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Analytical investigation for crack effect on unstable pipe conveying fluid

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Abstract
Pipe applications are important and widely used applications in all industrial fields because pipes are used in many fields such as water, oil, gas, etc. It was therefore necessary to highlight the study of the pipes and the problems faced by them. In this study, the problem of controlling the vibration produced by the flow of fluids within the pipes was discussed. The research included studying the amount of stability of the pipes, in addition to, the effect of crack was investigated on the level of stability Confidentiality and vibration of the pipes in various regions and dimension the crack and depth of the crack. The results of this study were obtained by deriving the differential equations of the pipes with crack and without crack and for the different types of fixation (simply supported, cantilever, and fixed from one edge and spring and damper from other edge). Through the derivation of the differential equations of the movement and dealing with them for the purpose of reaching the general equation so as to facilitate conversion within the state space and analysis of equations. The final equations are programmed and the results are found for the pipes. Vibration control is performed by increasing the pressures through the differential pressure connect to the system. To conclude of the results show that the addition of damping vibration increase in the stability of the pipes during the flow. In addition, the adoption of the frequency (imaginary), which indicates the stability of the pipes where the occurrence of the lead leads to a decrease in frequency and this, leads to increased instability of the pipes with crack. It appears that the increasing stability by adding damping vibrations to the pipe network.

Keywords: Control pipe; Unstable pipe; Crack pipe; Analytical vibration pipe.

1. Introduction
Circular and noncircular fluid flow in pipes is frequently encountered in practice. The cold and hot water used in our houses are pumped in through the pipes. Tap water in a city spreads out through extensive networks of piping. By large pipelines natural gas and oil flow around hundreds of miles' flow in the same way that blood is carried throughout our bodies by veins and arteries. Similarity, the cooling water in a machine or an engine flows by hoses into the pipes where it is cooled as it flows in the radiator. Fluid flow could be either internal or external, according to whether the fluid is enforced to flow in a conduit or over a surface. External and internal flows demonstrate very different properties. By passing the fluid inside the pipes, the following effects may appear. All or some effects can be achieved depending on the state variable [1],

1. Effect of pressure: It is produced by the difference in fluid pressure inside the pipe. This effect can be shown as a static compressive force working transversely on the pipe.
2. Effect of centrifugal force: These effects of the pipe curvature which cause change in the momentum of the fluid.
3. Effects of Coriolis force: this effect comes from rotation liquid element when translates horizontally to change the movement of the pipe. In this situation, the liquid acts as a damping technique. Within certain limits of the speed of this fluid damping is positive and tends to decay vibration. At huge speeds, the damping becomes negative and increases the oscillation leading to significant growth in capacitance, so the pipe loses its stability by another type of sensitivity called flutter.

Vibration is defined as a continuous from -and -to motion from a position of equilibrium. In a piping system, there are many reasons that lead to vibration. Few basic reasons could cause the vibration. There is a variety of mechanisms of excitation which can exist in a piping system that could lead to vibration and eventually fail due to fatigue. Some of these causes are, as each type of flow has a mechanism in vibration and the following Figure 1, shows the vibration mechanism,

1. Vibration induced by flow: appear from the flowing fluid.
2. Equipment mechanical forces: caused by the forces of excitation of rotary and reciprocating equipment like compressors, pumps, etc.
3. Pulsations of pressure of reciprocating equipment.
4. Excitations of high Acoustic frequency generated by valves of relief, orifice plates or control valves.
5. Momentum changes or water hammer (Surge) due to closure of sudden valve.
6. Cavitation or collapse of vapor bubble via pressure localized drop.
7. Because of sudden fluid flashing.
8. Disturbances of periodic pressure during a flow passing the branch dead end instrumental/connection items.

There are number of researchers who study the situation and reach a general equation for the movement of the fluid inside the pipe. At the same time there are researchers, who study crack effects extensively, R.T. Faal et. al, 2011, [2], the basic conclusions of this paper can be listed as the first natural frequencies of dimensionless coupled system are augmented by stiffness growth of elastic support, pipe rigidity and by decreasing internal density of fluid and velocity, diameter of inner pipe (with continuous thickness) length of beam and the second in the analysis of stability of pipe vibration where the parameter β plays a significant role. After these, M. J. Jweeg et. al., 2015, [3], one of the problems is the active control of the parametric resonance. To overcome it, units or elements of active control are designed under the barometric resonance. The stability of the pipes is determined primarily by the velocity of the liquid. The linear part is designed as a controller based on the linear quadratic theory and then the work of numerical control unit is examined. Also, at same year, M. J. Jweeg et. al., 2015, [4], using different fixations at a different speed in the pipe experiments to offer an alternative analysis of the dynamic behavior and stability of the Galerkin-based fluid transfer pipes to find natural frequencies. A special instrument is used for this purpose to verify the results and a numerical technique using FEM was used on ANSYS. The experimental results show a satisfactory agreement with those obtained from analytical and numerical methods.

M. Al-Waily et. al., 2017, [5], investigated the effect of the rack angle and flow velocity on the vibration characterizations of pipe. There, the investigation included calculated the natural frequency for the pipe with various crack angle and velocity flow by using numerical and experimental techniques. Then, M. J. Jweeg et. al., 2018, [6] this research investigates the effect of cracks into a simply supported pipe conveying fluid on the frequency and the response of the pipe. A laminar flow established according to Reynold's number calculation between 500 to 1500 with a flow velocity 1 m/sec. The MATLAB program is used to solve equations and using the finite element method (FEM) adopting COMSOL 5.2 program to verify the analytical results. And then it turns out that the increase in the size of crack decrease the frequency and the position of the crack into pipe has an effect on the frequency value of the pipe. This value becomes smaller when the crack position gets closer to the middle position. The developed analytical solution is powerful and cheap solution in dealing with the prediction of frequency for cracked and un-cracked pipes conveying fluid. The comparison of the results has shown a maximum percentage of discrepancy between the developed solution and the numerical prediction is (5.53%).

At 2019 to 2020, [7-11], D. S. Hussein and M. Al-Waily, studied the control and pipe stability for pipe with different pipe supported and used various stability techniques. There, the state-space, bode diagram, Nyquist's, and root locus were used to evaluated the control stability for pipe induced vibration, in

addition to, used the experimental techniques to calculate the natural frequency and pipe response with various flow parameters effect.
From the above-mentioned literature, it is obvious that a considerable amount of research in this area has been done in recent years. All these researchers provided a basic understanding of the vibrations and control that occur in the pipeline systems used in different fields. In this work, the theoretical study includes the derivation of the general equation of the pipe and applies the boundary condition various fixes (Simply support, cantilever, fixed with spring damper) with no crack and with a crack at different locations and different crack dimensions (depth and position). For find values response and natural frequency values for each of the installation cases, which were mentioned.

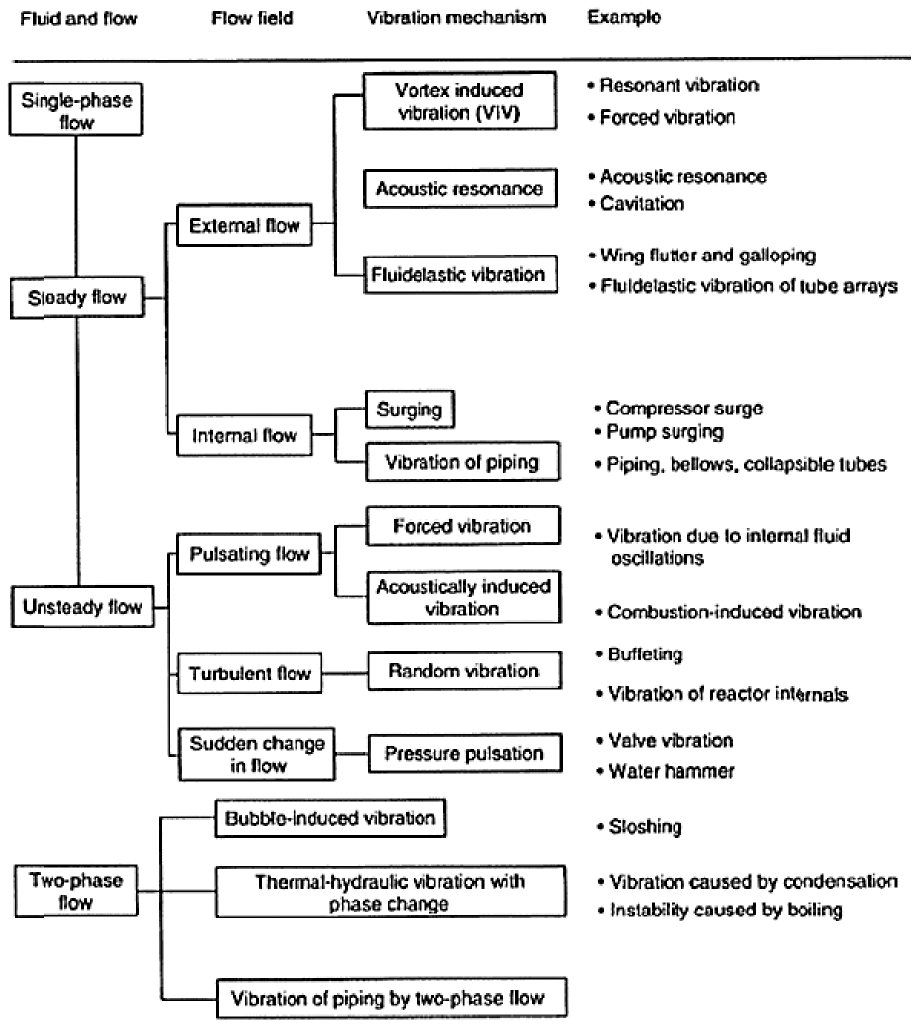


Figure 1. Vibration Mechanism.

2. Control System

A control system manages, commands, regulates, or directs the conduct of other systems or devices by means control loops. Control Systems can be classified as open loop control and closed loop control. In the system of open-loop control, the control action from the controller is self-governing of the variable of process. Central heating boiler that is only controlled by a timer is an example of this. The on/off switching is the control action. The variable of process is the building temperature. Heat is applied in this controller for a constant time irrespective with the building temperature, [12]. In a system of closed-loop control, the action of control from the controller is reliant on the actual and desired variable of process. In the analogy of the boiler, this could use a thermostat for monitoring the temperature of building, and give a signal as feedback to make sure that the controller output keeps the temperature of building to the temperature set on thermostat device. A controller of closed loop has a feedback loop that makes sure the controller exerts an action of control to control a variable of process at the same value of that as the set point. Therefore, controllers of closed-loop are also known as feedback controllers, [12].

