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Analytical investigation of crack depth and position effect onto beam force vibration response with various harmonic frequency influence

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Abstract
The crack defect one of important parameters lead to failure the structure, then, it was necessary investigation the crack effect of the dynamic characterization of beam structure applied to harmonic load. So, in this paper investigation the effect of crack depth and location on the beam deflection response applied to harmonic load with various effect of load frequency influence. Therefore, the investigation included derive the general equation of motion for beam applied to harmonic load with crack effect, and then, solving it equation analytically to calculate the natural frequency and beam response with harmonic load effect. In addition, used numerical technique, by using finite element method, to calculate the dynamic characterizations for beam with cack effect and comparison results by analytical results calculated to given the discrepancy for results calculated by techniques used. So, the comparison between analytical and numerical technique shown a good agreement for results calculated, with maximum error did no exceed about (1.46%). There, the results calculated shown the effect for crack depth and position for beam supported with different boundary condition, in addition to, the results shown the effect for load frequency harmonic applied on the beam response. Then, from the results presented can be conclusion that the crack defect lead to decrease the stiffness for beam, then, decreasing for natural frequency and increasing beam deflection response, in addition to, the results shown that the effect for frequency harmonic load applied increase with increase the effect for crack defect.

Keywords: Crack beam vibration; Beam response; Beam vibration; Harmonic vibration; FEM vibration.

1. Introduction
The vibrations are basic and fundamental portion of our daily lives for environment engineering today, as it occur in different engineering applications such as vehicles, aeronautics, structures, machines, electronic apparatuses, electric motors, satellites and so forth. At the point when a system or framework is vibrating under the influence of higher frequencies which lead to higher deflection, higher displacement, heat generation and noise. Hence it is necessary to examine these influences of vibrations to enhance the stability of machines and motors. It is necessary to study the response by the vibration when approaching the resonance state and its influence on the natural frequency as each material has its own internal damping

function that works against vibrations. Most of the heat generation by vibration due to the loss of vibration energy resulting from the internal damping system.

The cracked structures' dynamics have been a subject of interest and research. When a structural origin of the cracks was exposed, the materials' intensity and stiffness are reduced, and so reducing natural frequency. The dynamic behavior of cracked structures had been researched through many analytical, numerical and experimental methods and the influence of crack in moving load on machines, instruments and structures is an important problem in the field of engineering and its applications.

Many important studies discussed the influence of crack depth and position on forced vibration for beam structure. While this subject is one of the main things especially in dynamic science and for any moving part. In spite of its problem, the vibration test was the subject of various tests due to its extensive application in many mechanical fields. There are researchers who have studied crack influences with a wide range especially in vibration beam, as,

At, 1999, S. M. Cheng et. al, [1], investigated the effect for crack on the vibration characterizations of cantilever beam by using breathing model. Where, the investigation included calculate the response load displacement of fatigue crack model. So, the theoretical technique was used to evaluate the beam response and frequency with crack effect. Then, at 2005, M. Behzad et. al., [2], presented a new approach to analysis the vibration behavior for beam with open edge crack effect. Where, the Hamilton principle used to analysis the vibration according to boundary condition of beam. The analytical solution and numerical technique were used to calculated and comparison the results. At, 2007, S. Orhan, [3], investigated the effect of crack defect on the free and force vibration behavior for beam structure supported as cantilever supported. Where, the study included used analytical solution to calculate the vibration behavior for free and forced vibration beam without crack effect and modeling the crack with beam and solution by using numerical technique, with using finite element method. Thus, the results calculated included the natural frequency and deflection of beam with various forced frequency.

With, 2009, M. S. Prabhakar, [4], presented numerical solution to calculate the vibration behavior for beam structure with effect of crack defect. In addition, the investigation included used neural networks technique to calculate the vibration characterizations for beam. At, 2012, L. S. Al-Ansari et. al, [5], studied the effect of crack depth and location on the natural frequency of simply supported beam. Where, the experimental and numerical technique were used to calculate the natural frequency for beam. Thus, the experimental technique included building vibration rig to calculate the natural frequency for beam, and the numerical technique included used finite element technique to calculate the beam frequency. Also, at same year, P. K. Jena et. al, [6], presented study for damage effect on the vibration of cantilever beam. Where, the study presented analysis for multi crack effect of the natural frequency and mode shape for cantilever beam by using analytical solution with driving general equation of motion for beam with multi crack effect.

Then, at 2013, M. Al-Waily, [7], presented analytical solution to calculate the natural frequency for simple supported beam with different crack depth and location effect. Where, the investigation included derived the general equation of motion for free vibration beam with crack effect. In addition, used finite element technique to caparison the results calculated. Also, same researcher at same year, [8], presented experimental technique to calculated the natural frequency of beam with different crack position and depth effect. Thus, the experimental results evaluated comparison by analytical results calculated. In addition, same researcher at 2015, [9], studied the effect of crack orientation on the natural frequency of simply supported beam analytically. Where, the analytical solution included derived the general equation of motion of simply supported beam with various crack depth, location and orientation effect. In addition, used numerical technique, with using finite element method, to comparison the results.

At, 2017, Y. Ma et. al, [10], investigated for natural vibration of beam with oblique crack effect. Thus, the investigation included modeling the oblique crack as spring model, in addition to, used finite element method to calculate the natural frequency of cantilever beam. Finally, at, 2019, D. H. J. Al-Zubaidi et. al, [11], presented study of crack effect on the heat generation due to vibration of beam under harmonic forced applied. Where, the analytical technique used to calculate the relation for heat generation with time for vibration beam structure with various crack effect. Thus, the drive of general equation of forced vibration beam, under harmonic load, was used to calculate the heat generation with various frequency load applied. In addition, the numerical technique, by using finite element method, used to comparison the results calculate.

In addition, other researchers investigation the effect of crack on various application with different structure, as effect of crack depth and location on vibration behavior of different composite plate structure, [12, 13], effect of crack orientation on the vibration behavior for plate structure, [14, 15], and effect of crack on the vibration of pipe induce fluid, [16]. Therefore, from researchers presented can be see that the crack lead to decrease the stiffness for structure, and decrease the dynamic characterizations for structure, decrease for natural frequency and increase structure response.

There, this paper investigation the effect of crack depth and position in the natural frequency and vibration response for beam supported with different boundary condition, by using analytical solution for general equation of motion for beam with crack effect. Where, the analytical solution included drive for general equation of motion and then, solving of this equation by using exact solution for different boundary condition for beam, with using Fourier series and separation of variables techniques in addition, used finite element technique to comparison results calculated.

2. Analytical Study

The analytical investigation is techniques used to give exact solution of problem with various parameters effect, [17-20], in addition to, the analytical solution is given agreement results comparison with other technique used, [21-24]. Thus, the analytical part included two section, first evaluated of natural frequency of beam, and then, derived the equation determine the beam response supplied frequency harmonic onto beam and evaluated the equation of beam response as a function of time and length beam.

2.1 Vibration of Beam without Crack

To evaluated the natural frequency of beam, solution of general equation of motion, [25-27], of free vibration uniform cross section and constant modulus beam, as, [28-30],

$$EI\frac{\partial^4 w(x,t)}{\partial x^4} + \rho A\frac{\partial^2 w(x,t)}{\partial t^2} = 0 \tag{1}$$

Where, $E.I.\rho.A$ are modulus of elasticity, moment of inertia, density, and cross section area of beam, respectively, x is beam direction through length of beam, t is time, and w is the deflection of beam through lateral direction of beam. Then, by using separation of variable as,

$$W(x,t) = W_n(x) \times W_n(t) \tag{2}$$

Then, by substation Eq. 2 in to Eq. 1, and solve equation, get,

$$W(x) = \begin{pmatrix} C_1(\cos\beta x + \cosh\beta x) + C_2(\cos\beta x - \cosh\beta x) + \\ C_3(\sin\beta x + \sinh\beta x) + C_4(\sin\beta x - \sinh\beta x) \end{pmatrix} \tag{3}$$

Where, $C_1, C_2, C_3, and\ C_4$ are constant. And, β can be determine from boundary conditions of beam. And, the natural frequency of beam can be determine from,

$$\omega_n = (\beta l)^2\sqrt{\frac{EI}{\rho A l^4}} \tag{4}$$

Where, l is the length of beam and n is the number of mode of beam. Therefore, to evaluated the value of natural frequency must be selected the boundary condition of beam, then, can be selected three types of boundary condition beam as,

1. Cantilever beam, the boundary condition of cantilever beam included first ends fixed and other edges free as shown in Fig. 1, as,

$$W = 0, \frac{\partial W(x)}{\partial x} = 0 \text{ at fixed end}$$
$$\frac{\partial^2 W(x)}{\partial x^2} = 0, \frac{\partial^3 W(x)}{\partial x^3} = 0 \text{ at free end.} \tag{5}$$

Then by substitution Eq. 5 into Eq. 3 and solution, get (first four natural frequencies),

$$\beta_1 l = 1.875104$$
$$\beta_2 l = 4.694091$$
$$\beta_3 l = 7.854757$$
$$\beta_4 l = 10.995541 \tag{6}$$

And, the normal mode functions of beam, as,

$$W_n(x) = \left((\sin\beta_n x - \sinh\beta_n x) - \left(\frac{\sin\beta_n l + \sinh\beta_n l}{\cos\beta_n l + \cosh\beta_n l}\right)(\cos\beta_n x - \cosh\beta_n x)\right) \tag{7}$$

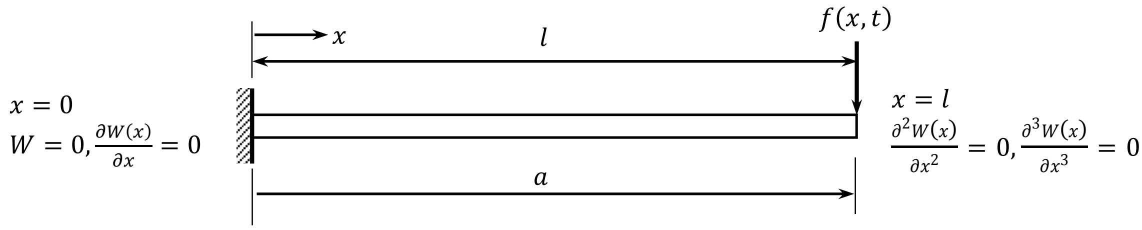


Figure 1. Boundary condition of cantilever beam supported.

2. Simply supported beam, the boundary condition of simply supported beam are pinned ends for two edges of beam, as shown in Fig. 2, as,

$W = 0, \frac{\partial^2 W(x)}{\partial x^2} = 0$ at two edges of beam. (8)

Also, by substation Eq. 8 in to Eq. 3, get (first four natural frequencies),

$\beta_1 l = 3.141593$, $\beta_2 l = 6.283185$, $\beta_3 l = 9.424778$, $\beta_4 l = 12.566371$ (9)

And, the normal mode functions of beam, as,

$W_n(x) = (\sin \beta_n x)$ (10)

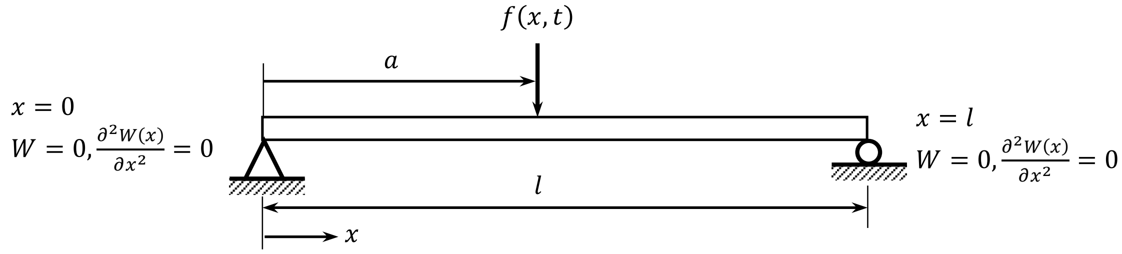


Figure 2. Boundary condition of simply supported beam.

3. Clamped beam, the boundary condition of clamped supported beam are fixed ends for two edges of beam, as shown in Figure (3), as,

$W = 0, \frac{\partial W(x)}{\partial x} = 0$ at two edges of beam. (11)

Also, by substation Eq. 11 in to Eq. 3, get (first four natural frequencies),

$\beta_1 l = 4.730041$
$\beta_2 l = 7.853205$
$\beta_3 l = 10.995608$
$\beta_4 l = 14.137165$ (12)

And, the normal mode functions of beam, as,

$W_n(x) = \left[(\sinh \beta_n x - \sin \beta_n x) + \left(\frac{\sinh \beta_n l - \sin \beta_n l}{\cos \beta_n l - \cosh \beta_n l}\right)(\cosh \beta_n x - \cos \beta_n x)\right]$ (13)

Then, applied the frequency on the beam less than the natural frequency of beam and evaluated the response on the beam. And then, applied frequency on the beam equal to natural frequency of beam (resonance of beam occurred) and comparison the different between the two case.

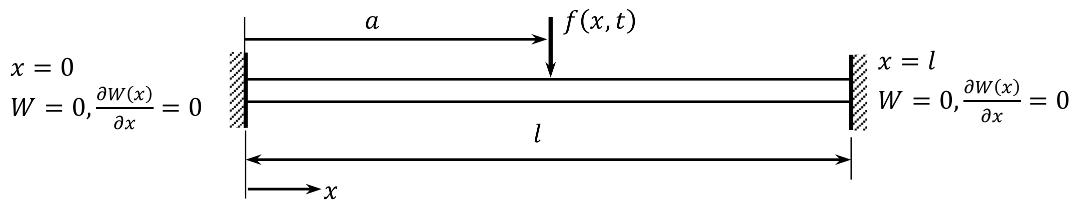


Figure 3. Boundary condition of clamped beam supported.

Therefore, the general equation of motion of force vibration beam can be used,

$$\omega_n^2 W_n(x) W_n(t) + W_n(x) \frac{d^2 W_n(t)}{dt^2} = \frac{1}{\rho A} f(x,t) \quad (14)$$

Where, $f(x,t)$ general force applied on the beam at distance a (N/m), as shown in Figures. 1, 2, and 3. By multiplying Eq. 14 throughout by $W_n(x)$, and integrating from $0\ to\ l$, and using the orthogonally, get,

$$\frac{d^2 W_n(t)}{dt^2} + \omega_n^2 W_n(t) = \frac{1}{\rho A b} Q_n(t) \quad (15)$$

Where, $Q_n(t)$ is generalized forced (N),

$Q_n(t) = \int_0^l f(x,t) W_n(x)\, dx.$

And, b is constant (m), and can be expressed as,

$$b = \int_0^l W_n^2(x)\, dx \quad (16)$$

Then, by using Duhamel integral, the solve of Eq. 16, for zero initial conditions (displacement and velocity), and substation the solution into Eq. 2, get,

$$w(x,t) = W_n(x) \left(\frac{1}{\rho A b \omega_n} \int_0^t Q_n(\tau) \sin \omega_n (t-\tau)\ d\tau \right) \quad (17)$$

Therefore, applied harmonic load $f(x,t) = f_o \sin \omega t$ with frequency of ω, at $x = a$, and substitution it's in Eqs. 16 and 17, to evaluating the general equation of beam deflection as a function of x and time.

