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Finite-time exergoeconomic performance of an endoreversible Carnot heat engine with complex heat transfer law

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Abstract
The finite time exergoeconomic performance of an endoreversible Carnot heat engine with a complex heat transfer law, including generalized convective heat transfer law and generalized radiative heat transfer law, $q \propto (\Delta T^n)^m$, is investigated in this paper. The finite time exergoeconomic performance optimization of the engine is investigated by taking profit optimization criterion as the objective. The focus of this paper is to search the compromised optimization between economics (profit) and the utilization factor (efficiency) for endoreversible Carnot heat engine cycles. The obtained results include those obtained in many literatures and can provide some theoretical guidance for the design of practical heat engines.

Keywords: Finite time thermodynamics, Endoreversible Carnot heat engine, Exergoeconomic performance.

1. Introduction

Classical thermodynamic processes are based on reversible assumption. However, reversible processes in practice are difficult to realize. Finite time thermodynamics [1-10] extends the reversible process to include rates constraints. In the analysis and optimization of heat engine cycles, the objective functions are often pure thermodynamic parameters including power, efficiency, entropy production, effectiveness, cooling load, heating load, coefficient of performance, and loss of exergy. Salamon and Nitzan [11] viewed the operation of the endoreversible heat engine as a production process with work as its output. They carried out the economic optimization of the heat engine with the maximum profit as the objective function [12].
A relatively new method that combines exergy with conventional concepts from long-run engineering economic optimization to evaluate and optimize the design and performance of energy systems is exergoeconomic (or thermoeconomic) analysis [13-18]. Salamon and Nitzan's work [11] combined the endoreversible model with exergoeconomic analysis. It was termed as finite time exergoeconomic analysis [19-21] to distinguish it from the endoreversible analysis with pure thermodynamic objectives and the exergoeconomic analysis with long-run economic optimization. Similarly, the performance bound at maximum profit was termed as finite time exergoeconomic performance bound to distinguish it from the finite time thermodynamic performance bound at maximum thermodynamic output. A similar idea was provided by Ibrahim *et al*. [22], De Vos [23, 24] and Bejan [25]. Zheng *et al*.[26] obtained the

maximum exergoeconomic performance of a class of universal steady flow endoreversible heat engine cycles with Newton heat transfer law. Chen *et al*. [27] further obtained the maximum exergoeconomic performance of generalized irreversible Carnot engine with Newton heat transfer law.

However, the heat engine's exergoeconomic performance is affected by the heat-transfer law [28-30]. Chen *et al*. [28] investigated the endoreversible thermoeconomic performance of heat engine with the linear phenomenological heat transfer law $q \propto \Delta(T^{-1})$ based on Ref. [23]. Wu *et al.*[29] derived the finite time exergoeconomic performance of an endoreversible Carnot heat engine with generalized radiative heat transfer law $q \propto \Delta(T^n)$. Zhu *et al.* [30] obtained the finite time exergoeconomic performance of an endoreversible Carnot heat engine with generalized convective heat transfer law $q \propto (\Delta T)^n$. Sahin *et al.* [31-34] provided a new thermoeconomic optimization criterion, thermodynamic output rates (power, cooling load or heating load for heat engine, refrigerator or heat pump) per unit total cost, investigated the performances of endoreversible heat engine [31], refrigerator and heat pump [32], combined cycle refrigerator [33], combined cycle heat pump [34] as well as irreversible heat engine [35], refrigerator and heat pump [9, 36], combined cycle refrigerator [37], combined cycle heat pump [38], three-heat-reservoir absorption refrigerator and heat pump [39].

Recently, Li *et al.* [40] investigated the fundamental optimal relationship between power output and efficiency of the endoreversible Carnot heat engine by using a complex heat transfer law, including generalized convective heat transfer law [$q \propto (\Delta T)^n$] and generalized radiative heat transfer law[$q \propto (\Delta T^n)$], $q \propto (\Delta T^n)^m$ [10] in the heat transfer processes between the working fluid and the heat reservoirs. This paper will extend the previous work to find the optimal exergoeconomic performance of the endoreversible Carnot heat engine by using the complex heat transfer law, $q \propto (\Delta T^n)^m$, in the heat transfer processes between the working fluid and the heat reservoirs of the heat engine.