2.1. State Space Control System

The state space model of Linear Time-Invariant (LTI) system can be represented as,

$$\dot{x} = Ax + Bu$$
$$y = cx + Du \qquad (1)$$

The first and the second formulas are termed as state equation and output equation. Where,
x and $\dot{x}$ are referring to vector of the state vector and the differential state vector respectively.

U and Y are referring to vector of the input and output respectively.

A is the matrix for the system.

C and B are the matrices for the output and the input.

D represent the matrix of feed-forward.

When,

X :is the n dimensional state vector $x = \begin{bmatrix} x_1 \\ x_2 \\ . \\ . \\ x_n \end{bmatrix}$,

U: is the m dimensional input vector $= \begin{bmatrix} u_1 \\ u_2 \\ . \\ . \\ u_n \end{bmatrix}$.

A: is the n × n system matrix $A = \begin{bmatrix} a_{11} & a_{12} & . & a_{1n} \\ a_{21} & a_{22} & . & a_{2n} \\ . & . & . & . \\ . & . & . & . \\ a_{n1} & a_{n2} & . & a_{nm} \end{bmatrix}$

B: is the n × M control matrix $B = \begin{bmatrix} b_{11} & . & . & b_{1m} \\ b_{21} & . & . & b_{2m} \\ . & . & . & . \\ . & . & . & . \\ b_{n1} & . & . & b_{nm} \end{bmatrix}$

C: is the output matrix $B = [c_1 \quad c_2 \quad . \quad cn]$

State equation from transfer functions Consider the general differential equation,

$$\frac{d^n y}{dt^n} + a_{n-1}\frac{d^{n-1}y}{dt^{n-1}} + \cdots + a_1\frac{dy}{dt} + a_0 y = b_{n-1}\frac{d^{n-1}u}{dt^{n-1}} + \cdots + b_1\frac{du}{dt} + b_0 y \qquad (2)$$

This equation can be represented by the transfer function shown in figure 2.

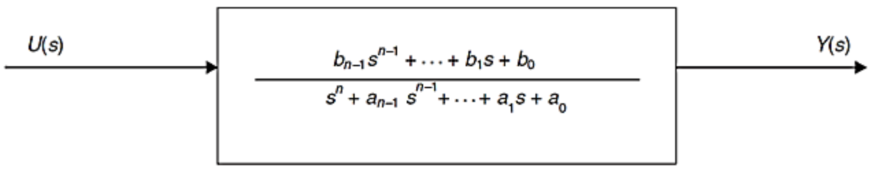


Figure 2. The transfer function.

And for conversion the transfer function to state space we follow the following by reaching an Eq. 3,
$x'_1 = x_2, x'_2 = x_3, x'_n = -a_0 x_1 - a_1 x_2 - \cdots - a_{n-1}x_n + u$

And the output equation,
$y = -b_0 x_1 - b_1 x_2 - \cdots - b_{n-1} x_n$
Then the state equation is,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_{n-1} \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & & & & \\ 0 & 0 & 0 & \cdots & 1 \\ -a_0 & -a_1 & -a_2 & \cdots & -a_{n-1} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} u \quad (3)$$

The state –space representation in The previous equation (3-29) is called the controllable canonical form and the output equation is,

$$y = \begin{bmatrix} b_0 & b_1 & b_2 & \cdots & b_{n-1} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix}, \text{ Here } D = 0.$$

Therefore, the state space parameters equation can be calculating by solution for general equation of motion for pipe with and without crack, for various boundary conditions, [13-25]. So, the solution for general equation of motion required applied for pipe boundary condition, [26-39], and then, calculate the pipe mode equation as a function of x-axis, [40-54], then subjected the mode pipe equation into general equation of motion for pipe with flow effect and using orthogonality technique to solve the natural frequency for pipe, [55-68], and finally subjected the general solution for equation of motion in Eq. 3, [69-78].

2.2. Pipe without Crack

For the purpose of reaching the state of space Eq. 3 requires the process of multiplying the equations that are derived for each case of installation (Simply support, cantilever, fixed with spring damper) taken in the y_x and then the integration process for the purpose of disposal of all existing X to the general equation for the purpose of easy conversion and be ready for use in the program to use the Matlab simulation and implementation of the concept of servo,

$$EI \frac{\partial^4 w}{\partial x^4} + 2m_f V \frac{\partial^2 w}{\partial t \partial x} + (m_f + m_p) \frac{\partial^2 w}{\partial t^2} = -(m_f V^2 + p_i A) \frac{\partial^2 w}{\partial x^2} \quad (4)$$

$$EI \frac{\partial^4 w}{\partial x^4} w_t + m_2 \frac{\partial^2 w}{\partial t \partial x} + m_3 w_x \frac{\partial^2 w}{\partial t^2} = -m_1 \frac{\partial^2 w}{\partial x^2} w_t \quad (5)$$

Where, $m_1 = (m_f V^2 + p_i A)$, $m_2 = 2m_f V$, $m_3 = (m_f + m_p)$
now,multiply by, (y_x),

$$EI \int_0^1 y_x \frac{\partial^4 w}{\partial x^4} w_t dx + m_2 \int_0^1 y_x \frac{\partial^2 w}{\partial t \partial x} dx + m_3 \int_0^1 w_x{}^2 \frac{\partial^2 w}{\partial t^2} dx = -m_1 \int_0^1 y_x \frac{\partial^2 w}{\partial x^2} w_t dx \quad (6)$$

$$O_1 w_t + O_2 \dot{w} + O_3 \ddot{w} = O_4 w_t$$
$$O_3 \ddot{w} + O_2 \dot{w} + (O_1 + O_4) w_t = 0 \text{ (general equation of motion for state space)} \quad (7)$$

Where,
$O_1 = EI \int_0^1 y_x \frac{\partial^4 w}{\partial x^4} dx$, $O_2 = m_2 \int_0^1 y_x \frac{\partial^2 w}{\partial t \partial x} dx$, $O_3 = m_3 \int_0^1 w_x{}^2 dx$, $O_4 = m_3 \int_0^1 w_x{}^2 \frac{\partial^2 y}{\partial t^2} dx$
O_1, O_2, O_3, O_4 have been extracted for each case of installation (Simply support, cantilever, fixed with spring damper) that has been taken and converted into a space state. The following equations can be used Eq. 8, for simply support and Eq. 9, for cantilever and Eq. 10, for fixed with spring damper to reach for eq. 7 and after that will convert easily to Eq. 3. Then, Eq. 7 similar to Eq. 2, therefore, analysis of its Equation, Eq. 7 by control analysis for stat space as shown in Eq. 3,

$$w(x) = A\left(e^{-iax}(\sinh b_1 x + \frac{S_1}{S_2}\cosh b_1 x) + e^{iax}(S_3 \sin b_2 x - \frac{S_1}{S_2}\cos b_2 x)\right) \quad (8)$$

Where, $S_1 = \left(e^{-ial}\sinh b_1 l - \frac{b_1}{b_2}e^{ial}\sin b_2 l\right)$, $S_2 = \left(e^{-ial}\cosh b_1 l - e^{ial}\cos b_2 l + \frac{b_2}{ia}e^{ial}\sin b_2 l\right)$
$S_3 = (\frac{S_1}{S_2}\frac{b_2}{ia} - \frac{b_1}{b_2})$, $S_4 = b_1^2 - a^2 - 2iab_1\frac{S_1}{S_2}$, $S_5 = \frac{S_1}{S_2}b_1^2 - a^2\frac{S_1}{S_2} - 2iab_1$,
$S_6 = 2\frac{S_1}{S_2}b_2 ia - S_3 b_2^2 - a^2 S_3$, $S_7 = 2iaS_3 b_2 + \frac{S_1}{S_2}b_2^2 + a^2\frac{S_1}{S_2}$
And,

$$w(x) = A\left(e^{-iax}\left(\sinh b_1 x + \frac{S_1}{S_2}\cosh b_1 x\right) + e^{iax}\left(S_3 \sin b_2 x - \frac{S_1}{S_2}\cos b_2 x\right)\right) \quad (9)$$

Where,
$S_1 = e^{-ial}\left((b_1^2 - a^2)\sinh b_1 l - 2iab_1\cosh b_1 l\right) + e^{ial}\left(\left(b_1 b_2 + \frac{b_1}{b_2}a^2\right)\sin b_2 l - 2iab_1\cos b_2 l\right)$
$S_2 = \left(e^{-ial}(2iab_1\sinh b_1 l - (b_1^2 - a^2)\cosh b_1 l) + e^{ial}\left(\left(\frac{2ia^3}{b_2}\right)\sin b_2 l - (b_2^2 - 3a^2)\cos b_2 l\right)\right)$
$S_3 = \left(\frac{S_1}{S_2}\frac{2ia}{b_2} - \frac{b_1}{b_2}\right)$, $S_4 = b_1^2 - a^2 - 2iab_1\frac{S_1}{S_2}$, $S_5 = \frac{S_1}{S_2}b_1^2 - a^2\frac{S_1}{S_2} - 2iab_1$,
$S_6 = 2\frac{S_1}{S_2}b_2 ia - S_3 b_2^2 - a^2 S_3$, $S_7 = 2iaS_3 b_2 + \frac{S_1}{S_2}b_2^2 + a^2\frac{S_1}{S_2}$
Also,

$$w(x) = A\left(e^{-iax}(\sinh b_1 x - \frac{S_1}{S_2}\cosh b_1 x) - (\frac{b_1}{b_2} + \frac{S_1}{S_2}\frac{2ia}{b_2})e^{iax}\sin b_2 x - \cos b_2 x\right) \quad (10)$$