2.2 Vibration of Beam with Crack

For beam with crack effect, By using Euler-Bernoulli beam theory, the equation of motion for beam, assumed to have uniform cross section, is given by,

$$EI \frac{\partial^4 W(x.t)}{\partial x^4} + \rho A \frac{\partial^2 W(x.t)}{\partial t^2} = f(x,t) \quad (18)$$

By letting the forcing term, $f(x,t)$ to be zero. Then, using the separable solutions, as in Eq. 2, in Eq. 18, leads to an associated eigenvalue problem, [11],

$$\frac{\partial^4 W(x)}{\partial x^4} - \lambda^4 W(x) = 0 \quad (19)$$

Where, $\lambda^4 = \frac{\rho A \omega^2}{EI}$.

The general solution of Eq. 19, for each segment, [11],

$for\ x = 0 \longrightarrow l_1$

$W_1(x) = A_1 \sin(\lambda x) + B_1 \cos(\lambda x) + C_1 \sinh(\lambda x) + D_1 \cosh(\lambda x)$

$for\ x = l_1 \longrightarrow l$

$$W_2(x) = A_2 \sin(\lambda(x-l_1)) + B_2 \cos(\lambda(x-l_1)) + C_2 \sinh(\lambda(x-l_1)) + D_2 \cosh(\lambda(x-l_1)) \quad (20)$$

Now, applying the boundary conditions for three cases to evaluate the general equation for beam response for harmonic vibration beam structure with crack effect, as,

I. Cantilever Beam

The equations for cantilever beam will be derived in the case of a crack, as shown in Fig. 4, by subject boundary conditions of beam, as,

$$W_1(0) = 0,\ \overline{W}_1(0) = 0,\ \overline{\overline{W}}_2(l) = 0,\ \overline{\overline{\overline{W}}}_2(l) = 0 \quad (21)$$

Applied boundary conditions, Eq. 21, on deflection beam equation, Eq. 20, get,

$W_1(x) = A_1(\sin(\lambda x) - \sinh(\lambda x)) + B_1(\cos(\lambda x) - \cosh(\lambda x))$

$$W_2(x) = \begin{pmatrix} A_2(\sin(\lambda(x-l_1)) + \alpha_1 \sinh(\lambda(x-l_1)) + \alpha_2 \cosh(\lambda(x-l_1))) + \\ B_2(\cos(\lambda(x-l_1)) + \alpha_3 \sinh(\lambda(x-l_1)) + \alpha_4 \cosh(\lambda(x-l_1))) \end{pmatrix} \quad (22)$$

Where,

$\alpha_1 = \cos(\lambda l_2).\cosh(\lambda l_2) - \sin(\lambda l_2).\sinh(\lambda l_2),$

$\alpha_2 = \sin(\lambda l_2).\cosh(\lambda l_2) - \cos(\lambda l_2).\sinh(\lambda l_2)$

$\alpha_3 = -\cos(\lambda l_2).\sinh(\lambda l_2) - \sin(\lambda l_2).\cosh(\lambda l_2),$

$\alpha_4 = \cos(\lambda l_2).\cosh(\lambda l_2) + \sin(\lambda l_2).\sinh(\lambda l_2)$

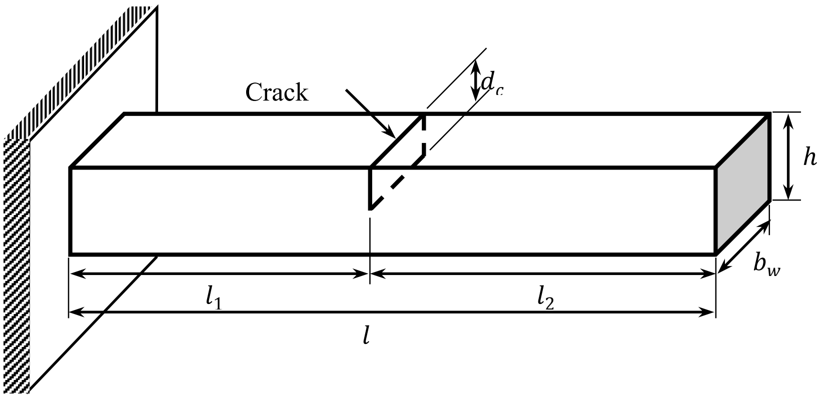


Figure 4. Cantilever Beam with crack.

Then, for evaluating other constant in Eq. 22, using boundary condition at crack location, as,

$$W_1(l_1) = W_2(l_1), \overline{\overline{W}}_1(l_1) = \overline{\overline{W}}_2(l_1), \overline{\overline{\overline{W}}}_1(l_1) = \overline{\overline{\overline{W}}}_2(l_1),$$
$$\overline{W}_2(l_1) - \overline{W}_1(l_1) = \theta l \overline{\overline{W}}_2(l_1) \quad (23)$$

For,

$$\theta = 6\pi \begin{pmatrix} 0.6384\left(\frac{d_c}{h}\right)^2 - 1.035\left(\frac{d_c}{h}\right)^3 + 3.7201\left(\frac{d_c}{h}\right)^4 - 5.1773\left(\frac{d_c}{h}\right)^5 + \\ 7.553\left(\frac{d_c}{h}\right)^6 - 7.332\left(\frac{d_c}{h}\right)^7 + 2.4909\left(\frac{d_c}{h}\right)^8 \end{pmatrix}\left(\frac{h}{l}\right)$$

There, substation boundary conditions in Eq. 23 into Eq. 22, get,

$$W_1(x) = A_1\big(sin(\lambda x) - sinh(\lambda x) + \alpha_7(cos(\lambda x) - cosh(\lambda x))\big)$$
$$W_2(x) = A_1\begin{pmatrix} \alpha_8\, sin\big(\lambda(x-l_1)\big) + \alpha_9\, cos\big(\lambda(x-l_1)\big) \\ +\alpha_{10}\, sinh\big(\lambda(x-l_1)\big) + \alpha_{11}\, cosh\big(\lambda(x-l_1)\big) \end{pmatrix} \quad (24)$$

Where,

$$\alpha_5 = -\left(\frac{\alpha_4}{\alpha_2} sin(\lambda l_1) + \frac{1}{\alpha_2} sinh(\lambda l_1)\right),$$
$$\alpha_6 = -\left(\frac{\alpha_4}{\alpha_2} cos(\lambda l_1) + \frac{1}{\alpha_2} cosh(\lambda l_1)\right)$$
$$\alpha_7 = \frac{(cos(\lambda l_1) + cosh(\lambda l_1) - \alpha_5 + \alpha_5\alpha_1 + \alpha_3\, sin(\lambda l_1))}{(sin(\lambda l_1) - sinh(\lambda l_1) + \alpha_6 - \alpha_6\alpha_1 - \alpha_3\, cos(\lambda l_1))},$$
$$\alpha_8 = (\alpha_5 + \alpha_6\alpha_7),$$
$$\alpha_9 = (sin(\lambda l_1) + \alpha_7\, cos(\lambda l_1))$$
$$\alpha_{10} = (\alpha_5\alpha_1 + \alpha_3\, sin(\lambda l_1) + \alpha_6\alpha_1\alpha_7 + \alpha_3\alpha_7\, cos(\lambda l_1))$$
$$\alpha_{11} = (\alpha_5\alpha_2 + \alpha_4\, sin(\lambda l_1) + \alpha_6\alpha_2\alpha_7 + \alpha_4\alpha_7\, cos(\lambda l_1))$$

Then, with using of boundary conditions, $\overline{W}_2(l_1) - \overline{W}_1(l_1) = \theta l \overline{\overline{W}}_2(l_1)$, get the general characterizations equation to evaluate natural frequency of beam with crack effect, as,

$$\alpha_8 + \alpha_{10} - cos(\lambda l_1) + cosh(\lambda l_1) + \alpha_7\, sin(\lambda l_1) + \alpha_7\, sinh(\lambda l_1) = \theta l \lambda(\alpha_{11} - \alpha_9) \quad (25)$$

By solving Eq. (25), the value of λ can be evaluated, then the value of ω_n can be evaluated.

II. Simply Supported Beam

The equations for simply supported beam will be derived in the case of a crack, as shown in Fig. 5, by subject boundary conditions of beam, as,

$$W_1(0) = 0, \overline{\overline{W}}_1(0) = 0, W_2(l) = 0, \overline{\overline{W}}_2(l) = 0 \quad (26)$$

Therefore, by substitution boundary conditions for simply supported beam, Eq. 26, into general equation for mode beam, Eq. 20, get,

$$W_1(x) = (A_1 \sin(\lambda x) + C_1 \sinh(\lambda x))$$
$$W_2(x) = A_2\big(\sin\big(\lambda(x-l_1)\big) + \alpha_1 \cos\big(\lambda(x-l_1)\big)\big) + C_2\big(\sinh\big(\lambda(x-l_1)\big) + \alpha_2 \cosh\big(\lambda(x-l_1)\big)\big) \quad (27)$$

Where, $\alpha_1 = (-\tan(\lambda l_2))$, $\alpha_2 = (-\tanh(\lambda l_2))$

Then, by substitution boundary condition for crack location, Eq. 23, into Eq. 27, get,

$$W_1(x) = A_1(\sin(\lambda x) + \alpha_5 \sinh(\lambda x))$$
$$W_2(x) = A_1\big(\alpha_4 \sin\big(\lambda(x-l_1)\big) + \alpha_6 \cos\big(\lambda(x-l_1)\big) + \alpha_7 \sinh\big(\lambda(x-l_1)\big) + \alpha_8 \cosh\big(\lambda(x-l_1)\big)\big) \quad (28)$$

Where,

$\alpha_3 = \frac{\sinh(\lambda l_1)}{\alpha_2}$, $\alpha_4 = \frac{\sin(\lambda l_1)}{\alpha_1}$, $\alpha_5 = \frac{[\cos(\lambda l_1) - \alpha_4]}{[\cosh(\lambda l_1) - \alpha_3]}$, $\alpha_6 = (\alpha_1\alpha_4)$, $\alpha_7 = (\alpha_3\alpha_5)$, $\alpha_8 = (\alpha_2\alpha_3\alpha_5)$

Also, by applying boundary condition, $\overline{W}_2(l_1) - \overline{W}_1(l_1) = \theta l \overline{\overline{W}}_2(l_1)$, get the general characterizations equation to evaluate the natural frequency for beam with various crack effect, as,

$$\alpha_4 + \alpha_7 - \cos(\lambda l_1) - \alpha_5 \cosh(\lambda l_1) = \theta l \lambda(\alpha_8 - \alpha_6) \quad (29)$$

By solving equation (29), the value of λ can be evaluated, then the value of ω_n can be evaluated.

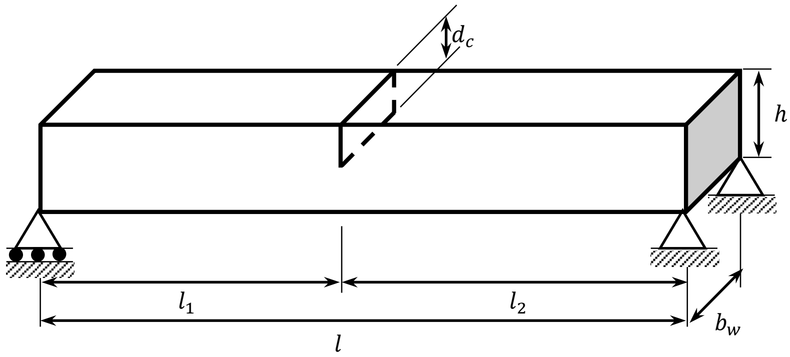


Figure 5. Simply supported Beam with crack.

III. Clamped Supported Beam

The equations for clamped supported beam will be derived in the case of a crack, as shown in Fig. 6, by subject boundary conditions of beam, as,

$$W_1(0) = 0,$$
$$\overline{W}_1(0) = 0,$$
$$W_2(l) = 0,$$
$$\overline{W}_2(l) = 0 \quad (30)$$

Then, by subject Eq. 30 into Eq. 20, get,

$$W_1(x) = (A_1(\sin(\lambda x) - \sinh(\lambda x)) + B_1(\cos(\lambda x) - \cosh(\lambda x)))$$

$$W_2(x) = \begin{pmatrix} A_2\left(\sin\left(\lambda(x - l_1)\right) + \alpha_1 \sinh\left(\lambda(x - l_1)\right) + \alpha_2 \cosh\left(\lambda(x - l_1)\right)\right) + \\ B_2\left(\cos\left(\lambda(x - l_1)\right) + \alpha_3 \sinh\left(\lambda(x - l_1)\right) + \alpha_4 \cosh\left(\lambda(x - l_1)\right)\right) \end{pmatrix} \quad (31)$$

Where,

$$\alpha_1 = (\sin(\lambda l_2).\sinh(\lambda l_2) - \cos(\lambda l_2).\cosh(\lambda l_2)),$$
$$\alpha_2 = (-\sin(\lambda l_2).\cosh(\lambda l_2) + \cos(\lambda l_2).\sinh(\lambda l_2))$$
$$\alpha_3 = (\cos(\lambda l_2).\sinh(\lambda l_2) + \sin(\lambda l_2).\cosh(\lambda l_2)),$$
$$\alpha_4 = -(\cos(\lambda l_2).\cosh(\lambda l_2) + \sin(\lambda l_2).\sinh(\lambda l_2))$$

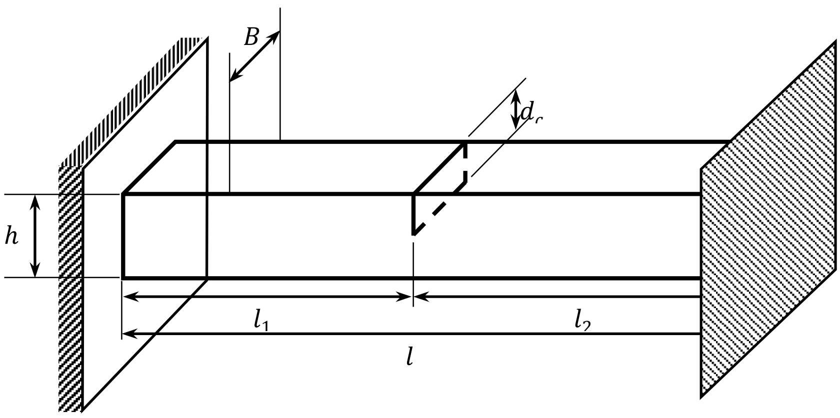


Figure 6. Clamped beam geometry with crack.