2. Endoreversible Carnot heat engine model

An endoreversible Carnot engine and its surroundings to be considered in this paper are shown in Figure l. The following assumptions are made for this model:

(i). The working fluid flows through the system in a steady-state fashion. The cycle consists of two isothermal and two adiabatic processes.

(ii). Because of the heat resistance, the working fluid temperatures (T_{HC} and T_{LC}) are different from the reservoir temperatures (T_H and T_L). The four temperatures are of the following decreasing order: $T_H > T_{HC} > T_{LC} > T_L$. The heat transfer obeys a complex heat transfer law $q \propto (\Delta T^n)^m$. The heat transfer surface areas (F_1 and F_2) of the high- and low-temperature heat exchangers are finite. The total heat transfer surface area (F) of the two heat exchangers is assumed to be a constant: $F = F_1 + F_2$.

(iii). The rate of heat transfer supplied by the heat source is Q_H and the rate of heat transfer released to the heat sink is Q_L.

(iv). The heat engine is an endoreversible one, i.e. the only irreversibility of finite rate heat transfer is considered.

3. Generalized optimal characteristics

The endoreversible performance of the cycle requires that

$$Q_H / T_{HC} = Q_L / T_{LC} \tag{1}$$

The first law of thermodynamics gives that the power output and the efficiency of the heat engine are

$$P = Q_H - Q_L = Q_H (1 - T_{LC} / T_{HC}) \tag{2}$$

$$\eta = P / Q_H = 1 - T_{LC} / T_{HC} \tag{3}$$

Considering that the heat transfer between the heat engine and its surroundings follows a complex law $q \propto (\Delta T^n)^m$. Then

$$Q_H = \alpha F_1 (T_H^n - T_{HC}^n)^m, Q_L = \beta F_2 (T_{LC}^n - T_L^n)^m \tag{4}$$

where α is the overall heat transfer coefficient of the high-temperature-side heat exchanger and β is the overall heat transfer coefficient of the low-temperature-side heat exchanger. Defining the heat transfer surface area ratio (f) and the working fluid temperature ratio (x) as follows: $f = F_1/F_2$, $x = T_{HC}/T_{LC}$, where $1 \le x \le T_H/T_L$. From Equations (1)-(4), one can obtain

$$P = \frac{\alpha f F}{1+f}\left[\frac{T_H^n x^{-n} - T_L^n}{(fr/x)^{1/m} + x^{-n}}\right]^m (1 - x^{-1}) \tag{5}$$

$$\eta = 1 - 1/x \tag{6}$$

where $r = \alpha/\beta$. Assuming the environment temperature is T_0 and the rate of exergy input of the heat engine is

$$A_{rev} = Q_H(1 - T_0/T_H) - Q_L(1 - T_0/T_L) = Q_H\eta_1 - Q_L\eta_2 \tag{7}$$

where $\eta_1 = 1 - T_0/T_H$ and $\eta_2 = 1 - T_0/T_L$ are Carnot coefficient of the high- and low-temperature reservoirs, respectively. The profit of the heat engine is

$$\Pi = \psi_1 P - \psi_2 A_{rev} \tag{8}$$

where ψ_1 is the value price of power output, ψ_2 is the value price of exergy input rate. Substituting Equations (1)-(5) and (7) into Equation (8), one can obtain

$$\Pi = \frac{\psi_1 \alpha f F (T_H^n x^{-n} - T_L^n)^m}{(1+f)[(fr/x)^{1/m} + x^{-n}]^m}[1 - 1/x - (\psi_2/\psi_1)(\eta_1 - \eta_2/x)] \tag{9}$$

Equation (9) indicates that the profit of the endoreversible Carnot heat engine is a function of the heat transfer surface area ratio (f) for the given T_H, T_L, T_0, α, β, n, m and x. Taking the derivatives of Π with respect to f and setting it equal to zero ($d\Pi/df = 0$) yields

$$f_a = (x^{1-nm}/r)^{1/(m+1)} \tag{10}$$

The corresponding profit is

$$\Pi = \frac{\psi_1 \alpha F (T_H^n - T_L^n x^n)^m}{[1 + (x^{mn-1} r)^{1/(m+1)}]^{m+1}}[1 - 1/x - (\psi_2/\psi_1)(\eta_1 - \eta_2/x)] \tag{11}$$