Where, $s_1 = \left(e^{-iax}\begin{pmatrix} b_1^2\sinh b_1 x - iab_1\cosh b_1 x - \\ b_1 ia\cosh b_1 x - a^2\sinh b_1 x \end{pmatrix} - e^{iax}\begin{pmatrix} -b_1 b_2\sin b_2 x + b_1 ia\cos b_2 x + \\ iab_1\cos b_2 x - \frac{b_1}{b_2}a^2\sin b_2 x \end{pmatrix}\right)$

$s_2\left(e^{-iax}\begin{pmatrix} b_1^2\cosh b_1 x - iab_1\sinh b_1 x - \\ b_1 ia\sinh b_1 x - a^2\cosh b_1 x \end{pmatrix} + e^{iax}\begin{pmatrix} -iab_2\sin b_2 x + (b_2^2 - 2a^2)\cos b_2 x - \\ a^2\cos b_2 x + \left(iab_2 - \frac{2ia^3}{b_2}\right)\sin b_2 x \end{pmatrix}\right)$

$s_3 = \left(b_1{}^2 + a^2 - \frac{s_1}{s_2}(b_1 ia - b_2 ia)\right)$, $s_4 = \left(\frac{s_1}{s_2}(a^2 - b_1 b_2) - 2b_1 ia\right)$

$s_5 = \left(-\frac{b_1}{b_2}a^2 - b_1 b_2 - \frac{s_1}{s_2}\left(-2\frac{a^3}{b_2}i - 2b_2 ia\right)\right)$, $s_6 = \left(2b_1 i - a^2 + b_2{}^2\right)$

2.3. Pipe with Crack

Because of the crack, there are two equations y for each case (Simply support, cantilever, fixed with spring damper) because of the division of the pipe where we need to use Fourier series equations for the purpose of representing one equation, so A Fourier series is an infinite series expansion in terms of trigonometric functions,

$$f(x) = y_x = a_0 + \sum_{n=1}^{\infty}(a_n\cos(nx) + b_n\sin(nx)) \quad (11)$$

Where,
$a_0 = \frac{1}{2\pi}\left(\int_0^{l_1} w_{x1}d_x + \int_{l_1}^{l} w_{x2}d_x\right)$
$a_n = \frac{1}{\pi}\left(\int_0^{l_1} w_{x1}\cos(nx)d_x + \int_{l_1}^{l} w_{x2}\cos(nx)d_x\right)$
$b_n = \frac{1}{\pi}\left(\int_0^{l_1} w_{x1}\sin(nx)d_x + \int_{l_1}^{l} w_{x1}\sin(nx)d_x\right)$
It is used Eq. 12 for simply support and Eq. 13 for cantilever and Eq. 14 for fixed with spring damper to Compensation in Eq. 11 after that Compensation to reach for Eq. 7 and after that will convert easily to Eq. 3. Then, subject Eq. 11 in Eq. 6 to get the general equation of motion for vibration beam with crack effect, and then solving its equation by stat space control techniques as presented in Eq. 3. Using the MATLAB program to apply Fourier equations series and then get one equation and then use this

equation orthogonality to be in a suitable format to convert to the equation of the state of space and as mentioned in the previous paragraph,

$$w_1(x) = A\begin{pmatrix} \left(e^{-iax}\sinh b_1 x - \frac{b_1}{b_2} e^{iax}\sin b_2 x\right) + \\ S_{30}\left(e^{-iax}\cosh b_1 x - e^{iax}\cos b_2 x + \frac{b_2}{ia} e^{iax}\sin b_2 x\right) \end{pmatrix}$$

$$w_2(x) = A\begin{pmatrix} S_{28}\begin{pmatrix} (e^{-ia(x-l_1)}\sinh b_1(x-l_1) + S1\, e^{ia(x-l_1)}\cos b_2(x-l_1)) - \\ \frac{S_4}{S_6}(e^{ia(x-l_1)}\sin b_2(x-l_1) + S3\, e^{ia(x-l_1)}\cos b_2(x-l_1)) \end{pmatrix} + \\ S_{30}S_{29}\begin{pmatrix} (e^{-ia(x-l_1)}\sinh b_1(x-l_1) + S1\, e^{ia(x-l_1)}\cos b_2(x-l_1)) - \\ \frac{S_4}{S_6}(e^{ia(x-l_1)}\sin b_2(x-l_1) + S3\, e^{ia(x-l_1)}\cos b_2(x-l_1)) \end{pmatrix} + \\ \frac{S_7}{S_{10}}\begin{pmatrix} (e^{-ia(x-l_1)}\cosh b_1(x-l_1) + S2\, e^{ia(x-l_1)}\cos b_2(x-l_1)) - \\ \frac{S_5}{S_6}(e^{ia(x-l_1)}\sin b_2(x-l_1) + S3\, e^{ia(x-l_1)}\cos b_2(x-l_1)) \end{pmatrix} + \\ S_{30}\frac{S_8}{S_{10}}\begin{pmatrix} (e^{-ia(x-l_1)}\cosh b_1(x-l_1) + S2\, e^{ia(x-l_1)}\cos b_2(x-l_1)) - \\ \frac{S_5}{S_6}(e^{ia(x-l_1)}\sin b_2(x-l_1) + S3\, e^{ia(x-l_1)}\cos b_2(x-l_1)) \end{pmatrix} - \\ S_{28}\frac{S_9}{S_{10}}\begin{pmatrix} (e^{-ia(x-l_1)}\cosh b_1(x-l_1) + S2\, e^{ia(x-l_1)}\cos b_2(x-l_1)) - \\ \frac{S_5}{S_6}(e^{ia(x-l_1)}\sin b_2(x-l_1) + S3\, e^{ia(x-l_1)}\cos b_2(x-l_1)) \end{pmatrix} + \\ S_{30}S_{29}\frac{S_9}{S_{10}}\begin{pmatrix} (e^{-ia(x-l_1)}\cosh b_1(x-l_1) + S2\, e^{ia(x-l_1)}\cos b_2(x-l_1)) - \\ \frac{S_5}{S_6}(e^{ia(x-l_1)}\sin b_2(x-l_1) + S3\, e^{ia(x-l_1)}\cos b_2(x-l_1)) \end{pmatrix} \end{pmatrix} \quad (12)$$

Where, $S1 = -\frac{e^{-ial_2}\sinh b_1 l_2}{e^{ial_2}\cos b_2 l_2}$, $S2 = \frac{e^{-ial_2}\cosh b_1 l_2}{e^{ial_2}\cos b_2 l_2}$, $S3 = \frac{\sin b_2 l_2}{\cos b_2 l_2}$

$$S_4 = \begin{pmatrix} (-ia(-iae^{-ial_2}\sinh b_1 l_2 + b_1 e^{-ial_2}\cosh b_1 l_2) + b_1(-iae^{-ial_2}\cosh b_1 l_2 + b_1 e^{-ial_2}\sinh b_1 l_2) \\ +S1(ia(iae^{ial_2}\cos b_2 l_2 - b_2 e^{ial_2}\sin b_2 l_2) - b_2(e^{ial_2}\sin b_2 l_2 + b_2 e^{ial_2}\cos b_2 l_2)) \end{pmatrix}$$

$$S_5 = \begin{pmatrix} -ia(-iae^{-ial_2}\cosh b_1 l_2 + b_1 e^{-ial_2}\sinh b_1 l_2) + b_1(e^{-ial_2}\sinh b_1 l_2 + b_1 e^{-ial_2}\cosh b_1 l_2) \\ +S2(ia(iae^{ial_2}\cos b_2 l_2 - b_2 e^{ial_2}\sin b_2 l_2) - b_2(iae^{ial_2}\sin b_2 l_2 + b_2 e^{ial_2}\cos b_2 l_2)) \end{pmatrix}$$

$$S_6 = \begin{pmatrix} (ia(e^{ial_2}\sin b_2 l_2 + b_2 e^{ial_2}\cos b_2 l_2) + b_2(e^{ial_2}\cos b_2 l_2 - b_2 e^{ial_2}\sin b_2 l_2)) + \\ S3((e^{ial_2}\cos b_2 l_2 - b_2 e^{ial_2}\sin b_2 l_2) - b_2(e^{ial_2}\sin b_2 l_2 + b_2 e^{ial_2}\cos b_2 l_2)) \end{pmatrix}$$

$S_7 = (e^{-ial_1}\sinh b_1 l_1 - \frac{b_1}{b_2} e^{ial_1}\sin b_2 l_1)$, $S_8 = \left(e^{-ial_1}\cosh b_1 l_1 - e^{ial_1}\cos b_2 l_1 + \frac{b_2}{ia} e^{ial_1}\sin b_2 l_1\right)$

$S_9 = \left(S1 - \frac{S_4}{S_6} S3\right)$, $S_{10} = \left((1 + S2) - \frac{S_5}{S_6} S3\right)$, $S_{11} = (iaS1 - b_2\frac{S_4}{S_6} - ia\frac{S_4}{S_6} S3)$

$S_{12} = (b_2\frac{S_4}{S_6} S3 - ia\frac{S_4}{S_6} - b_2 S1)$, $S_{13} = (iaS2 - b_2\frac{S_5}{S_6} - ia\frac{S_5}{S_6} S3)$, $S_{14} = (b_2\frac{S_5}{S_6} S3 - ia\frac{S_5}{S_6} - b_2 S2)$

$S_{15} = ({b_1}^2 - a^2)$, $S_{16} = (iaS_{11} + b_2 S_{12})$, $S_{17} = (iaS_{12} + b_2 S_{11})$, $S_{18} = (iaS_{13} + b_2 S_{14})$

$$S_{19} = (S_{14} - b_2 S_{13} S_{14}),\ S_{20} = \begin{pmatrix} (-a^2 + {b_1}^2)e^{-ial_1}\sinh b_1 l_1 - 2iab_1 e^{-ial_1}\cosh b_1 l_1 - \\ 2iab_1 e^{ial_1}\cos b_2 l_1 + (b_1 + a^2\frac{b_1}{b_2})e^{ial_1} b_2 \sin b_2 l_1 \end{pmatrix}$$

$$S_{21} = \begin{pmatrix} ({b_1}^2 + a^2)e^{-ial_1}\cosh b_1 l_1 - 2ia\, b_1 e^{-ial_1}\sinh b_1 l_1 + \\ b_2 e^{ial_1}\sin b_2 l_1 + (\frac{{b_2}^2}{ia} - 3a^2 + {b_2}^2)e^{ial_1}\cos b_2 l_1 \end{pmatrix},$$