Then, by subjecting boundary condition for crack defect, Eq. 23, into Eq. 31, get,

$$W_1(x) = A_1\left((\sin(\lambda x) - \sinh(\lambda x)) + \alpha_7(\cos(\lambda x) - \cosh(\lambda x))\right)$$

$$W_2(x) = A_1 \begin{pmatrix} \alpha_8 \sin(\lambda(x-l_1)) + \alpha_9 \cos(\lambda(x-l_1)) + \\ \alpha_{10} \sinh(\lambda(x-l_1)) + \alpha_{11} \cosh(\lambda(x-l_1)) \end{pmatrix} \tag{32}$$

Where,

$\alpha_5 = -\left(\frac{\alpha_4}{\alpha_2}\sin(\lambda l_1) + \frac{1}{\alpha_2}\sinh(\lambda l_1)\right)$,

$\alpha_6 = -\left(\frac{\alpha_4}{\alpha_2}\cos(\lambda l_1) + \frac{1}{\alpha_2}\cosh(\lambda l_1)\right)$

$\alpha_7 = \frac{(\cos(\lambda l_1)+\cosh(\lambda l_1)-\alpha_5+\alpha_5\alpha_1+\alpha_3\sin(\lambda l_1))}{(\sin(\lambda l_1)-\sinh(\lambda l_1)+\alpha_6-\alpha_6\alpha_1-\alpha_3\cos(\lambda l_1))}$,

$\alpha_8 = (\alpha_5 + \alpha_6\alpha_7)$,

$\alpha_9 = (\sin(\lambda l_1) + \alpha_7\cos(\lambda l_1))$

$\alpha_{10} = (\alpha_5\alpha_1 + \alpha_3\sin(\lambda l_1) + \alpha_6\alpha_1\alpha_7 + \alpha_3\alpha_7\cos(\lambda l_1))$

$\alpha_{11} = (\alpha_5\alpha_2 + \alpha_4\sin(\lambda l_1) + \alpha_6\alpha_2\alpha_7 + \alpha_4\alpha_7\cos(\lambda l_1))$

Also, by using boundary condition, $\bar{W}_2(l_1) - \bar{W}_1(l_1) = \theta l \bar{\bar{W}}_2(l_1)$, for Eq. 32, get the general characterization equation for evaluating the natural frequency of cantilever beam with various crack size and position effect, as,

$$\alpha_8 + \alpha_{10} - \cos(\lambda l_1) + \cosh(\lambda l_1) + \alpha_7\sin(\lambda l_1) + \alpha_7\sinh(\lambda l_1) = \theta l\lambda(\alpha_{11} - \alpha_9) \tag{33}$$

To applying the equation of deflection beam as a function of x-direction, Eq. 24, 28, 32 for cantilever, simply supported and clamped beam, respectively, in general equation of motion of beam with crack effect, must be transformation the equations of beam to continuous equation by using Fourier series formulation, and then subjecting the resulting equation into Eq. 18, as,

$$W(x) = A_o + \sum_{n=1}^{\infty} A_n \cos\frac{2n\pi x}{L} + \sum_{n=1}^{\infty} B_n \sin\frac{2n\pi x}{L} \tag{34}$$

Where,

$A_o = \frac{1}{l}\int_0^l W(x)dx = \frac{1}{l}\left(\int_0^{l_1} W_1(x)dx + \int_{l_1}^{l} W_2(x)dx\right)$

$A_n = \frac{2}{l}\int_0^l W(x)\cos\frac{2n\pi x}{L}dx = \frac{2}{l}\left(\int_0^{l_1} W_1(x)\cos\frac{2n\pi x}{l}dx + \int_{l_1}^{l} W_2(x)\cos\frac{2n\pi x}{l}dx\right)$

$B_n = \frac{2}{l}\int_0^l W(x)\sin\frac{2n\pi x}{l}dx = \frac{2}{l}\left(\int_0^{l_1} W_1(x)\sin\frac{2n\pi x}{l}dx + \int_{l_1}^{l} W_2(x)\sin\frac{2n\pi x}{l}dx\right)$

Then, by substation Eqs. 24, 28, or 32, into Eq. 34, according of beam supported, and integration the resulting equation, get the constant Fourier series, then subjecting Eq. 34 onto Eq. 18, get,

$$\begin{pmatrix} EI\left(\sum_{n=1}^{\infty} A_n \left(\frac{2n\pi}{L}\right)^4 \cos\frac{2n\pi x}{L} + \sum_{n=1}^{\infty} B_n \left(\frac{2n\pi}{L}\right)^4 \sin\frac{2n\pi x}{L}\right) w(t) + \\ \rho A\left(A_o + \sum_{n=1}^{\infty} A_n \cos\frac{2n\pi x}{L} + \sum_{n=1}^{\infty} B_n \sin\frac{2n\pi x}{L}\right)\frac{\partial^2 w(t)}{\partial t^2} \end{pmatrix} = f(x,t) \tag{35}$$

Then, by using orthogonally technique, with multiplying Eq. 35 by Eq. 34, and then, integrating from $0 \ to \ l$, get,

$$\omega^2 W(t) + \frac{\partial^2 W(t)}{\partial t^2} = \frac{\int_0^l \left(A_o + \sum_{n=1}^{\infty} A_n \cos\frac{2n\pi x}{L} + \sum_{n=1}^{\infty} B_n \sin\frac{2n\pi x}{L}\right) f(x.t)\,dx}{\rho A\int_0^l \left(A_o + \sum_{n=1}^{\infty} A_n \cos\frac{2n\pi x}{L} + \sum_{n=1}^{\infty} B_n \sin\frac{2n\pi x}{L}\right)^2 dx} = \frac{\int_0^l W(x)f(x,t)dx}{\rho A\int_0^l W^2(x)dx} = \frac{1}{\rho Ab}Q_n(t) \tag{36}$$

Where, $Q_n(t)$, is the forced generalized (N), and can be evaluating from,

$$Q_n(t) = \int_0^l W(x)f(x,t)dx \tag{37}$$

And, b: orthogonally constant, can be evaluating from Eq. 16. Then, by using Duhamel integral, the solve of Eq. 36, for zero initial conditions (displacement and velocity), and substation the solution into Eq. 2, get,

$$w(x,t) = W_n(x)\left(\frac{1}{\rho Ab\omega_n}\int_0^t Q_n(\tau)\sin\omega_n(t-\tau)\ d\tau\right) \tag{38}$$

Therefore, applied harmonic load $f(x,t) = f_o \sin\omega t$ with frequency of ω, at $x = a$, and substitution it's in Eqs. 37 and 38, to evaluating the general equation of beam deflection as a function of x and time.

3. Numerical Technique

The numerical investigation is technique used to evaluation approximate solution for problem, [31-33], thus, its technique in more application used to analysis the difficult structure its cannot solution by analytical techniques, [34, 35]. In addition, the numerical technique used to comparison the numerical results by other results are evaluated with other techniques, to given the agreement of results, [36-38].

Therefore, the COMSOL program will be used, which will depend on the finite element method (FEM) for design and analysis of the models, and give the outcomes required by simulations using computer programming to reach the outputs and outcomes of the completion of the research and development of the mechanism of work, and compared to other outcomes extracted with the analytical solution, and given the percent error of its results. The COMSOL program is used to find precise numerical solutions and different designs and structures and to find the results of complex mathematical equations that are difficult to solve analytically, [39-40]. In this work, the COMSOL/CFD program was used to find the response by the vibration of the beam exposed to the harmonic force at different frequencies for different cases of supported. Follow the main steps when using the software in the computer as,

1. Design of the models according to properties, dimensions and geometries of the material to be used.
2. Defining the meshing geometry.
3. Applying the boundary conditions, frequency and force.
4. Solving and visualizing your results.

The numerical examination is strategy utilized to assessment estimated answer for issue, in this way, the its method in more application utilized to investigation the troublesome structure its can't arrangement by scientific strategies. Likewise, the numerical procedure used to correlation the numerical outcomes by different outcomes are assessed with different methods, to given the understanding of results, [41-42]. Therefore, the COMSOL program will be utilized, which will rely upon the finite element method (FEM) for outline and examination of the models, and give the results required by reproductions and simulations utilizing PC programming to achieve the yields and results of the fruition of the innovative work of the system of work, and contrasted with different results removed with the scientific arrangement, and given the percent error of its outcomes. This program is utilized to discover exact numerical arrangements and diverse outlines and structures and to discover the outcomes of complex scientific conditions that are hard to solve analytically, [43]. The beam design length (84 cm) and cross-section (2.5 cm * 2.5 cm) as in the Fig. 7. The material used is carbon steel (carbon=1.5%) with a density of (7750 kg/m^3) and modulus of elasticity (200 Gpa). The supported is determined by the type of beam where the use of three types including (cantilever beam, simply supported beam, clamped beam) and each type of supported has special boundary conditions, the required frequency and force are applied to the beam, after which the program is executed and the desired results and figures are included. The design of the model form on the mesh, which follows a finite element method (FEM) to give the results the elements, the results will be more accurate as the mesh is accurate if the distance between the nearest mesh lines, the mesh was represented symmetrically for all types of supported, so that the accuracy of the results is uniform for all cases to be used in comparison.

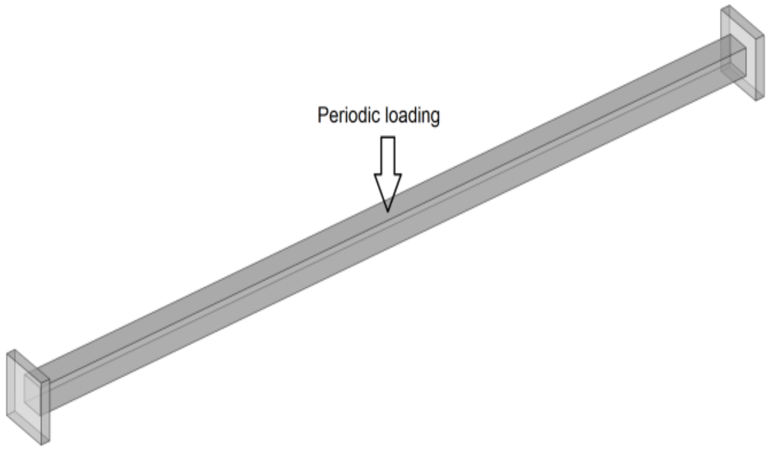
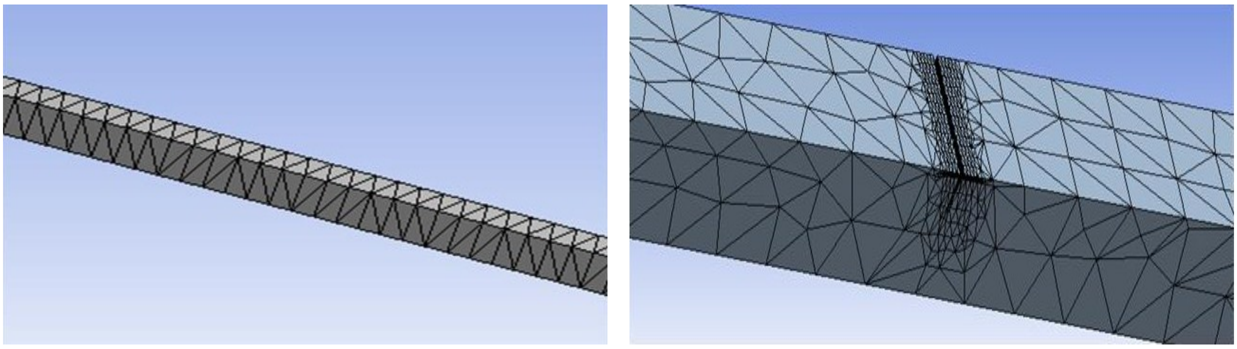


Figure 7. Three-dimensional computational domain.

Default mesh setups were utilized. The model solution at transient boundaries. It is plotted measured values of displacement utilizing domain plotting parameters. The Eigen natural frequency analysis outcomes initially then, the resonance are appeared in the distorted shape with sub- domain outcomes. The beam response plot outcomes are appeared of normal sub- domain outcomes. The results obtained through the use of the COMSOL program by simulating similar cases in the analytical solution. The results shown in the forms were obtained to determine the relationship between the variables and the behavior of the curves of the model in the cases of assuming that the harmonic force causes vibration very near to the resonance state of each case to show the variables clearly.

In this work, the COMSOL/CFD program was utilized to discover the beam response values by the vibration of the beam presented to the harmonic force at various frequencies for various instances of supported with crack and to know the effect of its location and depth on the beam response. Thus, to solution the problem by using its numerical technique, firstly, selecting the best number of element and node can be used, [44-45], then, different cases are studied for the computer mesh where the comparison is made for the design of the mesh using the COMSOL program in the absence of a crack in the beam as in Fig. 8.a. Also, a computer mesh is designed in the case of a crack in the beam and what is the effect of the crack on the mesh as in Fig. 8.b, and using the finite element method (FEM) where the model is divided into small parts and elements and simple geometric shapes that make it easy to distinguish between the two cases to give different results.



a. without crack b. with crack

Figure 8. Beam mesh with and without crack.

4. Results and Discussion

The results investigation in this work shown the effect of crack depth and location on the beam deflection, supported with various boundary conditions. Where, the investigation included used analytical solution for derived general equation of motion for beam structure with crack effect, with various boundary condition for beam. In addition, using numerical technique, by using finite element method, to calculate the beam deflection with different crack effect. Then, comparison the results calculated by analytical and numerical together. Therefore, the results presenting dividing to four parts, first, the natural frequency for bean with and without crack effect, second, analytical beam deflection results without crack effect for different supported beam, third, analytical beam deflection results with various crack location and depth effect for different beam supported, and finally, comparison for beam deflection analytical and numerical results and given the discrepancy for result calculating.

4.1 Beam Natural Frequency

The natural frequency of the assumed beam (length (L=0.84 m), width, height (w = h = 2.5 cm) is calculated analytically by using the Matlab program during the vibration of the beam with crack to different depths and positions under the influence of harmonic force for three cases of supported. Table 1 shows the natural frequency of cantilever beam changes according to crack depths and positions ($d_c = (0.25, 0.5, 0.75, 1, 1.25, 1.5)$ cm, and $L_1 = (0.1, 0.2, 0.3, 0.4, 0.42, 0.44, 0.54, 0.64, 0.74)$ m, where, the natural frequency of the cantilever beam without crack $(\omega_n) = 182.6862 \frac{rad}{sec}$. Table 2 shows the natural frequency of beam changes according to crack depths and positions $d_c = (0.25, 0.5, 0.75, 1, 1.25, 1.5)$ cm and $L_1 = (0.1, 0.2, 0.3, 0.4, 0.42, 0.44, 0.54, 0.64, 0.74)$ m, where, the natural frequency of the simply supported beam without crack $(\omega_n) = 512.808 \frac{rad}{sec}$. also, Table 3, shows the natural frequency of beam changes according to crack depths and positions, as,

1. $d_c = (0.25, 0.5, 0.75, 1, 1.25, 1.5)$ cm
2. $L_1 = (0, 0.1, 0.2, 0.3, 0.4, 0.42, 0.44, 0.54, 0.64, 0.74, 0.84)$ m

Where, the natural frequency of the clamped beam without crack $(\omega_n) = 1162.478 \frac{rad}{sec}$.

Table 1. Natural Frequency for Cantilever Beam with Various Crack Depth and Position Effect.

No	L_1 (m)	ω (rad/s)					
		dc=0.2 cm	dc=0.5 cm	dc=0.75 cm	dc=1 cm	dc=1.25 cm	dc=1.5 cm
1	0.1	181.8780	179.6186	175.7675	169.9866	162.1470	152.6556
2	0.2	182.1602	180.6772	178.1054	174.1394	168.5536	161.4624
3	0.3	182.3772	181.4999	179.9564	177.5208	173.9739	169.2693
4	04	182.5292	182.0811	181.2846	180.0061	178.0959	175.4708
5	0.42	182.5522	182.1695	181.4881	180.3915	178.7466	176.4734
6	0.44	182.5729	182.2490	181.6717	180.7405	179.3385	177.3912
7	0.54	182.6452	182.5280	182.3179	181.9766	181.4565	180.7215
8	0.64	182.6770	182.6507	182.6036	182.5267	182.4092	182.2422
9	0.74	182.6855	182.6837	182.6804	182.6750	182.6667	182.6549

Table 2. Natural Frequency for Simply Supported Beam with Various Crack Depth and Position Effect.