From Equations (6) and (11), one can obtain the optimal relation between Π and η

$$\Pi = \frac{\psi_1 \alpha F [T_H^n - T_L^n (1-\eta)^{-n}]^m}{\{1 + [(1-\eta)^{1-mn} r]^{1/(m+1)}\}^{m+1}}\{\eta - (\psi_2/\psi_1)[\eta_1 - \eta_2(1-\eta)]\} \tag{12}$$

Equation (12) is the fundamental optimal relation between the profit and the efficiency of the endoreversible Carnot heat engine with the heat transfer law of $q \propto (\Delta T^n)^m$. Maximizing Π with respect to x by setting $\partial\Pi/\partial x = 0$ in Equation (11) directly yields the maximum profit rate and the corresponding optimal thermal efficiency η_o, that is, the finite-time thermodynamic exergoeconomic bound.

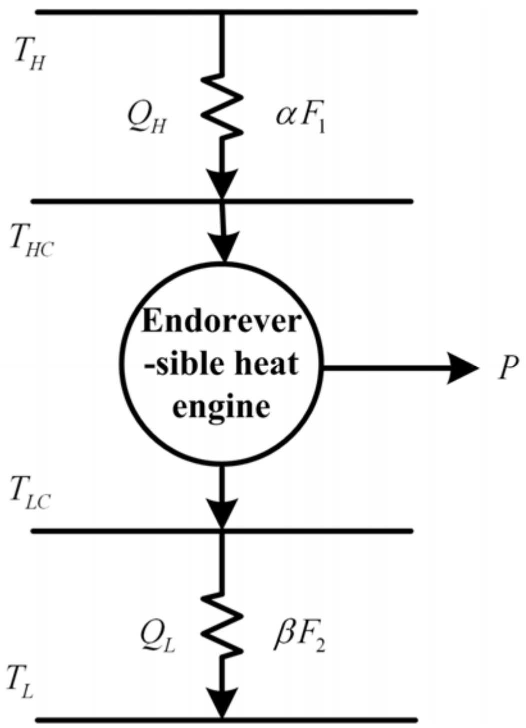

Figure 1. Endoreversible Carnot heat engine model

4. Discussions

(1). When $m=1$, Equation (12) becomes

$$\Pi = \frac{\psi_1 \alpha F[T_H^n - T_L^n(1-\eta)^{-n}]}{\{1+[(1-\eta)^{1-n} r]^{1/2}\}^2}\{\eta - (\psi_2/\psi_1)[\eta_1 - \eta_2(1-\eta)]\} \tag{13}$$

It is the same result as that obtained in Refs. [29]. The profit versus efficiency characteristic is a parabolic-like one. If $n=1$, it is the result of endoreversible heat engine with Newtownian heat transfer law [19, 23, 24, 27, 29, 30]. If $n=-1$, it is the result of endoreversible heat engine with linear phenomenological heat transfer law [28, 29]. If $n=4$, it is the result of endoreversible heat engine with radiative heat transfer law [29].

(2). When $n=1$, Equation (12) becomes

$$\Pi = \frac{\psi_1 \alpha F[T_H - T_L(1-\eta)^{-1}]^m}{\{1+[(1-\eta)^{1-m} r]^{1/(m+1)}\}^{m+1}}\{\eta - (\psi_2/\psi_1)[\eta_1 - \eta_2(1-\eta)]\} \tag{14}$$

It is the same result as that obtained in Refs. [30]. The profit versus efficiency characteristic is also a parabolic-like one. If $m=1$, it is the result of endoreversible heat engine with Newtownian heat transfer law [19, 23, 24, 27, 29, 30]. If $m=1.25$, it is the result of endoreversible heat engine with Dulong-Petit heat transfer law [41].