$S_{22} = (-2iab_1 + S_{16})$, $S_{23} = (S_{15} + S_{18})$

$$S_{24} = \begin{pmatrix} (ia(a^2 - {b_1}^2) - 2ia{b_1}^2)e^{-ial_1}\sinh b_1 l_1 + \left(({b_1}^3 - a^2 b_1) - 2a^2 b_1\right)e^{-ial_1}\cosh b_1 l_1 \\ +(2a^2 b_1 + b_2(b_1 + a^2\frac{b_1}{b_2}))e^{ial_1}\cos b_2 l_1 + (2iab_1 b_2 + ia(b_1 + a^2\frac{b_1}{b_2}))e^{ial_1}\sin b_2 l_1 \end{pmatrix}$$

$$S_{25} = \begin{pmatrix} (-ia({b_1}^2 + a^2) - 2ia{b_1}^2)e^{-ial_1}\cosh b_1 l_1 + (b_1({b_1}^2 + a^2) - 2a^2 b_1)e^{-ial_1}\sinh b_1 l_1 + \\ (iab_2 - (\frac{{b_2}^3}{ia} + {b_2}^3 - 3a^2 b_2))e^{ial_1}\sin b_2 l_1 + ((\frac{{b_2}^2}{i} + a{b_2}^2 - 3a^3)i + {b_2}^2)e^{ial_1}\cos b_2 l_1 \end{pmatrix}$$

$S_{26} = ((b_1 S_{15} - 2a^2 b_1) + (iaS_{16} + b_2 S_{17}))$, $S_{27} = \left(-(2ia{b_1}^2 + iaS_{15}) + (iaS_{18} + b_2 S_{19})\right)$

$$S_{28}=\frac{\left(\frac{S_{20}}{S_{22}}-\frac{S_7\,S_{23}}{S_{10}S_{22}}\right)}{\left(1+\frac{S_9}{S_{10}}\frac{S_{23}}{S_{22}}\right)},\ S_{29}=\frac{\left(\frac{S_8\,S_{23}}{S_{10}S_{22}}+\frac{S_{21}}{S_{22}}\right)}{\left(1+\frac{S_9}{S_{10}}\frac{S_{23}}{S_{22}}\right)},\ S_{30}=\frac{\left(S_{28}S_{26}-S_{28}\frac{S_9}{S_{10}}S_{27}+\frac{S_7}{S_{10}}S_{27}-S_{24}\right)}{\left(S_{25}-\frac{S_8}{S_{10}}S_{27}-S_{29}S_{26}-S_{29}\frac{S_9}{S_{10}}S_{27}\right)}$$

And,

$$w_1(x)=A\left(\left(e^{-iax}\sinh b_1x-\frac{b_1}{b_2}e^{iax}\sin b_2x\right)+\frac{\alpha_{20}}{\alpha_{19}}\left(e^{-iax}\cosh b_1x+e^{iax}\left(\frac{2ai}{b_2}\sin b_2x-\cos b_2x\right)\right)\right)$$

$$w_2(x)=A\begin{pmatrix} Z_1\left(e^{-iax}\sinh b_1(x-l_1)+\alpha_1 e^{iax}\sin b_2(x-l_1)+(\alpha_2-\alpha_3)e^{iax}\cos b_2(x-l_1)\right)+ \\ \frac{Z_2}{\alpha_{10}}\left(e^{-iax}\cosh b_1(x-l_1)+\alpha_4 e^{iax}\sin b_2(x-l_1)+(\alpha_5+\alpha_6)e^{iax}\cos b_2(x-l_1)\right)\end{pmatrix} \quad (13)$$

Where, $S1=\left((b_1^2-a^2)\sinh b_1 l_2-2iab_1\cosh b_1 l_2\right)$, $S2=\left((b_1^2-a^2)\cosh b_1 l_2-2iab_1\sinh b_1 l_2\right)$

$S3=\left((a^2+b_2^2)\sin b_2 l_2-2iab_2\cos b_2 l_2\right)$, $S4=\left((a^2+b_2^2)\cos b_2 l_2+2iab_2\sin b_2 l_2\right)$

$S5=((-2iab_1^2-ia(b_1^2-a^2))\sinh b_1 l_2+(b_1(b_1^2-a^2)-2a^2b_1)\cosh b_1 l_2)$

$S6=\left((b_1(b_1^2-a^2)-2a^2b_1)\sinh b_1 l_2-(ia(b_1^2-a^2)+iab_1^2)\cosh b_1 l_2\right)$

$S7=\left((ia(a^2+b_2^2)-2iab_2^2)\sin b_2 l_2+(b_2(a^2+b_2^2)+2a^2b_2)\cos b_2 l_2\right)$

$S8=((ia(a^2+b_2^2)+2iab_2^2)\cos b_2 l_2-(b_2(a^2+b_2^2)+2a^2b_2)\sin b_2 l_2)$

$S9=\left(S5-\frac{S1S8}{S4}\right)$, $S10=\left(S6-\frac{S2S8}{S4}\right)$, $S11=\left(S7-\frac{S3S8}{S4}\right)$, $\alpha_1=\frac{e^{-2ial_2}S9}{S11}$, $\alpha_2=\frac{e^{-ial_2}S1}{S4}$, $\alpha_3=\frac{e^{-ial_2}S3S9}{(S4)S11}$

$\alpha_4=\frac{e^{-2ial_2}S10}{S11}$, $\alpha_5=\frac{e^{-ial_2}S2}{S4}$, $\alpha_6=\frac{e^{-ial_2}S3S10}{(S4)S11}$, $\alpha_7=\left(e^{-ial_1}\sinh b_1 l_1-\frac{b_1}{b_2}e^{iax}\sin b_2x\right)$

$\alpha_8=\left(e^{-ial_1}\cosh b_1 l_1+e^{ial_1}\left(\frac{2ai}{b_2}\sin b_2 l_1-\cos b_2 l_1\right)\right)$, $\alpha_9=A_2(\alpha_2-\alpha_3)e^{ial_1}$

$$\alpha_{10}=(e^{-ial_1}+(\alpha_5+\alpha_6)e^{ial_1}),\ \alpha_{11}=\begin{pmatrix}(-a^2+b_1^{\,2})e^{-ial_1}\sinh b_1 l_1-2iab_1\,e^{-ial_1}\cosh b_1 l_1- \\ 2iab_1 e^{ial_1}\cos b_2 l_1+(b_1+a^2\frac{b_1}{b_2})e^{ial_1}b_2\sin b_2 l_1\end{pmatrix}$$

$$\alpha_{12}=\begin{pmatrix}(b_1^{\,2}+a^2)e^{-ial_1}\cosh b_1 l_1-2ia\,b_1 e^{-ial_1}\sinh b_1 l_1- \\ \frac{2a^3i}{b_2}e^{ial_1}\sin b_2 l_1+(-3a^2+b_2^{\,2})e^{ial_1}\cos b_2 l_1\end{pmatrix}$$

$\alpha_{13}=B_2((b_1^{\,2}-a^2)e^{-ial_1}+((2iab_2\alpha_4-a^2(\alpha_5+\alpha_6)-b_2^{\,2}(\alpha_5+\alpha_6))e^{ial_1})$

$\alpha_{14}=(-2iab_1e^{-ial_1}+(2ia\alpha_1 b_2-a^2(\alpha_2-\alpha_3)-b_2^{\,2}(\alpha_2-\alpha_3))e^{ial_1})$

$$\alpha_{15}=\begin{pmatrix}(ia(a^2-b_1^{\,2})-2iab_1^{\,2})e^{-ial_1}\sinh b_1 l_1+\left((b_1^{\,3}-a^2b_1)-2a^2b_1\right)e^{-ial_1}\cosh b_1 l_1+ \\ (2a^2b_1+b_2(b_1b_2+a^2b_1))e^{ial_1}\cos b_2 l_1+(2iab_1b_2+ia(b_1b_2+a^2b_1))e^{ial_1}\sin b_2 l_1\end{pmatrix}$$

$$\alpha_{16}=\begin{pmatrix}(-ia(b_1^{\,2}+a^2)-2iab_1^{\,2})e^{-ial_1}\cosh b_1 l_1+(b_1(b_1^{\,2}+a^2)-2a^2b_1)e^{-ial_1}\sinh b_1 l_1 \\ +(\frac{2a^4}{b_2}-(b_2^{\,3}-3a^2b_2))e^{ial_1}\sin b_2 l_1+((ab_2^{\,2}-3a^3)i-2ia^3b_2)e^{ial_1}\cos b_2 l_1\end{pmatrix}$$

$$\alpha_{17}=(b_1(b_1^{\,2}-a^2)-2a^2b_1)e^{-ial_1}+e^{ial_1}\begin{pmatrix}ia(2ia\alpha_1b_2-a^2(\alpha_2-\alpha_3)-b_2^{\,2}(\alpha_2-\alpha_3))+ \\ b_2(-a^2\alpha_1-\alpha_1b_2^{\,2}-iab_2(\alpha_2-\alpha_3))\end{pmatrix}$$

$$\alpha_{18}=((-2iab_1^{\,2}-ia(b_1^{\,2}-a^2))e^{-ial_1}+\begin{pmatrix}iae^{ial_1}(2iab_2\alpha_4-a^2(\alpha_5+\alpha_6)-b_2^{\,2}(\alpha_5+\alpha_6))+ \\ b_2(-a^2\alpha_4-b_2^{\,2}\alpha_4-2iab_2(\alpha_5+\alpha_6)))\end{pmatrix}$$

$$\alpha_{19}=\left(1-\frac{\left(\frac{\alpha_{17}}{\alpha_{16}}\right)\left(\frac{\alpha_{12}}{\alpha_{14}}+\frac{\alpha_8\alpha_{13}}{\alpha_{10}\alpha_{14}}\right)}{1+\frac{\alpha_9\alpha_{13}}{\alpha_{10}\alpha_{14}}}-\frac{\left(\frac{\alpha_9\,\alpha_{18}}{\alpha_{10}\alpha_{16}}\right)\left(\frac{\alpha_{12}}{\alpha_{14}}+\frac{\alpha_8\alpha_{13}}{\alpha_{10}\alpha_{14}}\right)}{1+\frac{\alpha_9\alpha_{13}}{\alpha_{10}\alpha_{14}}}-\frac{\alpha_8}{\alpha_{10}}\left(\frac{\alpha_{18}}{\alpha_{16}}\right)\right)$$

$$\alpha_{20}=\left(\frac{\left(\frac{\alpha_{11}}{\alpha_{14}}-\frac{\alpha_7\alpha_{13}}{\alpha_{10}\alpha_{14}}\right)}{1+\frac{\alpha_9\alpha_{13}}{\alpha_{10}\alpha_{14}}}+\frac{\alpha_7}{\alpha_{10}}\left(\frac{\alpha_{18}}{\alpha_{16}}\right)-\frac{\left(\frac{\alpha_{11}}{\alpha_{14}}-\frac{\alpha_7\alpha_{13}}{\alpha_{10}\alpha_{14}}\right)}{1+\frac{\alpha_9\alpha_{13}}{\alpha_{10}\alpha_{14}}}-\frac{\alpha_{15}}{\alpha_{16}}\right)$$