No	L_1 (m)	ω (rad/s)					
		dc=0.2 cm	dc=0.5 cm	dc=0.75 cm	dc=1 cm	dc=1.25 cm	dc=1.5 cm
1	0.1	512.5900	511.9663	510.8506	509.0411	506.2938	502.4313
2	0.2	512.0538	509.9063	506.1077	500.0605	491.1419	479.1168
3	0.3	511.4871	507.7526	501.2409	491.1132	476.6811	458.0953
4	04	511.1916	506.6399	498.7669	486.6755	469.7483	448.4303
5	0.42	511.1826	506.6061	498.6920	486.5424	469.5428	448.1477
6	0.44	511.1916	506.6399	498.7669	486.6755	469.7483	448.4303
7	0.54	511.4871	507.7526	501.2409	491.1132	476.6811	458.0953
8	0.64	512.0538	509.9063	506.1077	500.0605	491.1419	479.1168
9	0.74	512.5900	511.9663	510.8506	509.0411	506.2938	502.4313

Table 3. Natural Frequency for Clamped Supported Beam with Various Crack Depth and Position Effect.

No	L1 (m)	ω (rad/s)					
		dc=0.2 cm	dc=0.2 cm	dc=0.2 cm	dc=0.2 cm	dc=0.2 cm	dc=0.2 cm
1	0	1155.217	1135.876	1105.875	1066.861	1023.190	981.044
2	0.1	1160.991	1156.896	1150.118	1140.403	1128.061	1114.314
3	0.2	1162.456	1162.395	1162.290	1162.130	1161.907	1161.629
4	0.3	1161.105	1157.242	1150.578	1140.390	1126.244	1108.671
5	04	1159.805	1152.327	1139.562	1120.376	1094.355	1062.952
6	0.42	1159.760	1152.159	1139.189	1119.708	1093.309	1061.482
7	0.44	1159.805	1152.327	1139.562	1120.376	1094.355	1062.952
8	0.54	1161.105	1157.242	1150.578	1140.390	1126.244	1108.671
9	0.64	1162.456	1162.395	1162.290	1162.130	1161.907	1161.629
10	0.74	1160.991	1156.896	1150.118	1140.403	1128.061	1114.314
11	0.84	1155.217	1135.876	1105.875	1066.861	1023.190	981.044

4.2 Beam Response without Crack Effect

The amount of the response deflection in the assumed beam (length (L=0.84 m), width, height (w=h=2.5 cm)) is calculated analytically by using the Matlab program during the vibration of the beam under the influence of harmonic force at different frequencies compared to the natural frequency. Sometimes the response deflection is calculated for each location along the length of the beam on demonstrated constant time at other times it is calculated with time in a location where the response deflection the highest value in the beam for several cases of supported, as follows,

4.2.1 The response deflection for cantilever beam

Fig. 9, shows the response deflection in the beam changes according to the time and length of the beam, when the beam is vibrated at a frequency ($\omega = 0.9 \times \omega_n$). Fig. 10, the relationship between the response deflection with the length of the beam is explained at constant time for different frequencies (ω = (0.5, 0.6, 0.7, 0.8, 0.9, 1.1 and 1.2) ωn) as shown (a), (ω = 0.99 ωn) as shown in Fig. 10.b and (ω = ωn) as shown in Fig. 10.c. The Fig. 11, the relationship between the response deflection with the time is explained at the free end (x = 0.84 m) for the appearance of the highest response deflection as shown in the previous Fig. 10, for different frequencies (ω = (0.5, 0.6, 0.7, 0.8, 0.9, 0.99, 1, 1.1 and 1.2) ω_n).

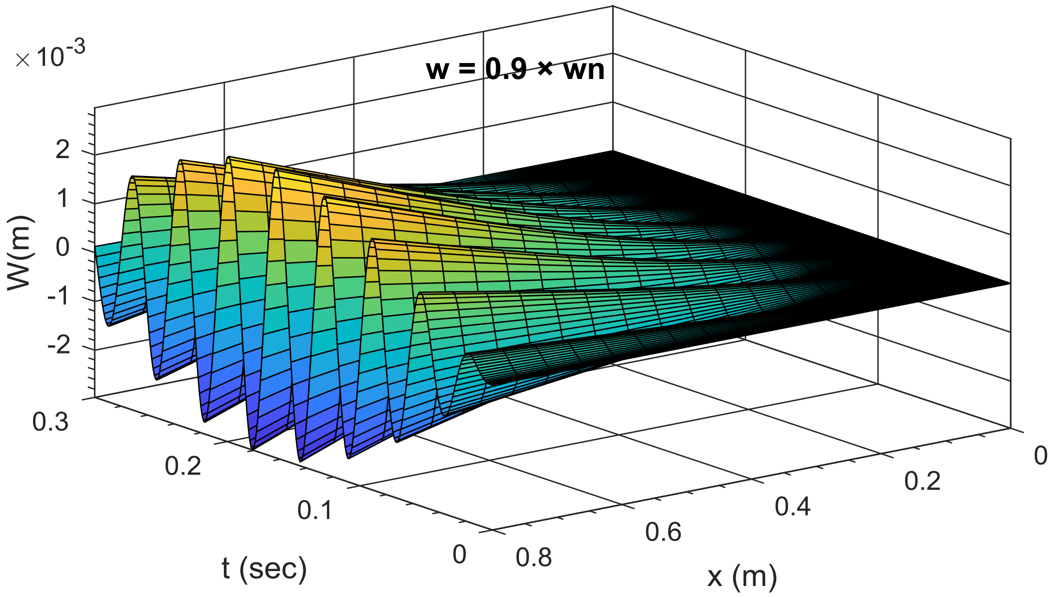
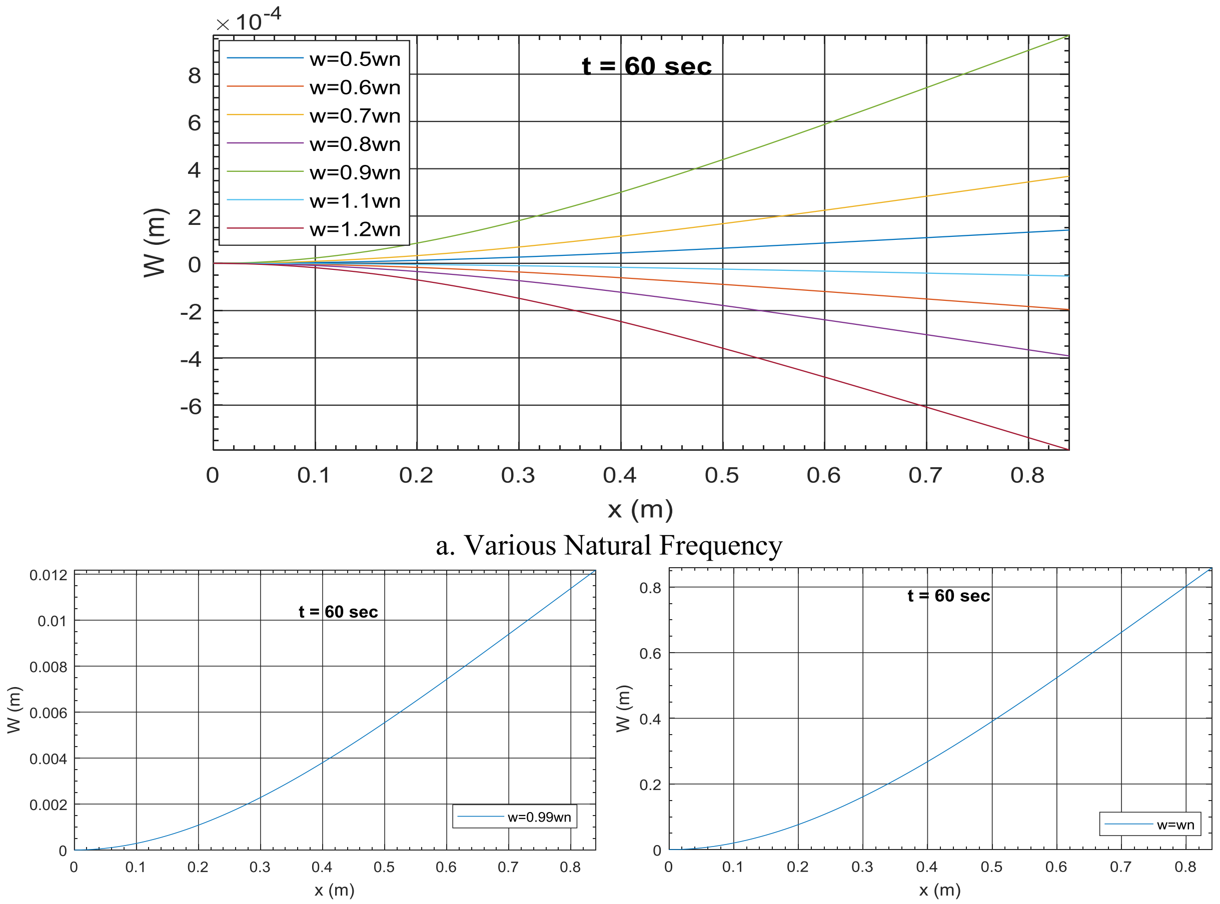


Figure 9. The deflection with time and length of the cantilever beam.



a. Various Natural Frequency

b. $\omega = 0.99\omega_n$ c. $\omega_n = \omega$

Figure 10. The deflection with length of the cantilever beam.

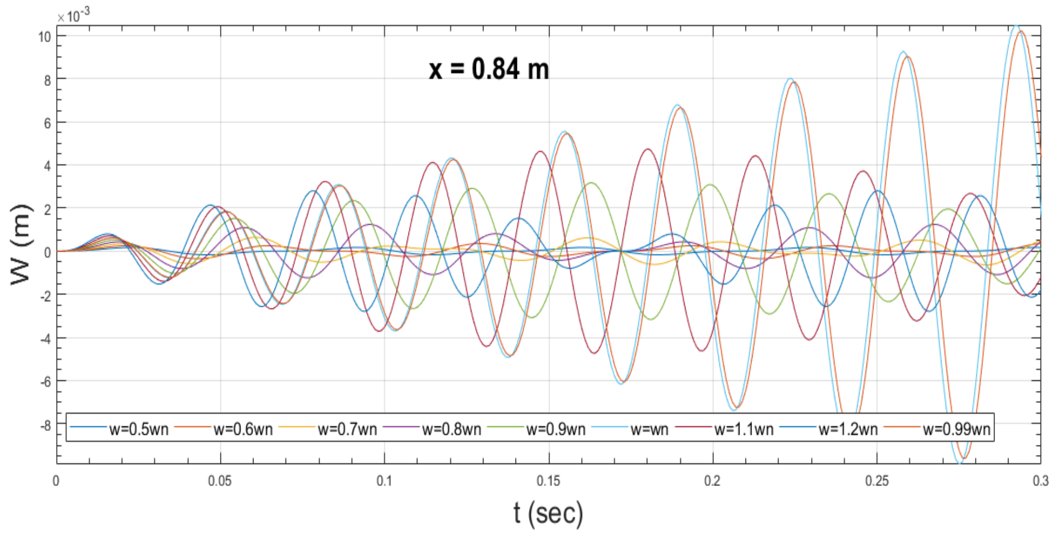


Figure 11. The cantilever Beam Response with time and various force frequency applied

4.2.2 The response deflection for simply supported beam

Fig. 12, shows the response deflection in the beam changes according to the time and length of the beam when the beam is vibrated at a frequency ($\omega = 0.9 \times \omega_n$). Now, in Fig. 13, the relationship between the response deflection in the beam with the length of the beam is explained at constant time for different frequencies (ω = (0.5, 0.6, 0.7, 0.8, 0.9, 0.99, 1.1 and 1.2) ωn) as shown in Fig. 13.a, and (ω = ωn) as shown in Fig. 13.b. The Fig. 14, the relationship between the response deflection with the time is explained at the middle of the beam (x = 0.42 m) for the appearance of the highest response deflection as shown in the Fig. 13, for different frequencies (ω = (0.5, 0.6, 0.7, 0.8, 0.9, 0.99, 1, 1.1 and 1.2) ωn).

4.2.3 The beam response of clamped beam

Fig.15, shows the response deflection in the beam changes according to the time and length of the beam when the beam is vibrated at a frequency ($\omega = 0.9 \times \omega_n$). Now, in Fig. 16, the relationship between the response deflection in the beam with the length of the beam is explained at constant time for different frequencies (ω = (0.5, 0.6, 0.7, 0.8, 0.9 and 1.1) ω_n) as shown in Fig. 16.a and (ω = (0.99, 1) ω_n) as shown in Fig. 16.b. The Fig. 17, the relationship between the response deflection with the time is explained at the middle of the beam (x = 0.42 m) for the appearance of the highest response deflection as shown in the previous Fig. 16, for different frequencies (ω = (0.5, 0.6, 0.7, 0.8, 0.9, 0.99, 1, 1.1 and 1.2) ω_n).

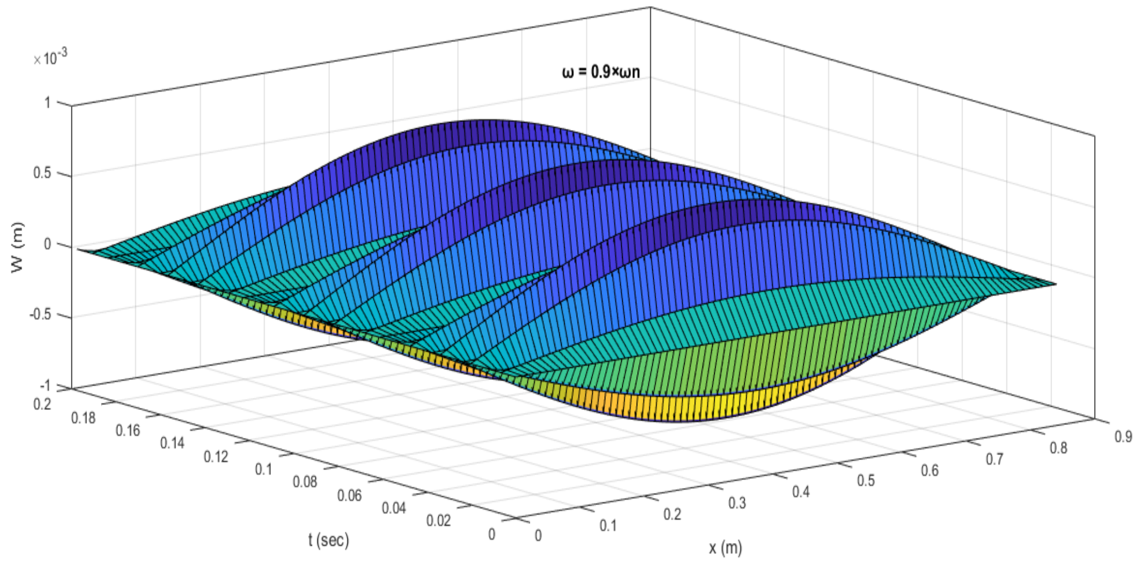
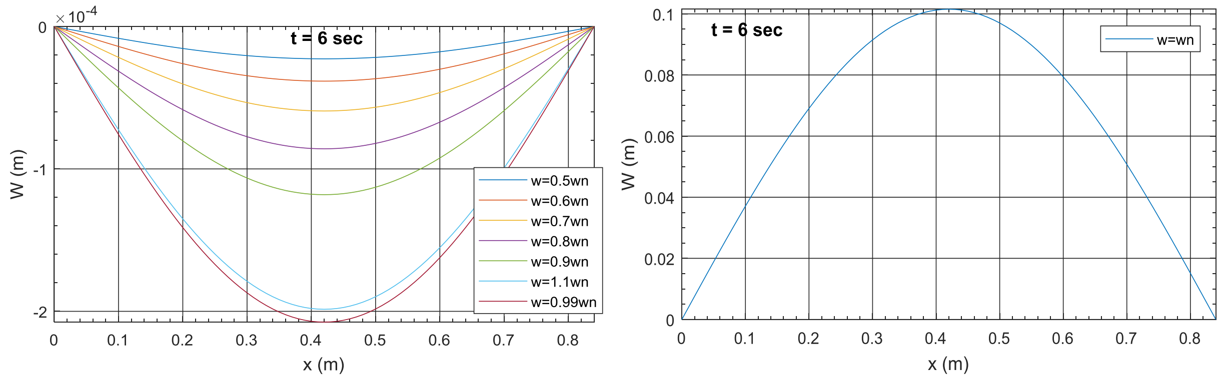


Figure 12. The deflection with time and length of the simply supported beam.



a. various applied load frequency b. $\omega = \omega_n$.