(3). From Equation (11), it can be seen that besides T_H, T_L and T_0, ψ_2/ψ_1 also has the significant influences on the profit of endoreversible Carnot heat engine. Note that for the process to be potential profitable, the following relationship must exist: $0<\psi_2/\psi_1<1$, because one unit of power input must give rise to at least one unit of exergy output rate. When the price of work output becomes very large compared with the price of the exergy input, i.e. $\psi_2/\psi_1 \to 0$, Equation (11) becomes

$$\Pi = \frac{\psi_1 \alpha F(T_H^n - T_L^n x^n)^m}{[1+(x^{mn-1} r)^{1/(m+1)}]^{m+1}}(1-1/x) = \psi_1 P \tag{15}$$

That is the profit rate maximization approaches the power maximization, where P is the power output of the endoreversible heat engine cycle [40].

When the price of work output approaches the price of the exergy input, i.e. $\psi_2/\psi_1 \to 1$, Equation (11) becomes

$$\Pi = \frac{\psi_1 \alpha F (T_H^n - T_L^n x^n)^m}{[1+(x^{mn-1} r)^{1/(m+1)}]^{m+1}}[1 - 1/x - (\eta_1 - \eta_2/x)] = -\psi_1 T_0 \sigma \tag{16}$$

where σ is the rate of entropy production of the heat engine. That is the profit maximization approaches the rate of entropy production minimization, in other word, the minimum waste of exergy. Equation (15) indicates that the heat engine is not profitable regardless of the thermal efficiency is at which the heat engine is operating. Only the engine is operating reversibly will the revenue equal the cost, and then the maximum profit will equal zero. The corresponding rate of entropy production is also zero.

Therefore, for any intermediate values of ψ_2/ψ_1, the finite-time exergoeconomic performance bound (η_o) lies between the finite-time thermodynamic performance bound and the reversible performance bound. η_o is related to the latter two through the price ratio, and the associated thermal efficiency bounds are the upper and lower limits of η_o.

5. Numerical example

To show the profit vs. efficiency characteristic of the endoreversible Carnot heat engine with the complex heat transfer law, one numerical example is provided. In the numerical calculations, $T_H = 1000K$, $T_L = 400K$, $T_0 = 300K$, $\alpha = \beta (r = 1)$, $\alpha F = 4W/K^{mn}$ and $\psi_1 = 1000\, yuan/kW$ are set. The effects of heat transfer laws on relation between the profit and efficiency are shown in Figure 2. In this case, $\psi_2/\psi_1 = 0.3$ is set. It shows that the relationship between profit and efficiency of the heat engine is a parabolic-like curve. It can be seen that heat transfer law changes the profit versus efficiency relation quantitatively and the bigger the value of mn, the smaller the efficiency at $\Pi = \Pi_{\max}$ point is.

Figure 3 shows the effects of the price ratio on the profit versus the efficiency for endoreversible heat engine, in this case, $n = 4$ and $m = 1.25$ are set. It can be seen that the price ratio has the significant influence on the relation between the profit and the efficiency and the price ratio changes the profit versus efficiency relation quantitatively.