$\gamma=(-iae^{-ial_1}+b_2\alpha_4e^{ial_1}+ia(\alpha_5+\alpha_6)e^{ial_1})$, $\gamma_1=(b_1e^{-ial_1}+\alpha_1b_2e^{ial_1}+ia(\alpha_2-\alpha_3)e^{ial_1})$

$$\gamma_2=\begin{pmatrix}((-iae^{-ial_1}\sinh b_1 l_1+b_1e^{-ial_1}\cosh b_1 l_1)-\frac{b_1}{b_2}(iae^{ial_1}\sin b_2 l_1+b_2e^{ial_1}\cos b_2 l_1))+ \\ \frac{\alpha_{20}}{\alpha_{19}}\quad((-iae^{-ial_1}\cosh b_1 l_1+b_1e^{-ial_1}\sinh b_1 l_1)+(\frac{2a^2i^2}{b_2}e^{ial_1}\sin b_2 l_1+2aie^{ial_1}\cos b_2 l_1) \\ -(iae^{ial_1}\cos b_2 l_1-b_2e^{ial_1}\sin b_2 l_1))\end{pmatrix}$$

$\gamma_3=(-2iab_1e^{-ial_1}+(2ia\alpha_1b_2-a^2(\alpha_2-\alpha_3)-b_2^{\,2}(\alpha_2-\alpha_3))e^{ial_1})$

$$\gamma_4 = ((b_1{}^2 - a^2)e^{-ial_1} + (2iab_2\alpha_4 - a^2(\alpha_5+\alpha_6) - b_2{}^2(\alpha_5+\alpha_6))e^{ial_1}$$

$$Z_1 = \frac{\left(\frac{\alpha_{11}}{\alpha_{14}} - \frac{\alpha_7\alpha_{13}}{\alpha_{10}\alpha_{14}}\right)}{\left(1+\frac{\alpha_9\alpha_{13}}{\alpha_{10}\alpha_{14}}\right)} + \frac{\frac{\alpha_{20}}{\alpha_{19}}\left(\frac{\alpha_{12}}{\alpha_{14}} + \frac{\alpha_8\alpha_{13}}{\alpha_{10}\alpha_{14}}\right)}{1+\frac{\alpha_9\alpha_{13}}{\alpha_{10}\alpha_{14}}},\ Z_2 = \left(\alpha_7 + \frac{\alpha_{20}}{\alpha_{19}}\alpha_8 - \frac{\left(\frac{\alpha_{11}}{\alpha_{14}} - \frac{\alpha_7\alpha_{13}}{\alpha_{10}\alpha_{14}}\right)}{\left(1+\frac{\alpha_9\alpha_{13}}{\alpha_{10}\alpha_{14}}\right)} + \frac{\frac{\alpha_{20}}{\alpha_{19}}\left(\frac{\alpha_{12}}{\alpha_{14}} + \frac{\alpha_8\alpha_{13}}{\alpha_{10}\alpha_{14}}\right)}{1+\frac{\alpha_9\alpha_{13}}{\alpha_{10}\alpha_{14}}\alpha_9}\right)$$

$$Z_3 = \frac{\left(\frac{\alpha_{11}}{\alpha_{14}} - \frac{\alpha_7\alpha_{13}}{\alpha_{10}\alpha_{14}}\right)}{1+\frac{\alpha_9\alpha_{13}}{\alpha_{10}\alpha_{14}}},\ Z_4 = \frac{\alpha_{20}}{\alpha_{19}}\frac{\left(\frac{\alpha_{12}}{\alpha_{14}} + \frac{\alpha_8\alpha_{13}}{\alpha_{10}\alpha_{14}}\right)}{1+\frac{\alpha_9\alpha_{13}}{\alpha_{10}\alpha_{14}}}$$

Also,

$$w_1(x) = A\left(\left(e^{-iax}\sinh b_1x - \frac{b_1}{b_2}e^{iax}\sin b_2x\right) + \frac{\alpha_{27}}{\alpha_{28}}\left(e^{-iax}\cosh b_1x + e^{iax}\left(\frac{2ai}{b_2}\sin b_2x - \cos b_2x\right)\right)\right)$$

$$w_2(x) = A\begin{pmatrix} \left(\alpha_{21} + \frac{\alpha_{27}}{\alpha_{28}}\alpha_{21}\right)\begin{pmatrix} e^{-iax}\sinh b_1(x-l_1) - \alpha_0 e^{iax}\sin b_2(x-l_1) + \\ \alpha_1 e^{iax}\cos b_2(x-l_1) + \alpha_2 e^{iax}\cos b_2(x-l_1) \end{pmatrix} + \\ \left(\alpha_{16} + \frac{\alpha_{27}}{\alpha_{28}}\alpha_{17} - \alpha_{21}\alpha_{18} - \frac{\alpha_{27}}{\alpha_{28}}\alpha_{21}\alpha_{18}\right)\begin{pmatrix} e^{-iax}\cosh b_1(x-l_1) - \alpha_4 e^{iax}\sin b_2(x-l_1) \\ +\alpha_3\cos b_2(x-l_1) + \alpha_5\cos b_2(x-l_1) \end{pmatrix} \end{pmatrix} \quad (14)$$

Where, $S1=\left((b_1^2-a^2)\sinh b_1 l_2 - 2iab_1\cosh b_1 l_2\right)$, $S2=\left((b_1^2-a^2)\cosh b_1 l_2 - 2iab_1\sinh b_1 l_2\right)$

$S3=\left((a^2+b_2^2)\sin b_2 l_2 - 2iab_2\cos b_2 l_2\right)$, $S4=\left((a^2+b_2^2)\cos b_2 l_2 + 2iab_2\sin b_2 l_2\right)$

$S5=EIe^{i\Omega t}e^{-ial_2}((-2iab_1^2 - ia(b_1^2-a^2))\sinh b_1 l_2 + (b_1(b_1^2-a^2) - 2a^2b_1)\cosh b_1 l_2)$

$S6=EIe^{i\Omega t}e^{-ial_2}\left((b_1(b_1^2-a^2) - 2a^2b_1)\sinh b_1 l_2 - (ia(b_1^2-a^2) + iab_1^2)\cosh b_1 l_2\right)$

$S7=EIe^{i\Omega t}e^{ial_2}\left((ia(a^2+b_2^2) - 2iab_2^2)\sin b_2 l_2 + (b_2(a^2+b_2^2) + 2a^2b_2)\cos b_2 l_2\right)$

$S8=EIe^{i\Omega t}e^{ial_2}((ia(a^2+b_2^2) + 2iab_2^2)\cos b_2 l_2 - (b_2(a^2+b_2^2) + 2a^2b_2)\sin b_2 l_2)$

$\vartheta_1 = \left(S5 - \vartheta_4 e^{-iax}\sinh b_1 l_2 - \vartheta_5 + \vartheta_6 e^{iax}\cos b_2 l_2\right)$,

$\vartheta_2 = \left(S6 - \vartheta_4 e^{-iax}\cosh b_1 l_2 + \vartheta_7 + \vartheta_8 e^{iax}\cos b_2 l_2\right)$

$\vartheta_3 = \left(S7 + \vartheta_4 e^{iax}\sin b_2 l_2 + \vartheta_9 - \vartheta_{10}e^{iax}\cos b_2 l_2\right)$

$\vartheta_4 = (ke^{i\Omega t} + ci\Omega e^{i\Omega t})$, $\vartheta_5 = -e^{-ial_2}S1\frac{S8}{S4}$, $\vartheta_6 = (e^{-ial_2}S1\frac{k}{S4}e^{i\Omega t} + e^{-ial_2}S1\frac{c}{S4}i\Omega e^{i\Omega t})$

$\vartheta_7 = \vartheta_4 = (ke^{i\Omega t} + ci\Omega e^{i\Omega t})$, $\vartheta_7 = -e^{-ial_2}S2\frac{S8}{S4}$, $\vartheta_8 = (e^{-ial_2}e^{i\Omega t}S2\frac{k}{S4} + e^{-ial_2}S2\frac{c}{S4}i\Omega e^{i\Omega t})$

$\vartheta_4 = (ke^{i\Omega t} + ci\Omega e^{i\Omega t})$, $\vartheta_9 = e^{ial_2}S3\frac{S8}{S4}$, $\alpha_0 = \frac{\vartheta_1}{\vartheta_3}$, $\alpha_1 = e^{-ial_2}\frac{S1}{S4}$, $\alpha_2 = \alpha_0 e^{ial_2}\frac{S3}{S4}$, $\alpha_3 = e^{-ial_2}\frac{S2}{S4}$