Figure 13. The simply supported response of beam with length.

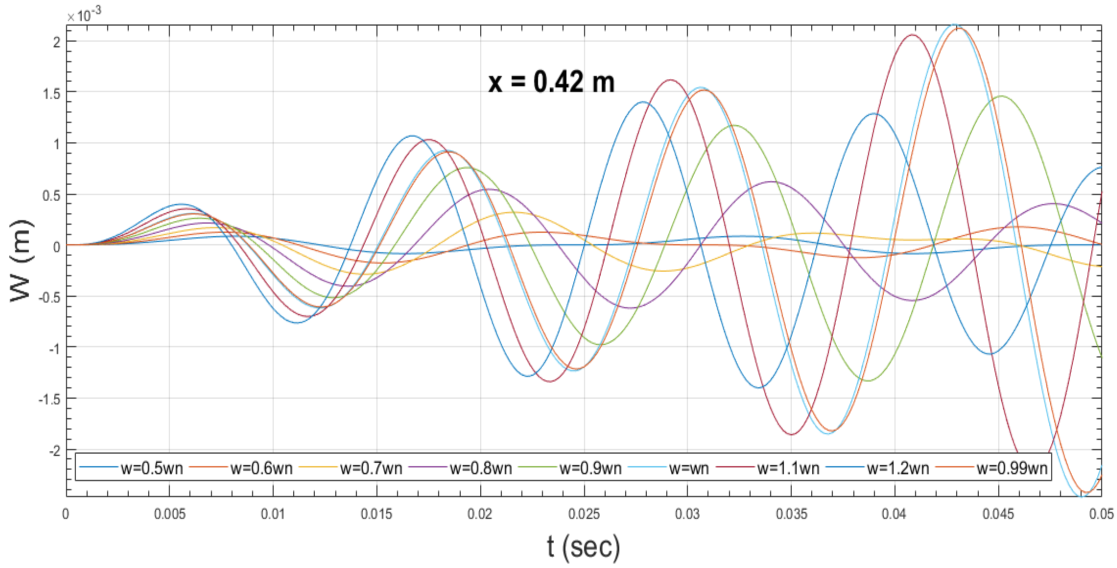


Figure 14. The simply supported Beam Response with time and various frequency force applied.

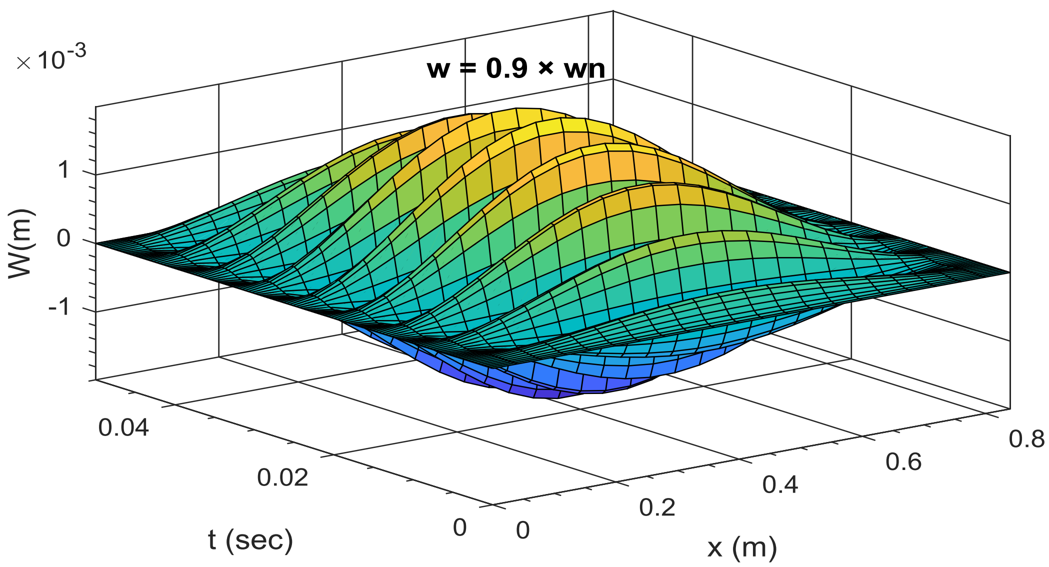
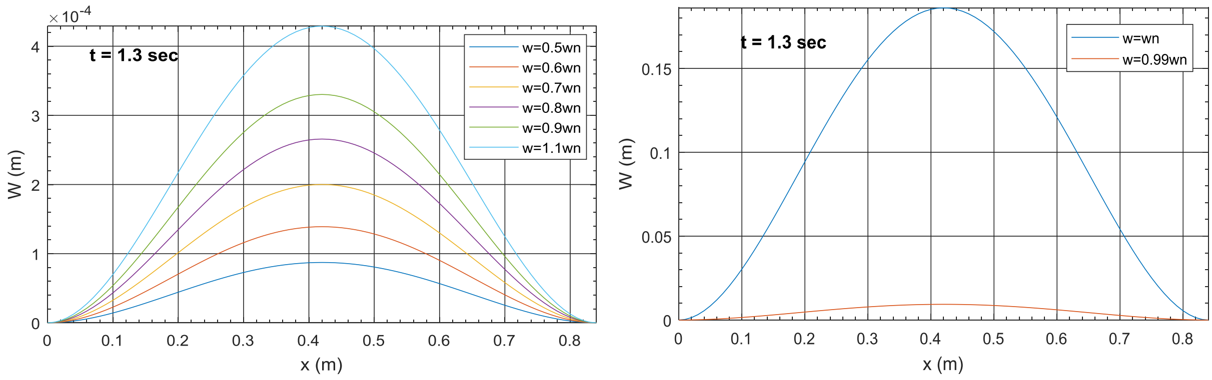


Figure 15. The deflection with time and length of the clamped beam.



a. various applied load frequency b. ω = (0.99, 1) ω_n

Figure 16. The deflection with length of the clamped beam and various frequency load applied.

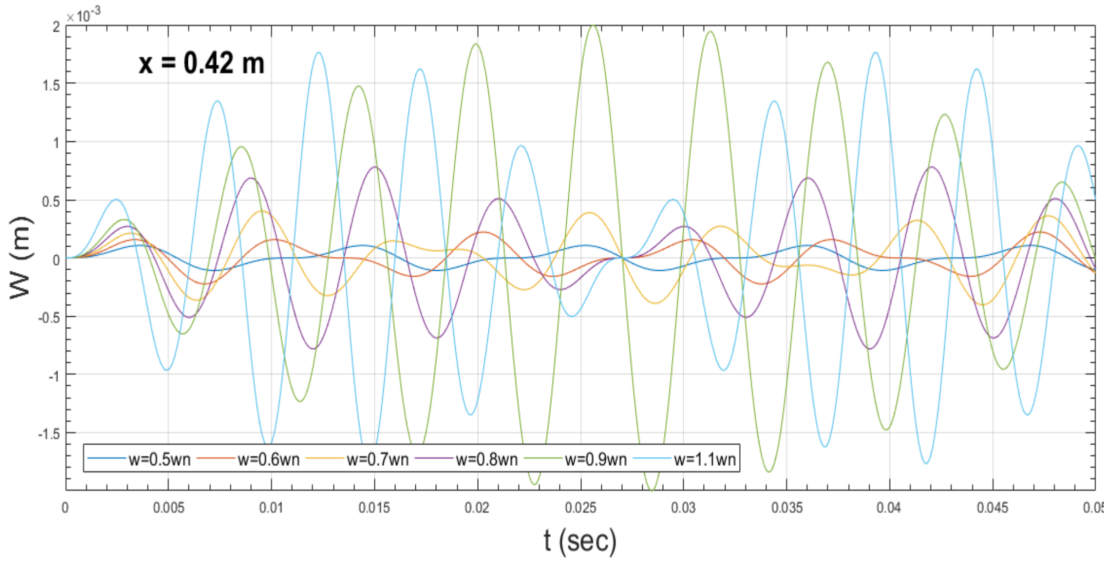


Figure 17. The clamped Beam Response with time and various frequency force applied.

4.3 Beam Response with Crack Effect

The amount of the response deflection in the assumed beam is calculated analytically during the vibration of the beam with crack under the influence of harmonic force at constant frequency ($\omega = 0.9 \times \omega_n$) this frequency is chosen for the appearance of response deflection clearly approaching the resonance state. Sometimes the response deflection is calculated for the existence of the crack to different depths and locations on the beam for each location along the length of the beam on demonstrated constant time at other times it is calculated with time in a location for several cases of supported, as follows,

4.3.1 The response deflection for cantilever beam

Fig. 18, shows the amount of the response deflection in the cantilever beam changes according to the time and length of the beam with crack ($L_1 = 0.5\ L = 0.42\ m;\ d_c = 0.5\ h = 1.25\ cm$). Figs. 19 and 20, shown the relationship between the response deflection for different depths of the crack dc = (0.5, 1, 1.25 and 1.5) cm and crack location (L1 = 0.2, 0.42, 0.64 m) with the length of the beam at constant time. In addition, in Figs. 21 and 22, shown same effect for crack parameters with various time.

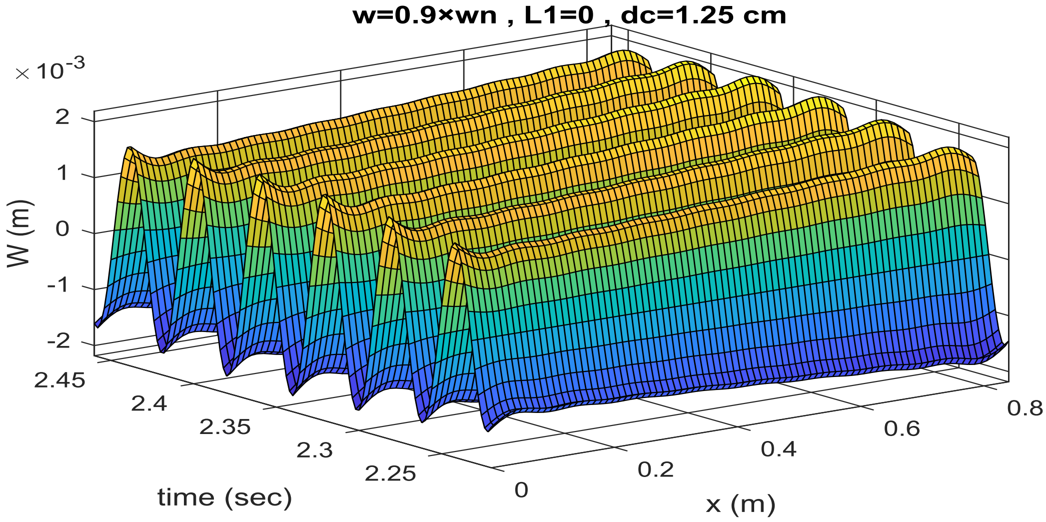
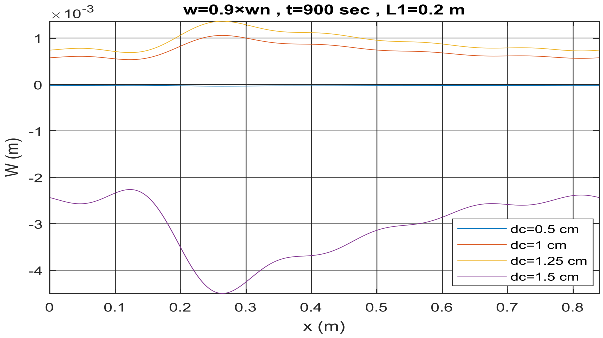
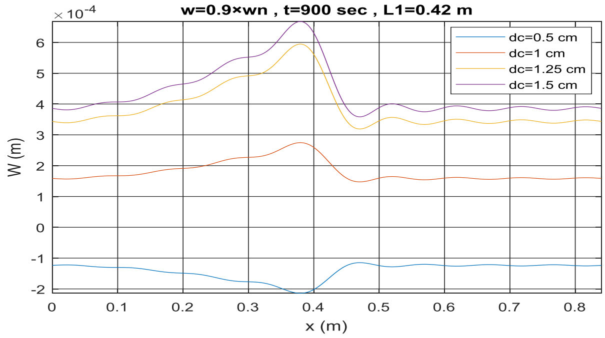


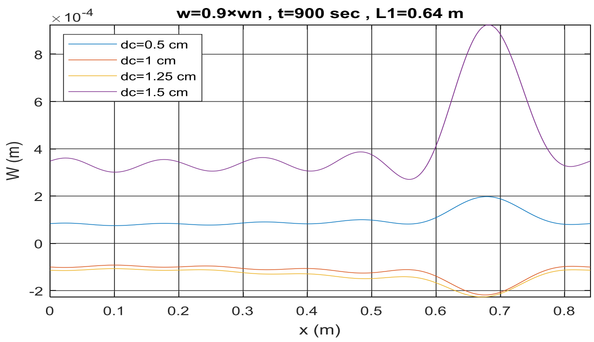
Figure 18. The deflection with time and length of the cantilever beam with crack effect.



a. Crack location L_1=0.2 m

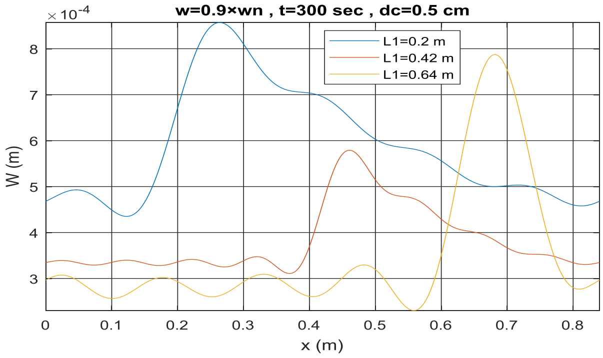


b. Crack location L_1=0.42 m

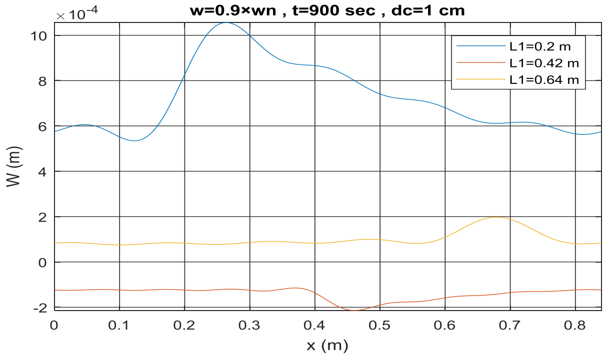


c. Crack location L_1=0.64 m

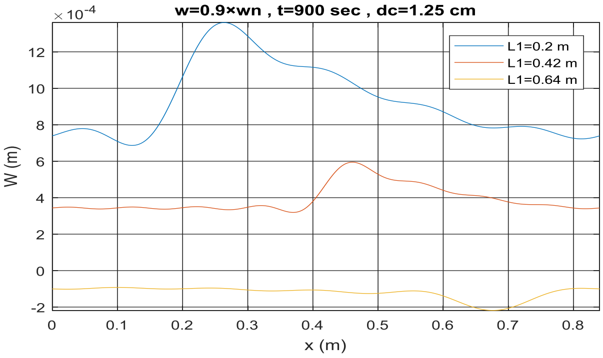
Figure 19. Cantilever Beam Response with Different Crack Position Effect.



