbon fiber-polyester $(30-70)\%$ at different velocity impact.



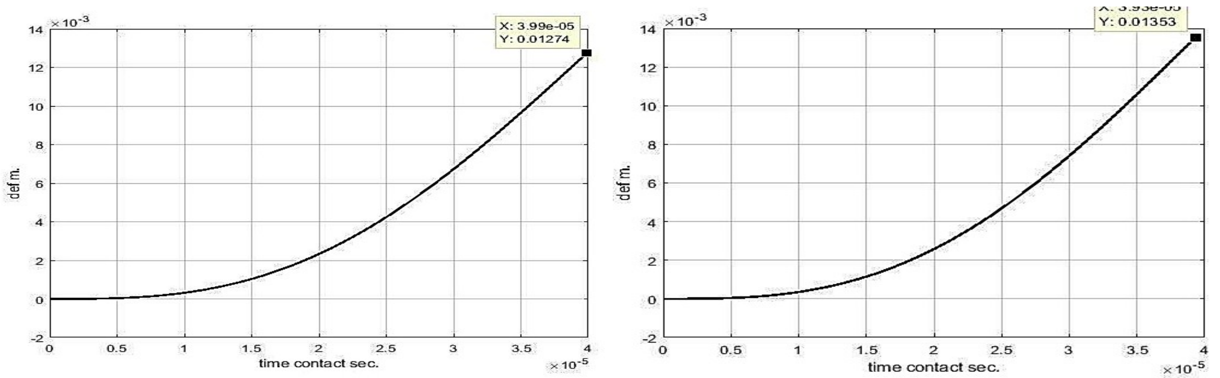
(a) 310 m/s (b) 325 m/s

Figure 9. Deformation for glass fiber-polyester $(30-70)\%$ at different velocity impact.



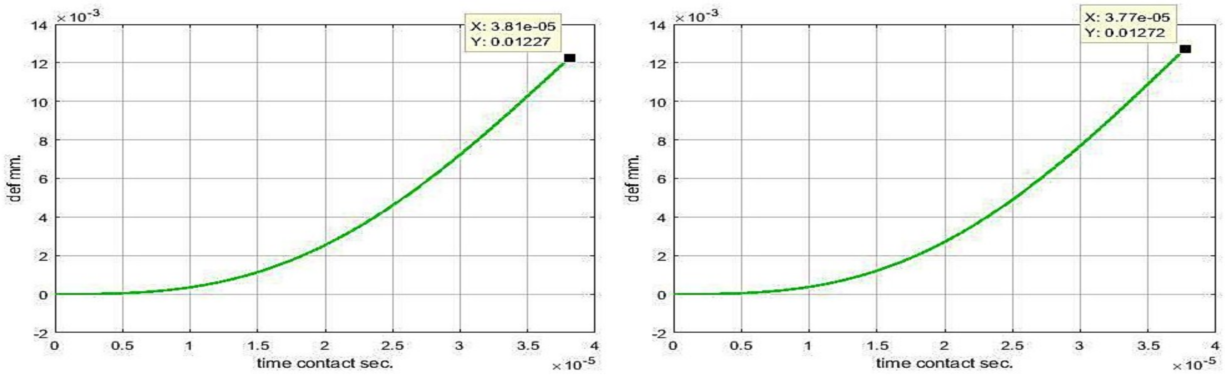
(a) 310 m/s (b) 325 m/s

Figure 10. Deformation for kevlar fiber-polyester $(40-60)\%$ at different velocity impact.



(a) 310 m/s (b) 325 m/s

Figure 11. Deformation for glass fiber-polyester $(40-60)\%$ at different velocity impact.



(a) 310 m/s (b) 325 m/s

Figure 12. Deformation for hybrid carbon-glass fiber-polyester $(40-60)\%$ at various velocity impact.

5. Conclusion

From theoretical investigation and numerical results, can be conclude the following points,

1. The analytical investigation is a perfect tool can be using to calculate the natural frequency and the elastic deformation for composite laminated plate, under high velocity impact load.
2. The comparison between analytical and numerical natural frequency results given a good agreement with maximum error did not exceed (1.23%).
3. Contact force is increased by the high modulus of elasticity for target, mass of the projectile and velocity of the projectile. Also, contact stiffness increases with a modulus of elasticity increases which leads to increased contact force. In addition, when the projectile velocity increases, the contact time decreases.
4. Elastic deformation decreases as the mass of the target increases, and, increase with increasing for velocity impact. In addition, the elastic deformation value of (K-P) is higher than that of (G-P) and finally (C-P), decrease with increase the mechanical properties for composite plate.

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