$\alpha_4 = \frac{\vartheta_2}{\vartheta_3}$, $\alpha_5 = \alpha_4 e^{ial_2}\frac{S3}{S4}$, $\alpha_6 = (b_1(b_1{}^2-a^2) - 2a^2b_1)$, $\alpha_7 = (2iab_1{}^2 + ia(b_1{}^2-a^2))$

$$\alpha_8 = \left(\left(\begin{matrix} ia\left((-a^2\alpha_0 + ib_2a\alpha_1 + ib_2a\alpha_2) + (ia\alpha_1b_2 - \alpha_0b_2{}^2 + ia\alpha_2b_2)\right) \\ -b_2(-a^2\alpha_0 + ib_2a\alpha_1 + ib_2a\alpha_2) + (ia\alpha_1b_2 - \alpha_0b_2{}^2 + ia\alpha_2b_2)))\end{matrix}\right.\right.$$

$$\alpha_9 = \begin{pmatrix} -b_2\left((-a^2\alpha_0 + ib_2a\alpha_1 + ib_2a\alpha_2) + (ia\alpha_1b_2 - \alpha_0b_2{}^2 + ia\alpha_2b_2)\right) - \\ ia((-a^2\alpha_0 + ib_2a\alpha_1 + ib_2a\alpha_2) + (ia\alpha_1b_2 - \alpha_0b_2{}^2 + ia\alpha_2b_2)) \end{pmatrix}$$

$\alpha_{10} = (ia(-\alpha_3b_2 - \alpha_5b_2 - ia\alpha_4) + \alpha_4b_2{}^2)$, $\alpha_{11} = (b_2(-\alpha_3b_2 - \alpha_5b_2 - ia\alpha_4) - ia\alpha_4b_2)$

$\alpha_{12} = (ia\alpha_{10} - b_2\alpha_{11})$ $\alpha_{13} = (\alpha_{10}b_2 + ia\alpha_{11})$, $\alpha_{14} = (ia(b_1{}^2-a^2) + 2iab_2{}^2)$

$$\alpha_{15} = (b_1(b_1{}^2-a^2) - 2a^2b_1),\ \alpha_{16} = \frac{\left(e^{-ial_1}\sinh b_1l_1 - \frac{b_1}{b_2}e^{ial_1}\sin b_2l_1\right)}{\left(e^{-ial_1} + \alpha_3 + \alpha_5\right)}$$

$$\alpha_{17} = \frac{\left(e^{-ial_1}\cosh b_1l_1 + e^{ial_1}\left(\frac{2ai}{b_2}\sin b_2l_1 - \cos b_2l_1\right)\right)}{\left(e^{-ial_1} + \alpha_3 + \alpha_5\right)},\ \alpha_{18} = \frac{\left(\alpha_1e^{ial_1} + \alpha_2e^{ial_1}\right)}{\left(e^{-ial_1} + \alpha_3 + \alpha_5\right)}$$

$$\alpha_{18b} = \frac{\left((-a^2+b_1{}^2)e^{-ial_1}\sinh b_1l_1 - 2iab_1e^{-ial_1}\cosh b_1l_1 - 2iab_1e^{ial_1}\cos b_2l_1 + (b_1 + a^2\frac{b_1}{b_2})e^{ial_1}b_2\sin b_2l_1\right)}{\left(-2iab_1e^{-ial_1} + ((-a^2\alpha_1 - ia\alpha_0b_2 - a^2\alpha_2) - (ia\alpha_0b_2 + b_2{}^2\alpha_1 + b_2{}^2\alpha_2))e^{ial_1}\right)}$$

$$\alpha_{19} = \frac{\left((b_1{}^2+a^2)e^{-ial_1}\cosh b_1l_1 - 2iab_1e^{-ial_1}\sinh b_1l_1 - \frac{2a^3i}{b_2}e^{ial_1}\sin b_2l_1 + (-3a^2+b_2{}^2)e^{ial_1}\cos b_2l_1\right)}{\left(-2iab_1e^{-ial_1} + ((-a^2\alpha_1 - ia\alpha_0b_2 - a^2\alpha_2) - (ia\alpha_0b_2 + b_2{}^2\alpha_1 + b_2{}^2\alpha_2))e^{ial_1}\right)}$$

$$\alpha_{20} = \frac{\left((b_2(-\alpha_3b_2 - \alpha_5b_2 - ia\alpha_4) - ia\alpha_4b_2)e^{ial_1} + (b_1{}^2-a^2)e^{-ial_1}\right)}{\left(-2iab_1e^{-ial_1} + ((-a^2\alpha_1 - ia\alpha_0b_2 - a^2\alpha_2) - (ia\alpha_0b_2 + b_2{}^2\alpha_1 + b_2{}^2\alpha_2))e^{ial_1}\right)}$$

$$\alpha_{21} = \frac{(\alpha_{18} - \alpha_{16}\alpha_{20})}{(1 - \alpha_{18}\alpha_{20})},\ \alpha_{22} = \frac{(\alpha_{19} - \alpha_{17}\alpha_{20})}{(1 - \alpha_{18}\alpha_{20})}$$

$$\alpha_{23}=\begin{pmatrix}\left(ia\left(a^2-{b_1}^2\right)-2iab_1{}^2\right)e^{-ial_1}\sinh b_1 l_1+\left(\left({b_1}^3-a^2b_1\right)-2a^2b_1\right)e^{-ial_1}\cosh b_1 l_1\\+(2a^2b_1+b_2(b_1b_2+a^2b_1))e^{ial_1}\cos b_2 l_1+(2iab_1b_2+ia(b_1b_2+a^2b_1))e^{ial_1}\sin b_2 l_1\end{pmatrix}$$

$$\alpha_{24}=\begin{pmatrix}\left(-ia\left({b_1}^2+a^2\right)-2iab_1{}^2\right)e^{-ial_1}\cosh b_1 l_1+\left(b_1\left({b_1}^2+a^2\right)-2a^2b_1\right)e^{-ial_1}\sinh b_1 l_1\\+\left(\frac{2a^4}{b_2}-({b_2}^3-3a^2b_2)\right)e^{ial_1}\sin b_2 l_1+((ab_2{}^2-3a^3)i-2ia^3b_2)e^{ial_1}\cos b_2 l_1\end{pmatrix}$$

$$\alpha_{25}=\left(\alpha_6 e^{-ial_1}+\alpha_8 e^{ial_1}\right)\quad \alpha_{26}=\left(\alpha_{13}e^{ial_1}-\alpha_{14}e^{-ial_1}\right)$$

$$\alpha_{27}=(\alpha_{21}\alpha_{25}+\alpha_{16}\alpha_{26}-\alpha_{21}\alpha_{18}\,\alpha_{26}-\alpha_{23})\quad \alpha_{28}=(\alpha_{21}\alpha_{18}\,\alpha_{26}+\alpha_{24}-\alpha_{21}\alpha_{25}-\alpha_{17}\alpha_{26})$$

2.4 Computer Program for Control System (Stability Test)

When examining all the cases that are taken to ensure that they are in a stable state or did not use the root locus method for the purpose of the statement of stability and it is found that all the cases of the installation (Simply support, cantilever, fixed with spring damper) taken in the case of instability and show below the case for cantilever in pressure (15bar) The following Figure 3 shows the instability.

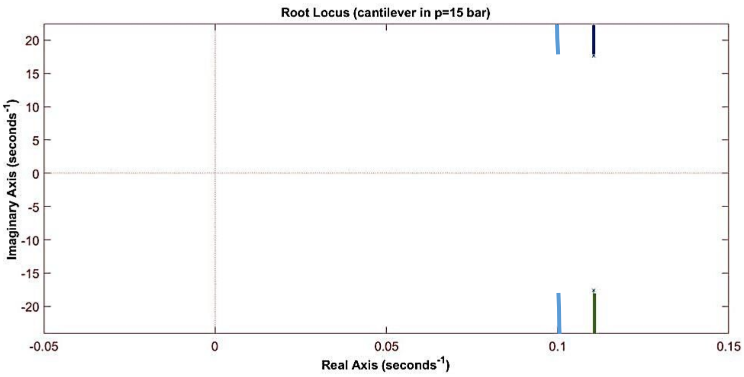
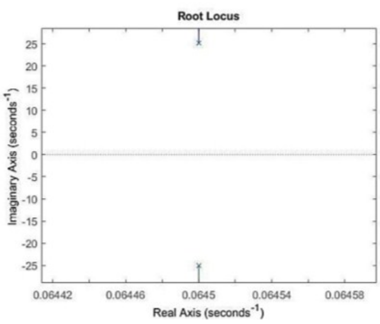


Figure 3. Shows the instability.

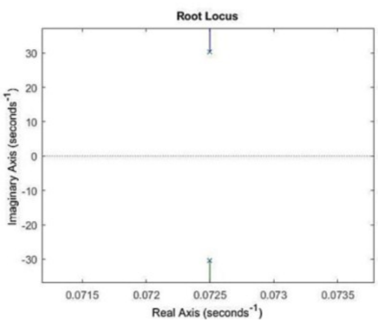
3. Results and Discussions

In this item we will discuss the change in the case of instability and change in position of instability using the root Locus with or without crack, the Figures 4 to 6 shows the change in the value of instability with various fixations (Simply support, cantilever, fixed with spring damper (K=77, C=5.73)) and the different types of pressure (5 bar, 10 bar, 15 bar) without crack. So, the Figure 4 Show instable in the simply support. It is noted that in this installation head for instability by increasing the pressure, where the values of mode response are large so that the vibration is clear in them as well as in cantilever also that show in Figure 5. Either in fixed with spring damper (K=77, C=5.73) as shown in Figure 6. In this installation, the increase in pressure leads to a small increase in instability and when comparing, there is a more stable state.