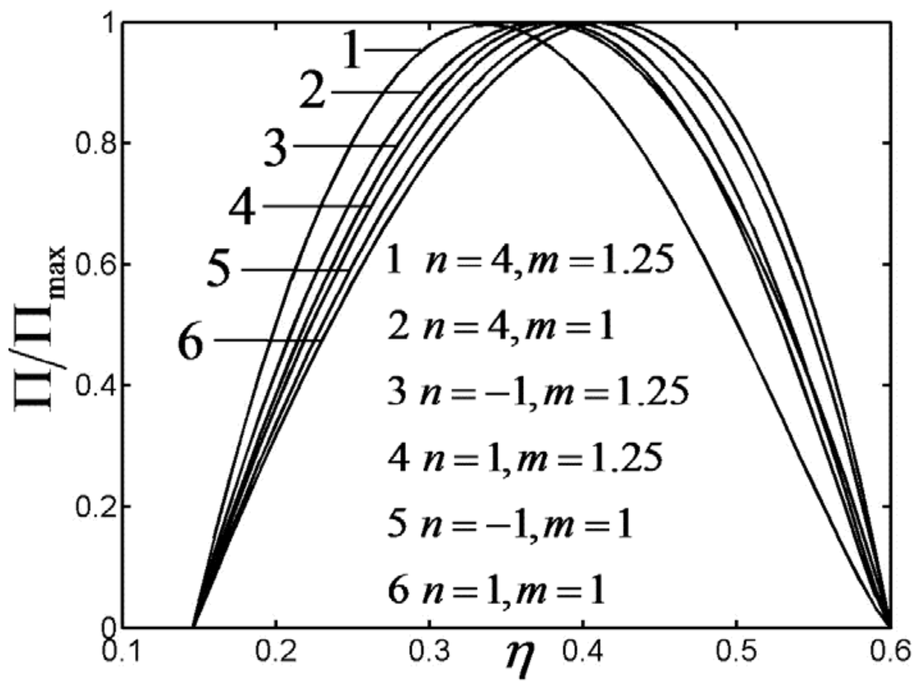

Figure 2. The effects of heat transfer laws on relation between the profit and efficiency

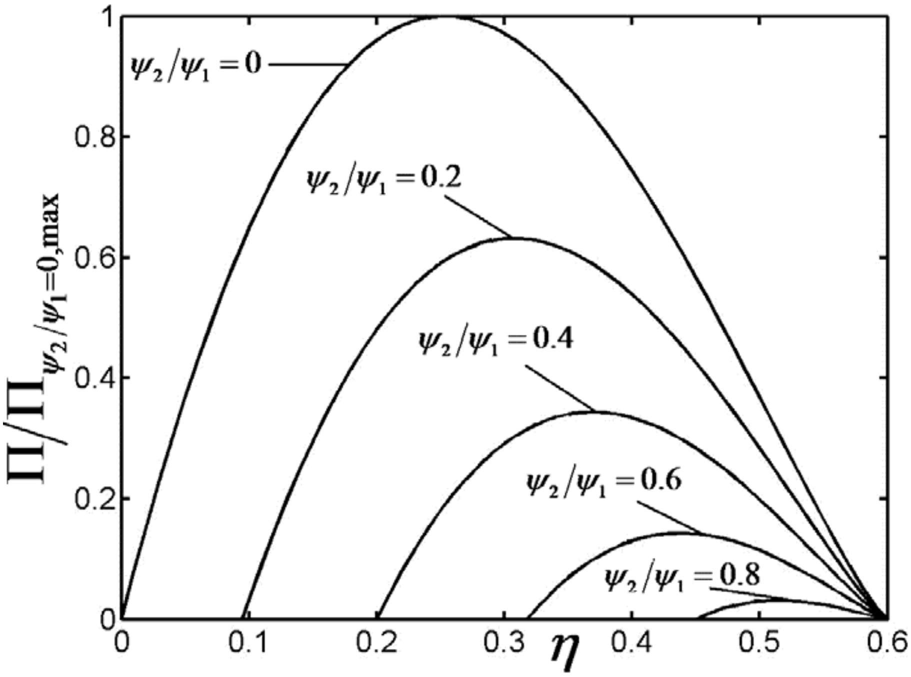

Figure 3. The effects of the price ratio on the profit versus the efficiency

6. Conclusion

This paper analyzes the exergoeconomic performance of the endoreversible Carnot heat engine with a complex heat transfer law, including generalized convective heat transfer law and generalized radiative heat transfer law, $q \propto (\Delta T^n)^m$, and obtained the fundamental optimal relation between the profit and the efficiency of the endoreversible Carnot heat engine. One seeks the economic optimization objective function instead of pure thermodynamic parameters by viewing the heat engine as a production process. It is shown that the economic and thermodynamic optimization converged in the limits $\psi_2/\psi_1 \to 0$ and $\psi_2/\psi_1 \to 1$. When the profit margin for exergy conversion is small, the maximum profit operation is near the minimum loss of exergy operation, while when the work is very cheap compared to the price of energy, the maximum profit operation is near the maximum power operation. The obtained results include those obtained in many literatures and can provide some theoretical guidance for the design of practical heat engines.

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