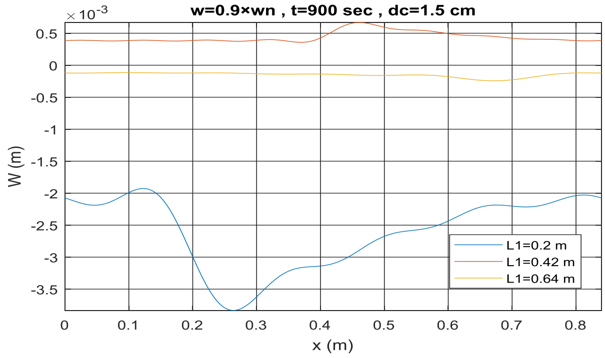
a. Crack Depth d_c=0.5 cm



b. Crack location d_c=1 cm

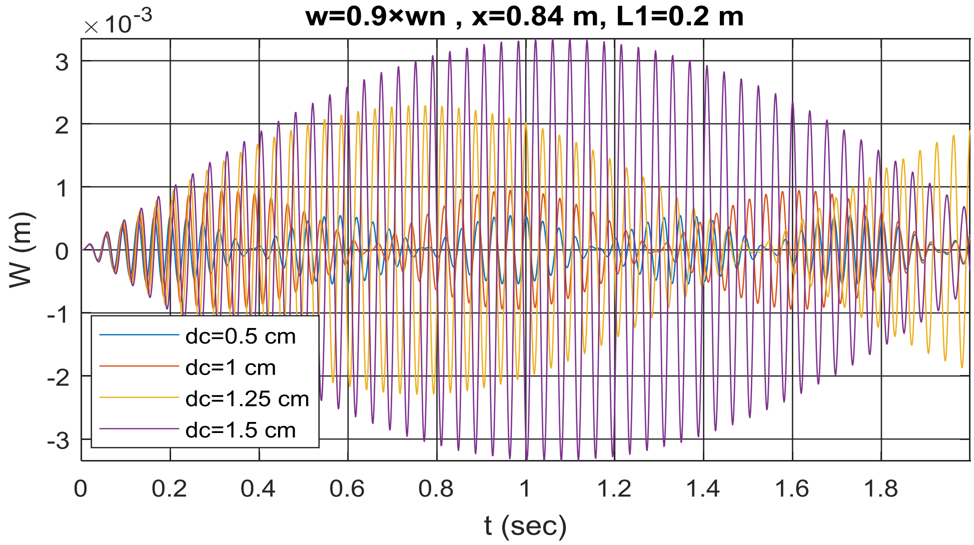


c. Crack Depth d_c=1.25 cm

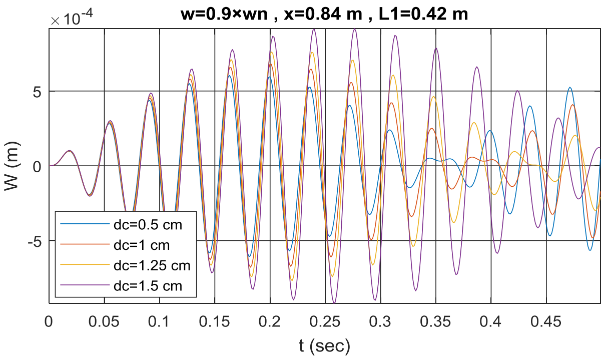


d. Crack location d_c=1.5 cm

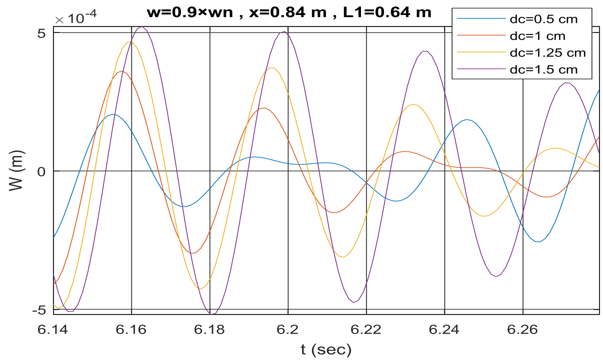
Figure 20. Cantilever Beam Response with Different Crack Depth Effect.



a. Crack location L_1=0.2 m

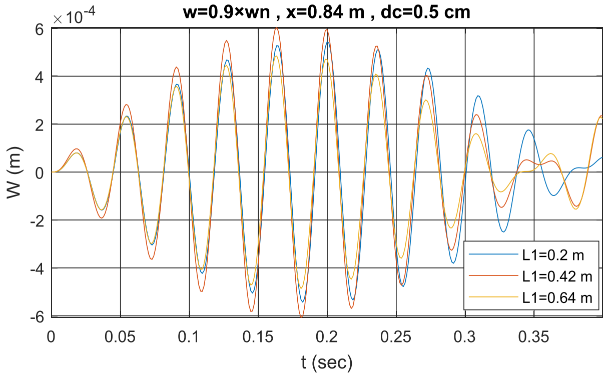


b. Crack location L_1=0.42 m

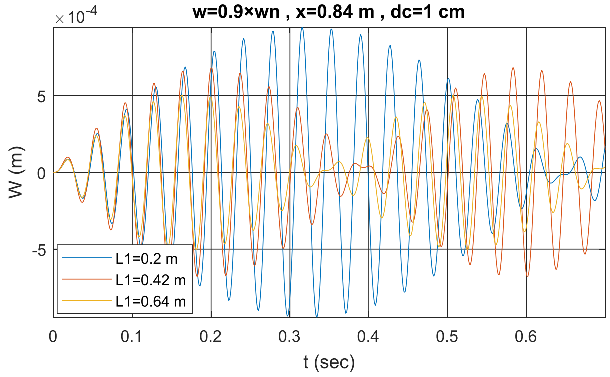


c. Crack location L_1=0.64 m

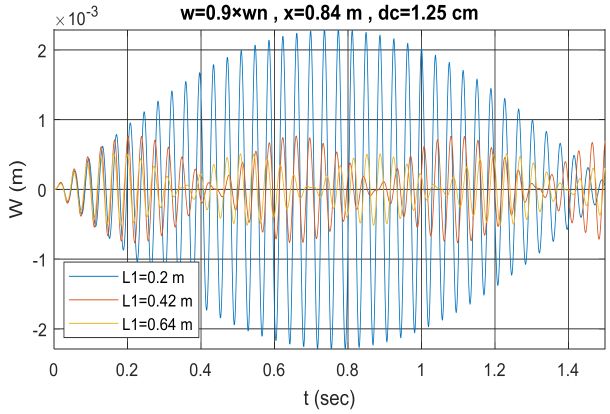
Figure 21. Cantilever Beam Response with Different Crack Position Effect at Various Time.



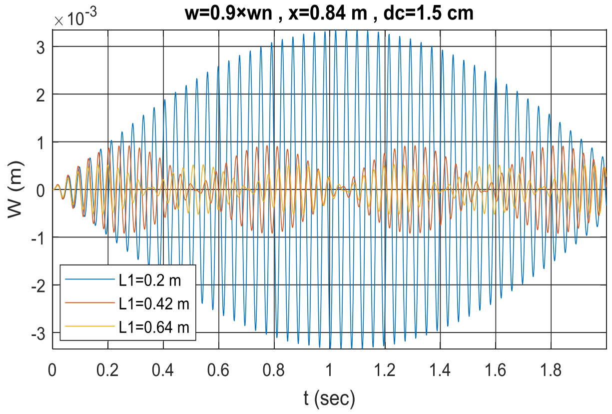
a. Crack Depth d_c=0.5 cm



b. Crack location d_c=1 cm



c. Crack Depth d_c=1.25 cm



d. Crack location d_c=1.5 cm

Figure 22. Cantilever Beam Response with Different Crack Depth Effect Various Time.

4.3.2 The response deflection for simply supported beam

Fig. 23, shows the amount of the response deflection in the simply supported beam changes according to the time and length of the beam with crack (L_1 = 0.5 L = 0.42 m; d_c = 0.5 h = 1.25 cm). Figs. 24 and 25, shown the relationship between the response deflection for different depths of the crack dc = (0.5, 1, 1.25 and 1.5) cm and crack location (L_1 = 0.2, 0.42) with the length of the beam at constant time. In addition, in Figs. 26 and 27, shown same effect for crack parameters with various time.

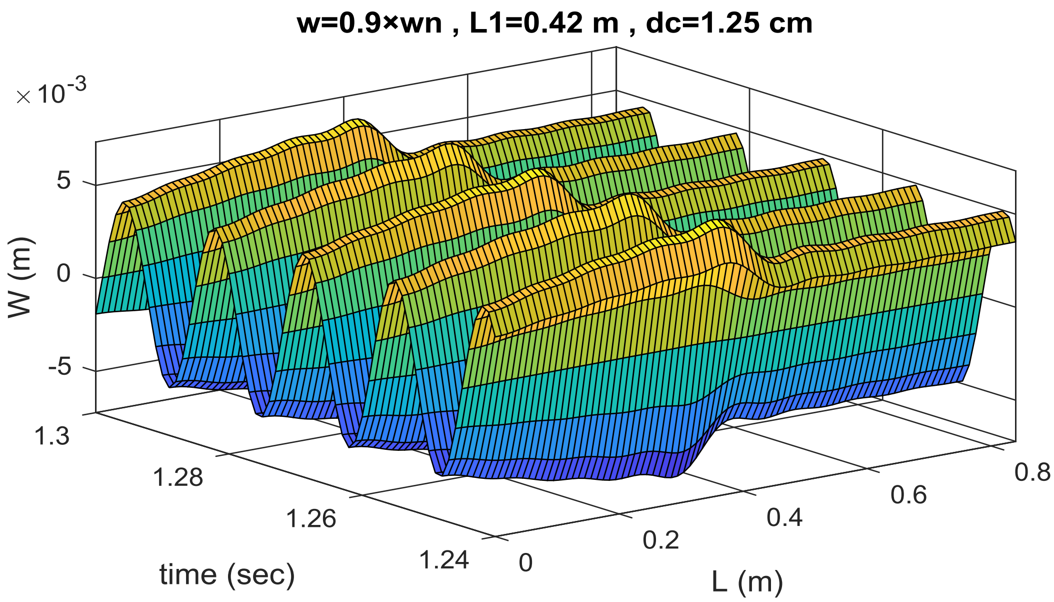
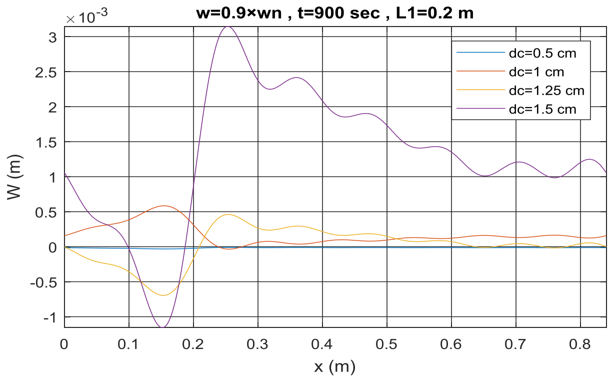
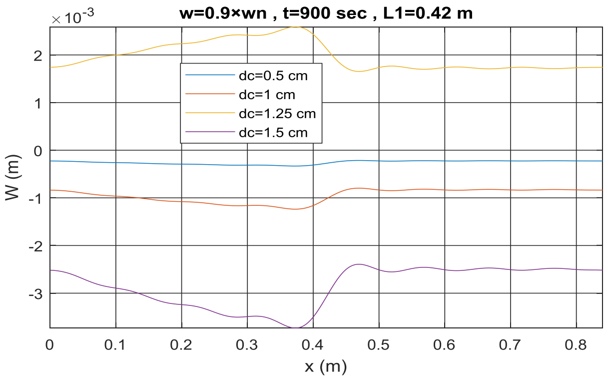


Figure 23. The deflection with time and length of the simply supported beam with crack effect.

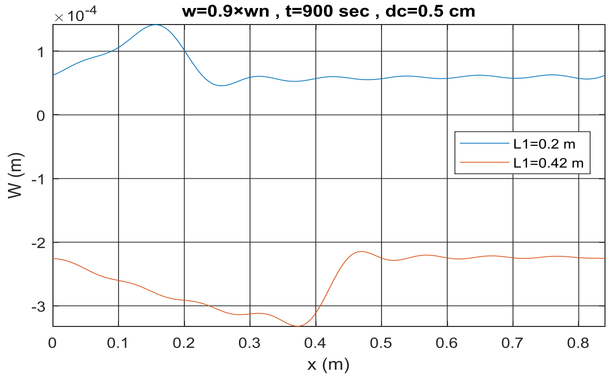


a. Crack location L_1=0.2 m

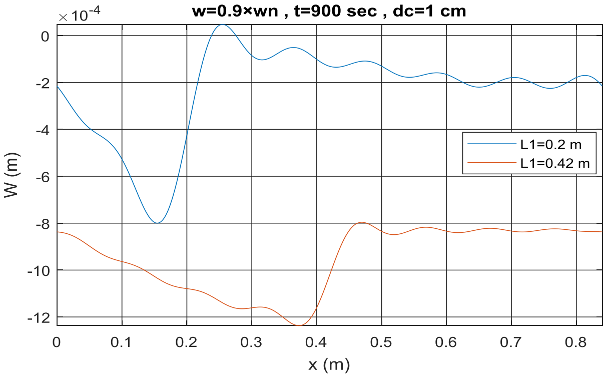


b. Crack location L_1=0.42 m

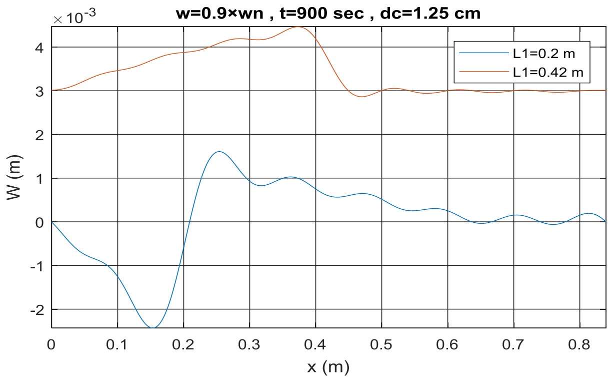
Figure 24. Simply Supported Beam Response with Different Crack Position Effect.