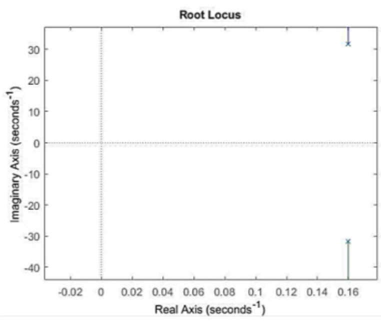
From Figure 7, it is clear that the instability state increases in the case of installation (simply support, cantilever) and the instability state decreases in the case of installation (fixed with spring damper). This indicates the large control that is done using spring and damper. The difference is clear and very large between the fixations and this is required to reach a more stable state.



a. P=5 bar

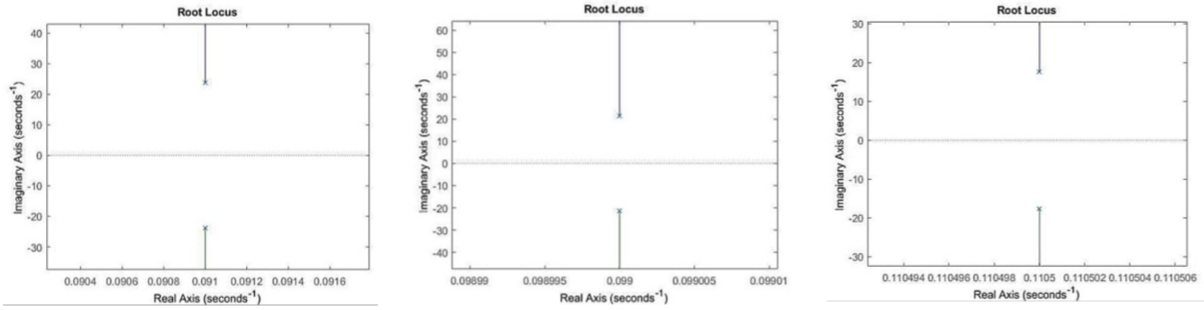


b. P=10 bar



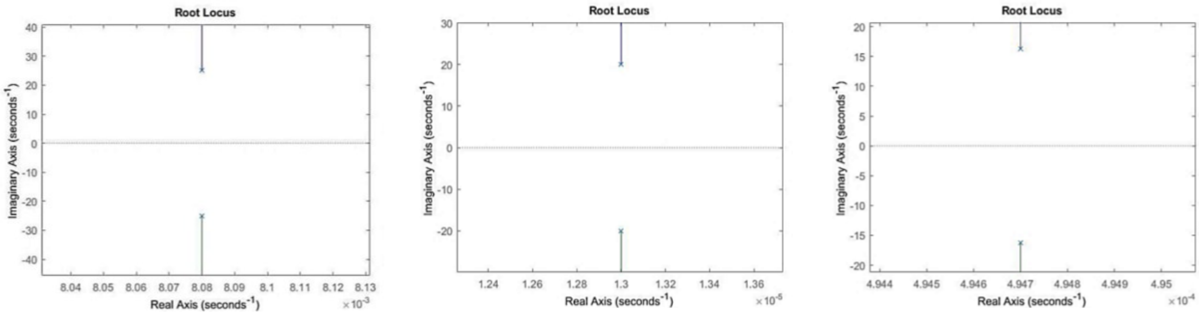
c. P=15 bar

Figure 4. Instable with simply supported.



a. P=5 bar b. P=10 bar c. P=15 bar

Figure 5. Instable with Cantilever.



a. P=5 bar b. P=10 bar c. P=15 bar

Figure 6. Instable with fixed with spring damper without crack.

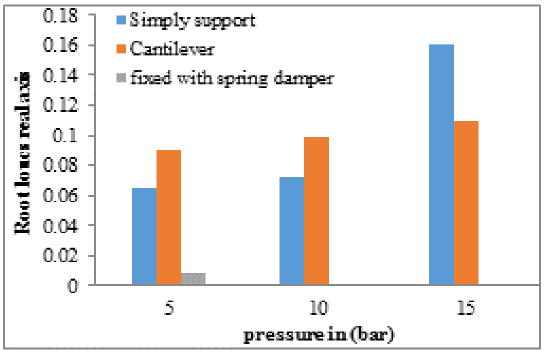
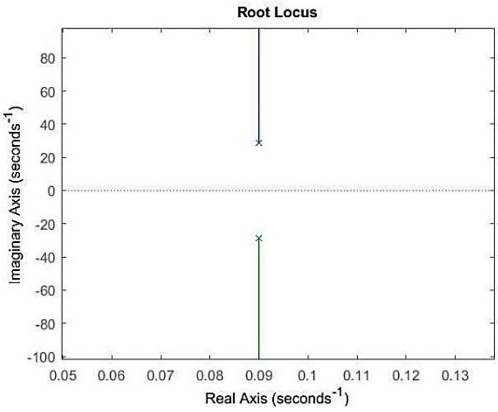
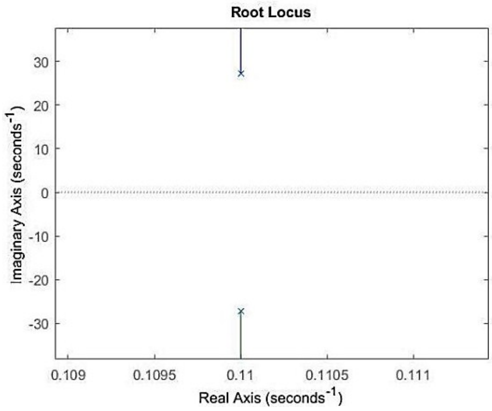


Figure 7. Comparison instable in different support without crack.

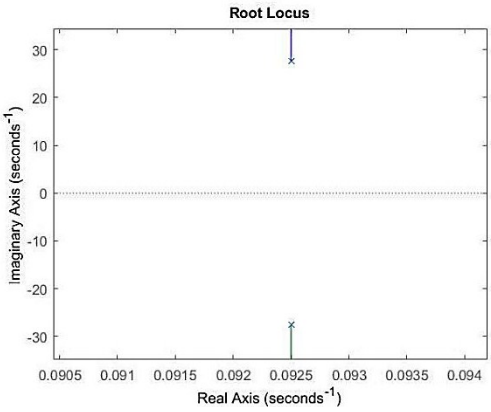
In the case of a crack, the Figure 8 shows the state of instability in the case of (10 bar) of pressure in simply support and for the crack is taken in different dimensions and depths (l=0.25L_p, l=0.5L_p,dc=0.267t ,dc=0.75t) Note that in this installation non-stability may increase by increasing pressure. The values of modulation response is so large that the vibration is clear and special at (dc=1.55mm) (l=150cm) as well as cantilever as shown in Figure 9 and in fixed with spring damper (K=77, C=5.73) as shown in Figure 10. In this installation, the increase in pressure leads to a small increase in non-stability and when comparing, there is a more stable state.
The comparison shows the instability to (simply support, cantilever and fixed with spring damper) that is shown in Figures 11 to 13 the presence of a crack in different dimensions. It is noticed that the stability decrease by increasing the depth of the crack and distance from the end of the installation as the response increases and the natural frequency the stiffness decreases. In either of the following figures 14 to 17, it appears that the comparison between the different supports (Simply support, cantilever, fixed with spring damper) in different fault dimensions (dc=0.75) (l=75), (dc=1.5) (l=75) (dc=0.75) (l=150) (dc=1.5) (l=150) respectively Note that the installation of (fixed with spring damper) is superior in all cases and is more stable than the other supports (Simply support, cantilever) where it is very close and very close to stability and this achieves the goal of controlling the vibrations that occurs in the pipeline systems Using this support.



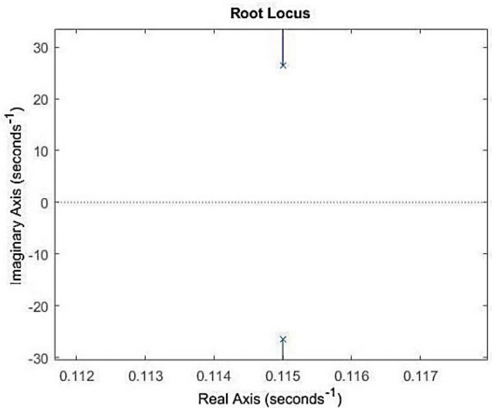
a. Crack (dc=0.75 mm, l=75 cm)



b. Crack (dc=1.5mm) (l=75cm)

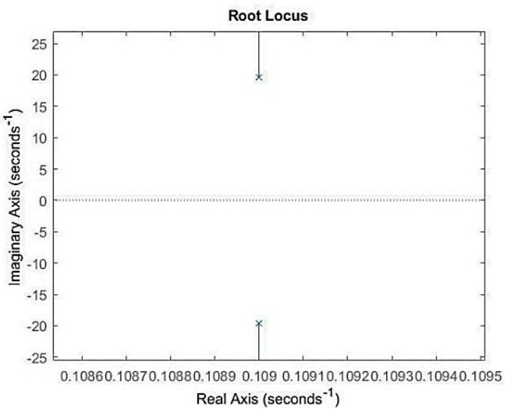


c. Crack (dc=0.75mm) (l=150cm)

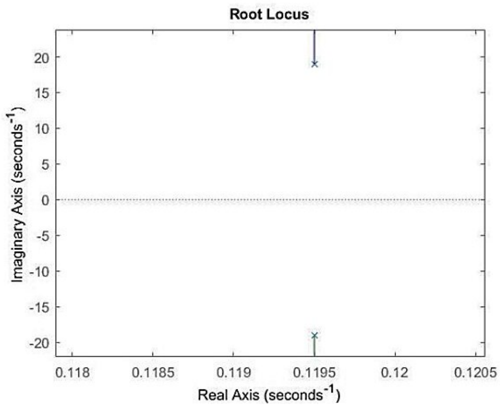


d. Crack (dc=1.55mm) (l=150cm)

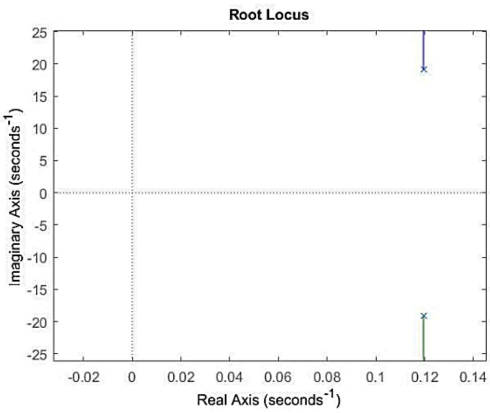
Figure 8. Instable with simply support with crack.



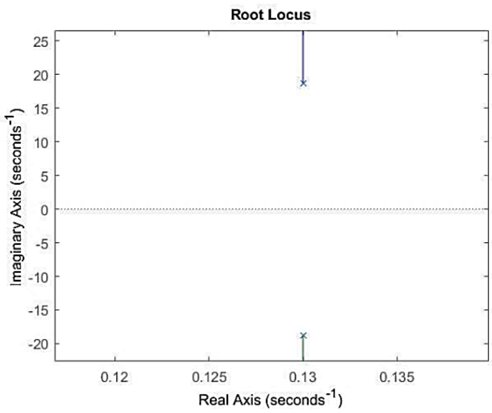
a. Crack (dc=0.75 mm, l=75 cm)



b. Crack (dc=1.5mm) (l=75cm)

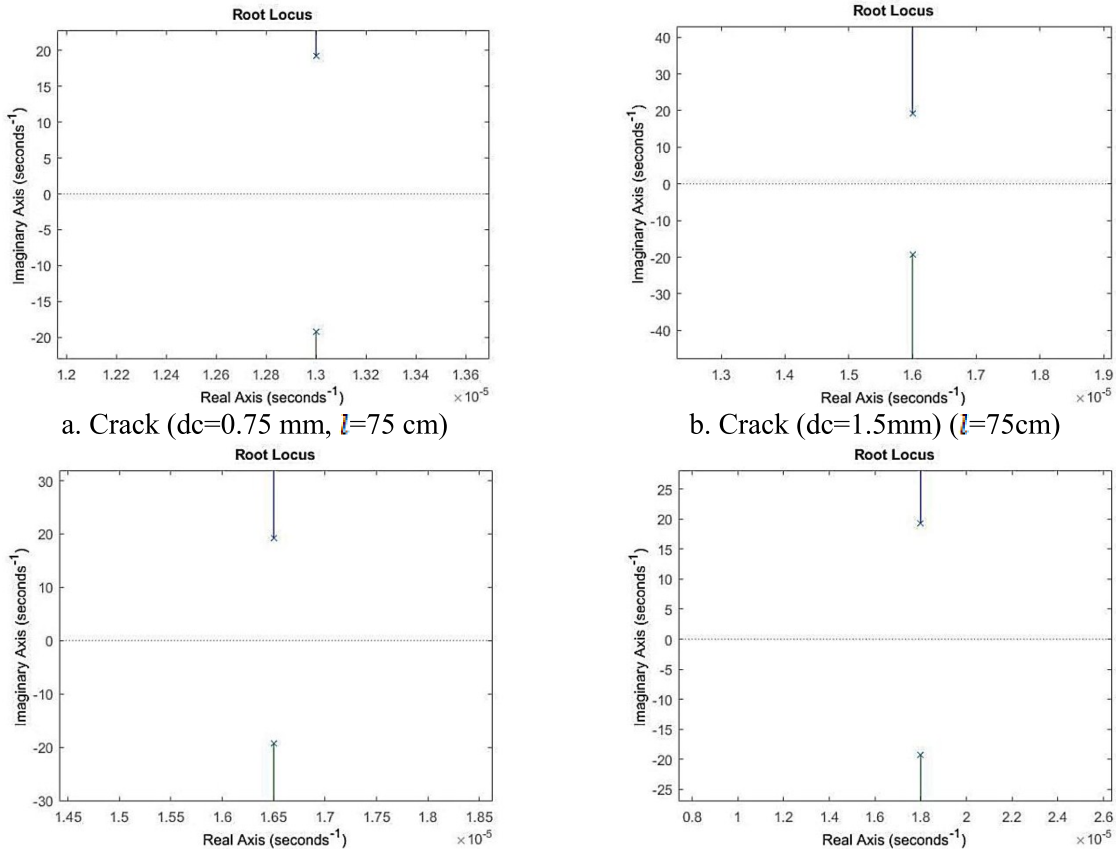


c. Crack (dc=0.75mm) (l=150cm)



d. Crack (dc=1.55mm) (l=150cm)

Figure 9. Instable with cantilever with crack.



a. Crack (dc=0.75 mm, l=75 cm)

b. Crack (dc=1.5mm) (l=75cm)

c. Crack (dc=0.75mm) (l=150cm)

d. Crack (dc=1.55mm) (l=150cm)

Figure 10. Instable with fixed with spring damper with crack.

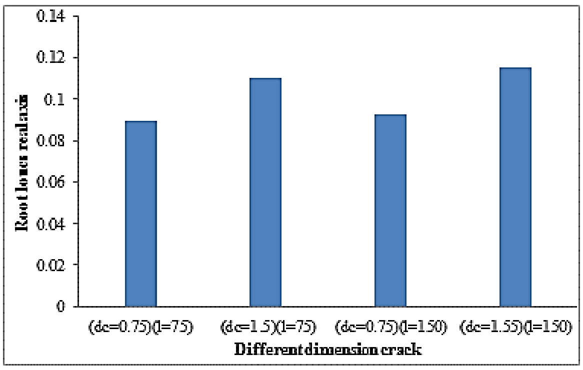


Figure 11. Comparison instable of Simply support with Different dimension crack.

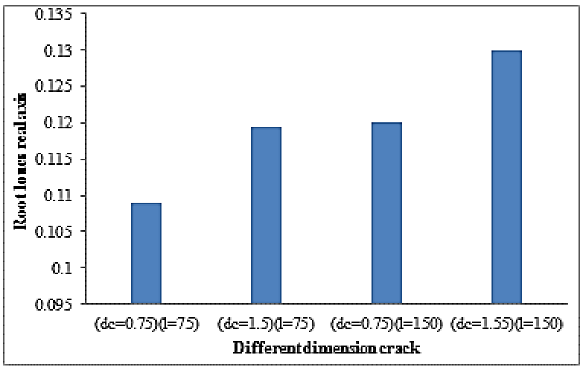


Figure 12. Comparison instable of cantilever with Different dimension crack.

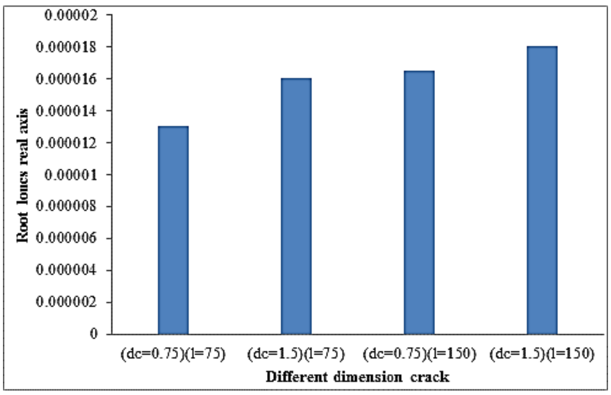


Figure 13. Comparison instable of fixed with spring damper with Different dimension crack.

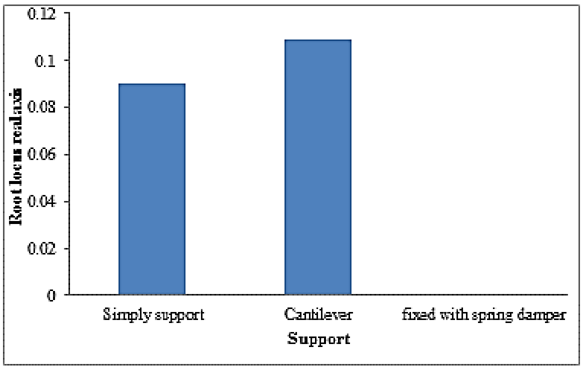


Figure 14. Comparison instable of Different support in (dc=0.75) (l=75) crack dimension.

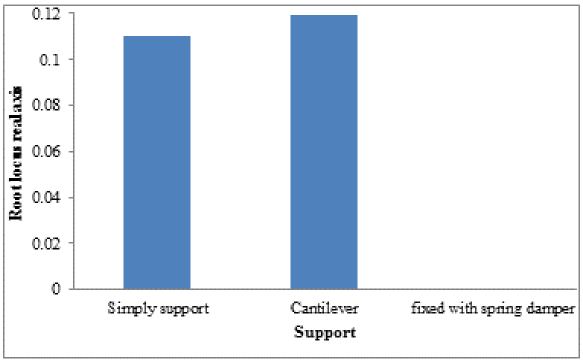


Figure 15. Comparison instable of Different support in (dc=1.5) (l=75) crack dimension.

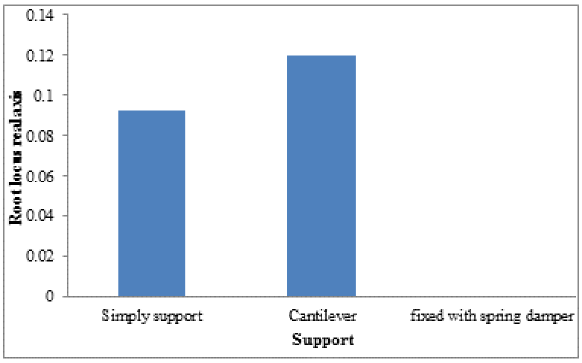


Figure 16. Comparison instable of Different support in (dc=0.75) (l=150) crack dimension.

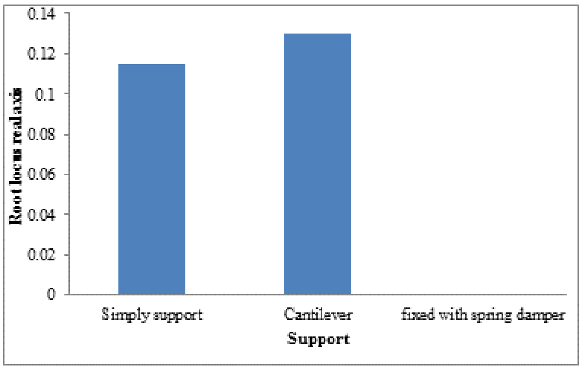


Figure 17. Comparison instable of Different support in (dc=1.5) (l=150) crack dimension.

4. Conclusion

The work presents an analytical solution for the stability investigation of pipe with crack effect and studding how to reduce the unstable behavior by adding a spring and damper system for the pipe. There, from the presented work can be concluded the following points, as,

1. The analytical solution was a perfect technique that can be used to solve the general equation of motion for pipe with and without crack effect and calculate the pipe stability with different parameters effect.
2. The instability state increases in the case of installation (simply support, cantilever) and the instability state decreases in the case of installation (fixed with spring damper). This indicates the large control that is done using spring and damper. The difference is clear and very large between the fixations and this is required to reach a more stable state.
3. The instability of pipe occurs due to crack defect of pipe and its increase with increasing the depth of the crack and distance from the end of the installation.

The installation of (fixed with spring damper) is superior in all cases and is more stable than the other supports (Simply support, cantilever) where it is very close and very close to stability and this achieves the goal of controlling the vibrations that occur in the pipeline systems Using this support

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