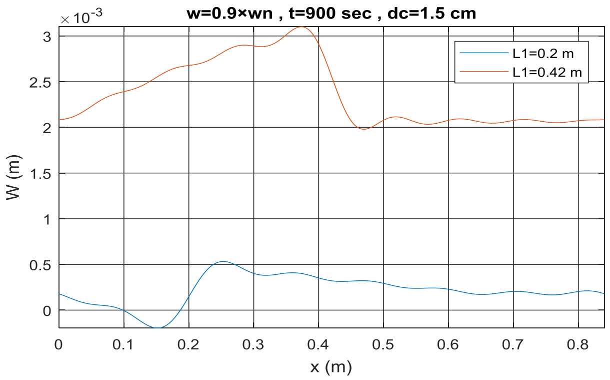
a. Crack Depth d_c=0.5 cm



b. Crack location d_c=1 cm

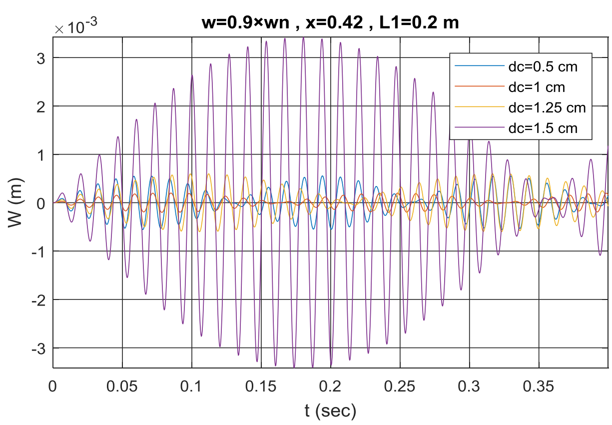


c. Crack Depth d_c=1.25 cm

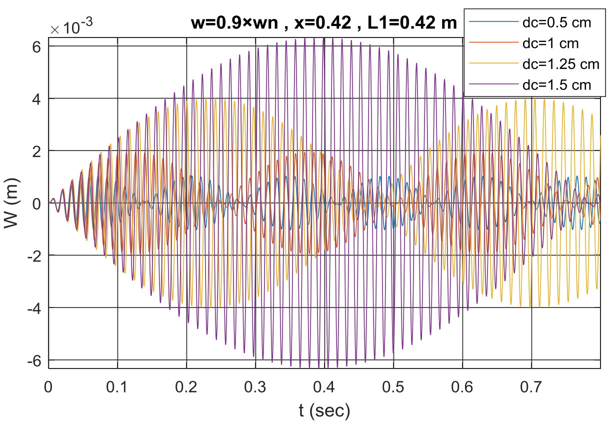


d. Crack location d_c=1.5 cm

Figure 25. Simply Supported Beam Response with Different Crack Depth Effect.

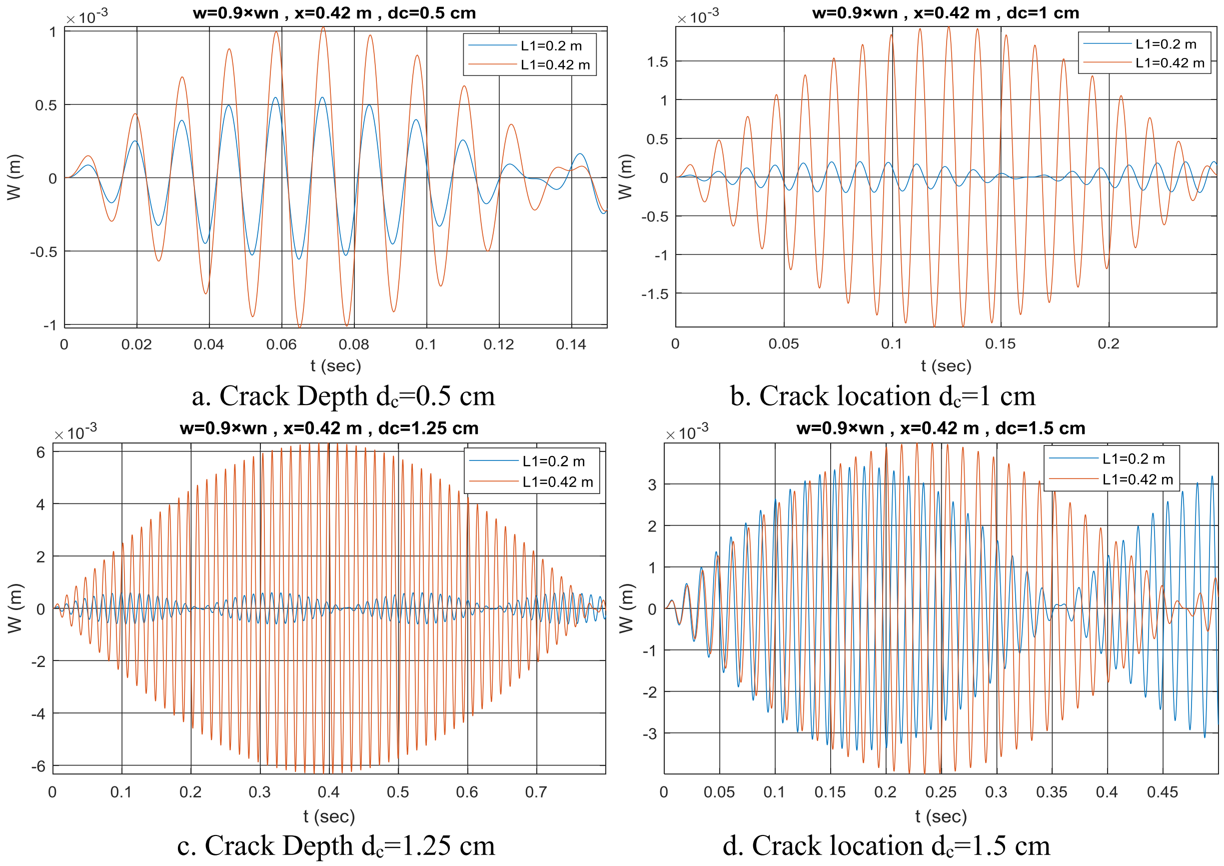


a. Crack location L_1=0.2 m



b. Crack location L_1=0.42 m

Figure 26. Simply Supported Beam Response with Different Crack Position Effect at Various Time.



a. Crack Depth d_c=0.5 cm

b. Crack location d_c=1 cm

c. Crack Depth d_c=1.25 cm

d. Crack location d_c=1.5 cm

Figure 27. Simply Supported Beam Response with Different Crack Depth Effect Various Time.

4.3.3 The response deflection for clamped beam

Fig. 28, shows the amount of the response deflection in the clamped beam changes according to the time and length of the beam with crack ($L_1 = 0.5\ L = 0.42$ m, $d_c = 0.5\ h = 1.25$ cm). Figs. 29 and 30, shown the relationship between the response deflection for different depths of the crack dc = (0.5, 1, 1.25 and 1.5) cm and crack location (L1 = 0.2, 0.42 m) with the length of the beam at constant time. In addition, in Figs. 31 and 32, shown same effect for crack parameters with various time.

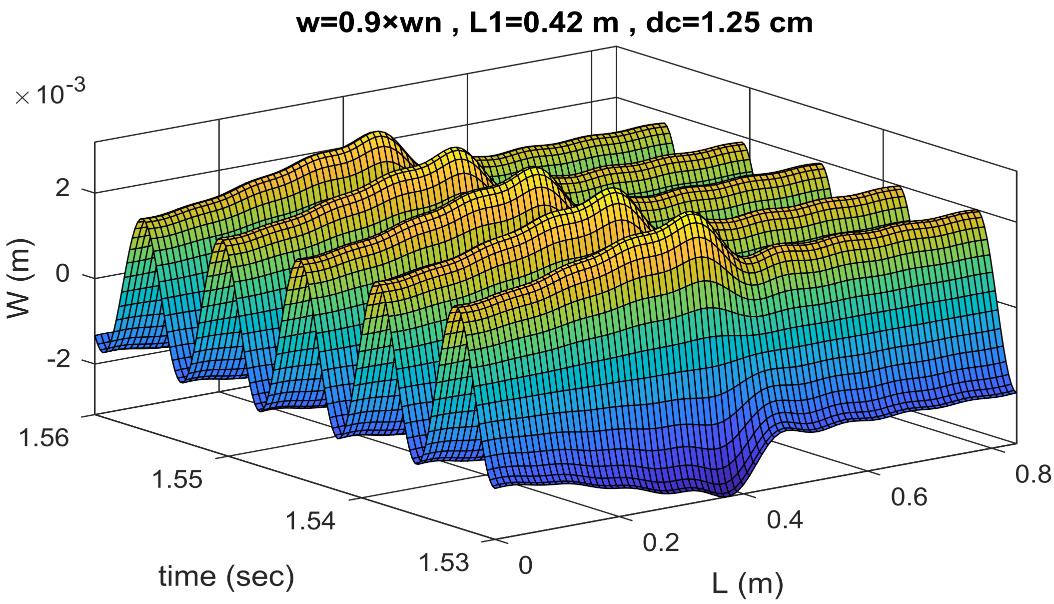
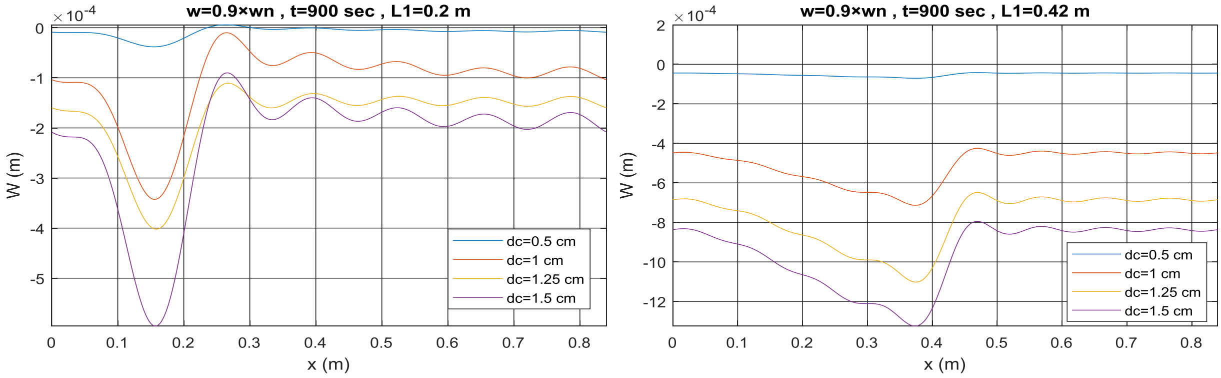
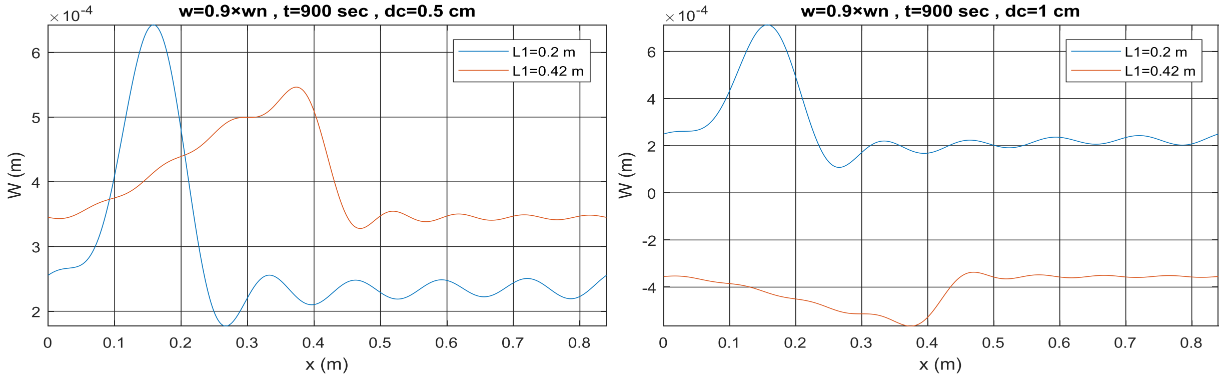


Figure 28. The deflection with time and length of the clamped supported beam with crack effect.

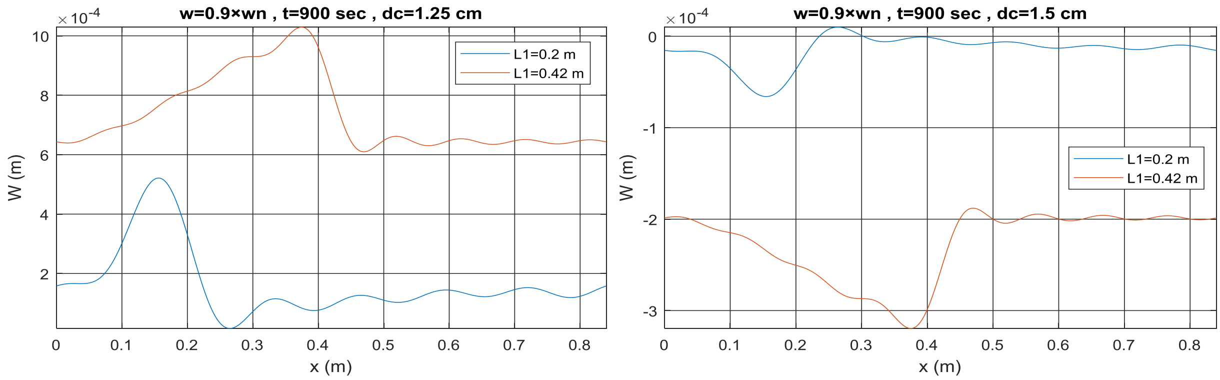


a. Crack location L_1=0.2 m b. Crack location L_1=0.42 m

Figure 29. Clamped Supported Beam Response with Different Crack Position Effect.

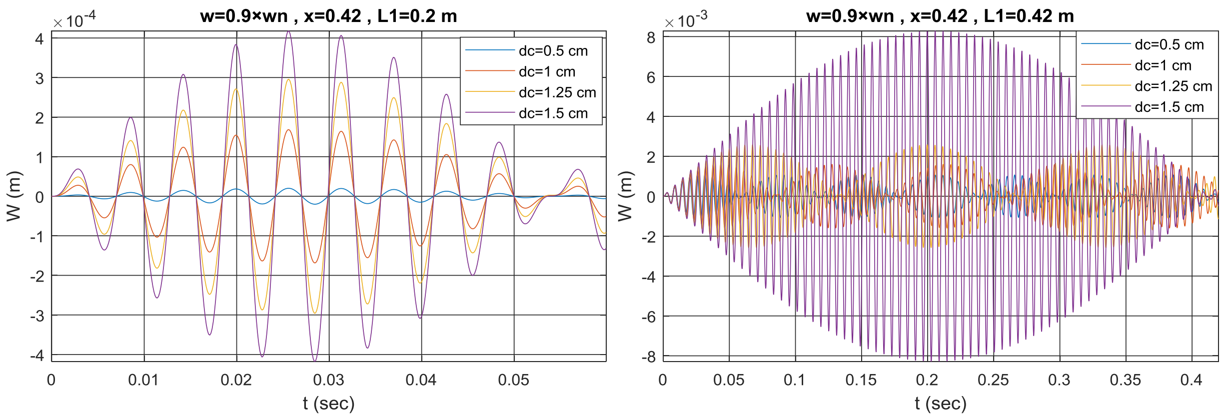


a. Crack Depth d_c=0.5 cm b. Crack location d_c=1 cm



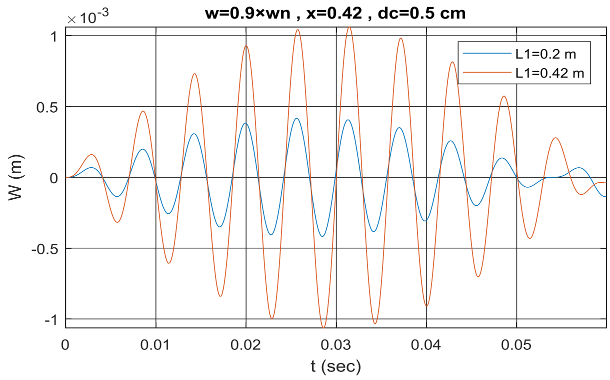
c. Crack Depth d_c=1.25 cm d. Crack location d_c=1.5 cm

Figure 30. Clamped Supported Beam Response with Different Crack Depth Effect.

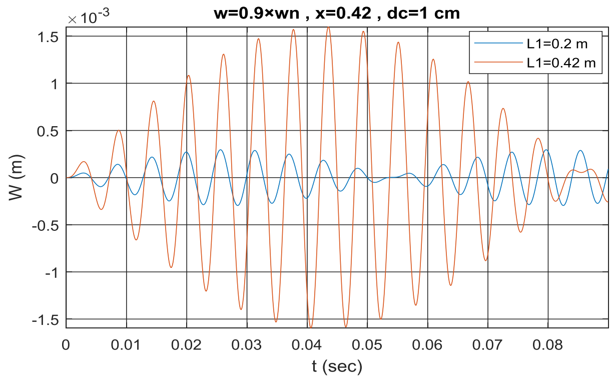


a. Crack location L_1=0.2 m b. Crack location L_1=0.42 m

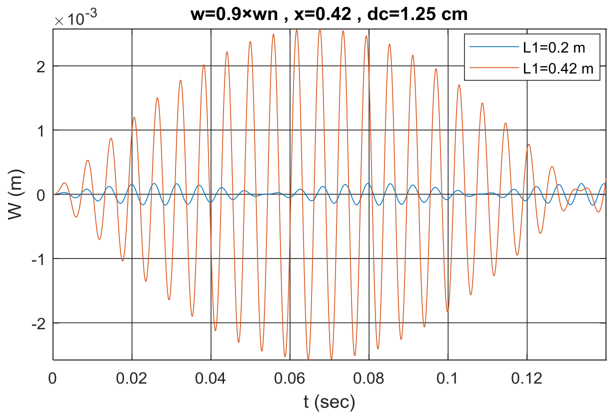
Figure 31. Clamped Supported Beam Response with Different Crack Position Effect at Various Time.



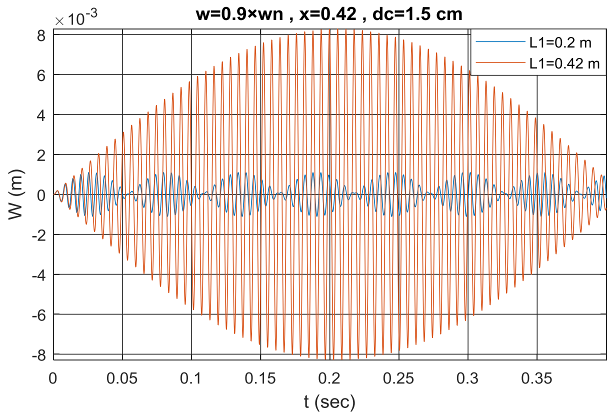
a. Crack Depth d_c=0.5 cm



b. Crack location d_c=1 cm



c. Crack Depth d_c=1.25 cm

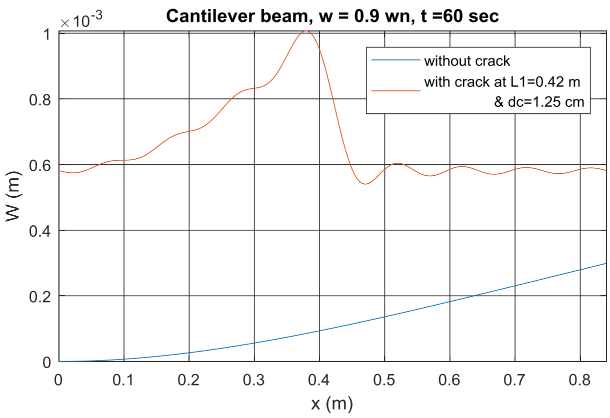


d. Crack location d_c=1.5 cm

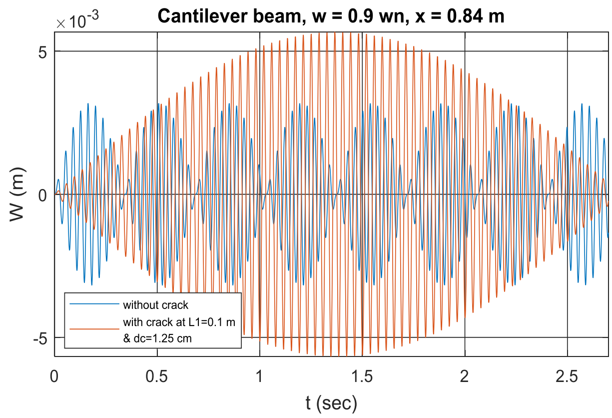
Figure 32. Clamped Supported Beam Response with Different Crack Depth Effect Various Time.

4.4 The comparison of beam response with and without crack effect

Fig. 33, shown the relationship between the response deflection of cantilever beam supported at constant frequency with the length of the beam is explained at constant time, as in Fig. 33.a or the time at (x = 0.84 m) as in Fig. 33.b, and the relationship between the response deflection of simply supported beam at constant frequency with the length of the beam is explained at constant time as shown in Fig, 34.a or the time at (x = 0.42 m) as shown in Fig. 34.b. In addition, the relationship between the response deflection of clamped beam supported at constant frequency with the length of the beam is explained at constant time as shown in Fig. 35.a, or the time at (x = 0.42 m) as shown in Fig. 35.b.

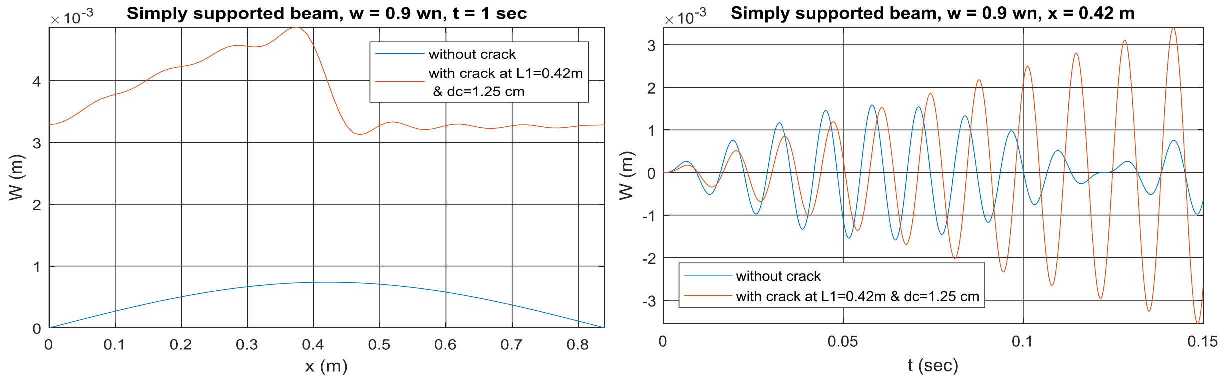


a. constant time



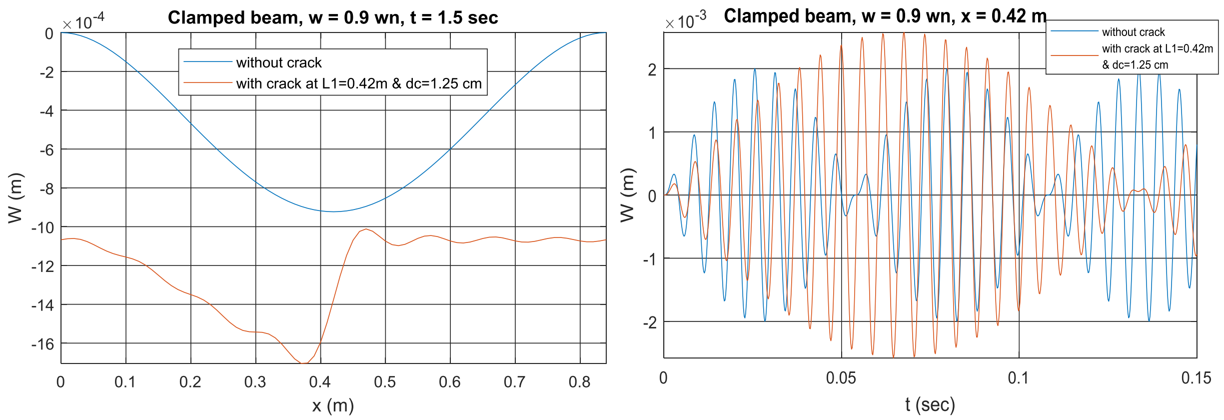
b. constant location

Figure 33. Comparison of Cantilever Beam Response with and without Crack Effect.



a. constant time b. constant location

Figure 34. Comparison of Simply Supported Beam Response with and without Crack Effect.



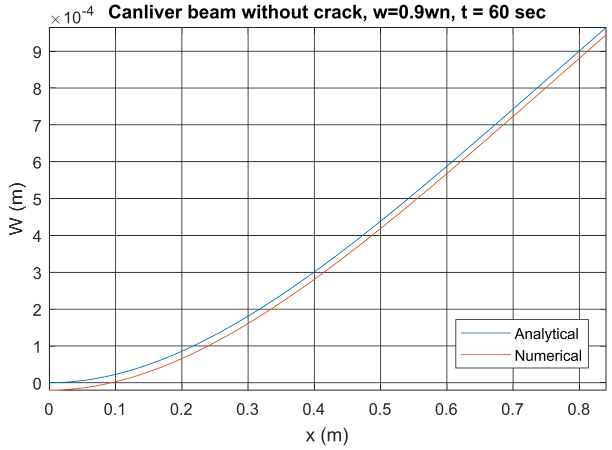
a. constant time b. constant location

Figure 35. Comparison of Clamped Beam Response with and without Crack Effect.

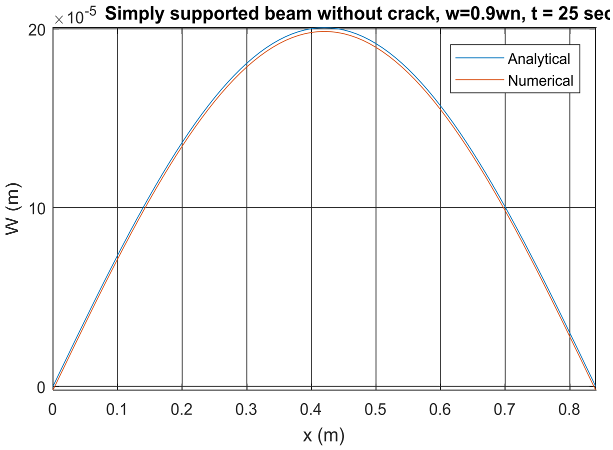
Then, from Fig. 9 to 17, can be shown the effect of frequency load applied on the beam response, with various supported effect. There, the deflection response for beam increase with increasing the frequency load applied, then, the maximum response occurred at the load frequency equal to natural frequency of beam. Finally, from Figs. 18 to 35, can be shown that the crack lead to decrease for the beam stiffness, therefore, the natural frequency decreasing, in addition, due to decrease both the beam stiffness and beam natural frequency, then, the beam response deflection increasing with crack. Thus, beam response increase with increase the crack depth and increasing with crack location occurred near the maximum beam moment position. Then, from the results presented can be shown that the frequency load applied decrease with crack, then, the resonance for beam with crack occurred at load frequency less than load frequency for beam without crack effect, due to decrease natural frequency for beam with crack effect as shown in Tables 1 and 2, with different beam supported.

4.5 The Comparison between Analytical and Numerical Solution

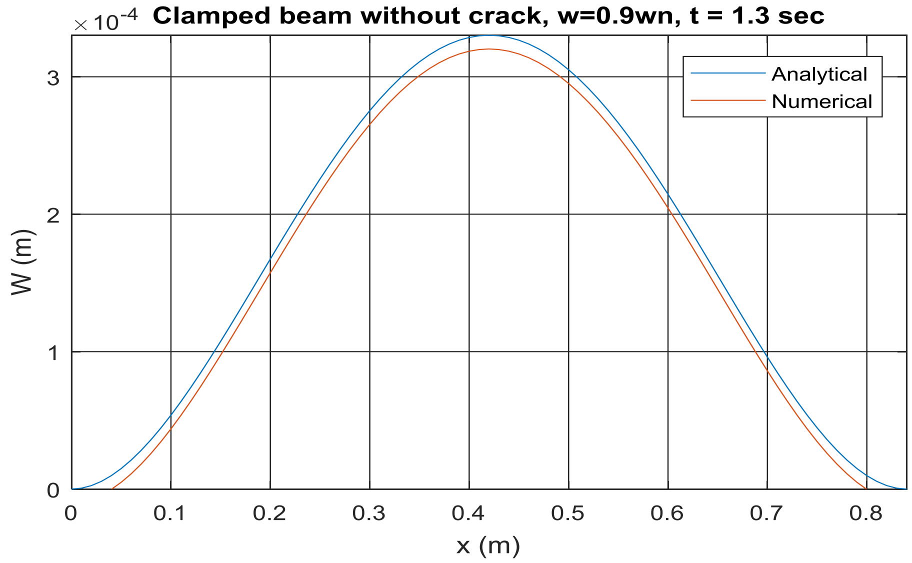
The comparison between analytical and numerical results included comparison for beam response results calculated by drive the general equation of motion for different beam supported, and results for beam response calculated by numerical technique with using finite element technique, to given the agreement for analytical solution for beam response with and without crack effect, [46-48]. Therefore, Fig. 36.a, shown the relationship between the response deflection with the length of the cantilever beam is explained for the analytical solution and the numerical solution at constant time and constant frequency, Fig. 36.b, shown the comparison between analytical and numerical results of simply supported beam deflection, and Fig. 36.c, shown the comparison for clamped beam response results calculated by analytical and numerical technique. Therefore, from Fig. 36 can be see that the maximum error for result evaluated by two techniques used did not exceed about (1.46%), then, from this results can be dependent on the analytical solution for different supported beam with various crack effect.



a. Cantilever Beam Supported



b. Simply Supported Beam



c. Clamped Beam Supported

Figure 36. Comparison between Analytical and Numerical Results for Beam Response.

5. Conclusions

The investigation included calculated the natural frequency and response deflection with various crack depth and location effect, for beam supported with various boundary conditions. Where, the investigation included used analytical and comparison with numerical technique. Therefore, form the presented results can be conclusion the following important points as,

1. The analytical solution for general equation of motion for beam with crack effect is perfect technique can be used to calculate the natural frequency and beam response with various effect of crack and different boundary condition supported beam subjected to harmonic load.
2. The comparison for analytical results, evaluated by solution of general equation of motion, with numerical results, calculated by using finite element technique, given a good discrepancy with maximum error did not exceed about (1.46%).
3. The crack defect lead to decrease the beam stiffness, then, crack defect lead to decrease the natural frequency for beam structure and increasing the vibration response for beam.
4. The effect of load frequency increase with the value near to the beam frequency, therefore, the value for maximum effect of load frequency was decrease with increase the effect of crack defect. Then, the effect for load frequency increasing with increasing crack effect.
5. The maximum effect for crack defect occur at the location for beam with maximum bending moment. Then, the maximum effect for crack location of simply supported beam occur at the middle section for beam, and the maximum effect for the crack occur at the supported section for cantilever beam.

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