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A capillary-based method determining the permeability of sand layer for geothermal applications

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Abstract
The hydraulic permeability is a major parameter for modelling the heat transfer of borehole heat exchangers (BHEs) in an aquifer, where the groundwater advection has a significant impact on the thermal performance of BHEs. This paper presented a novel method determining the hydraulic permeability of sand layer using the theory of capillary rise (CR). In this method, based on the observation on the dynamic CR process, the equilibrium height together with the fluid velocity was determined by the steepest descent method to calculate the hydraulic permeability. Further, a laboratory experimental setup was built to measure the hydraulic permeability of screened dry sand with different grain diameters. The measured results were in a good agreement with the predictions by other theoretical models. Finally, the effects of the filling height and immersion depth of the capillary tube on the test accuracy were discussed. The present method can also be applied for other porous materials with the grain diameter of 0.1-0.6mm.

Keywords: Hydraulic permeability; Capillary rise; Equilibrium height; Washburn equation.

1. Introduction

In recent years, water source heat pumps (WSHPs) have been widely used for space heating and air-conditioning of buildings in those regions with abundant ground water resources [1]. Usually, because of their high porosity and permeability, sand and gravel aquifers have higher hydraulic conductivity than clay or unfractured granite aquifers, which would thus be easier to extract water from (e.g., using a pumping well). Therefore, for WSHP systems, the hydraulic permeability is a major parameter for modelling the heat transfer of borehole heat exchangers (BHEs) in a sandy aquifer, where the groundwater advection has a significant impact on the thermal performance of BHEs [2-3].In addition, as one of the most complex and important of the properties of porous media, the permeability is also indispensable in many other applications, such as soil engineering, geology engineering, and nuclear waste engineering.

At present, the methods determining the hydraulic permeability of porous media can be classified into two categories: empirical or semi-empirical approach, and experimental approach. The first approach started from the late 1800s, when Hazen presented a simple estimation formula based on the representative grain diameter [4]. Later, considering other parameters including effective diameter, porosity or void ratio, specific surface, and grain shapes, tens of empirical models have been developed during the past decades, which can be seen in many previous studies [5-7]. For example, Kozeny-Carman equation is one of the most widely used [8]. This equation is not appropriate for either soils with

the effective size above 3mm or for clayey soils. More recently, Chapui [9] presented a new model to predict the saturated hydraulic conductivity of sand and gravel using effective diameter and void ratio. This model can be extended to natural nonplastic silty soils, but not to crushed soils or plastic silty soils. Lei et al [10] presented an updated regression model with good prediction accuracy for washed sand. The second method is based on the Darcy's law, which can be broadly classified into two types: laboratory test and field test. The former mainly includes constant-head method and falling-head method, and the latter includes pump test in wells, infiltration test, and slug test. However, the accuracy of field tests is easily affected by the complex hydrogeological conditions. In summary, although it has long been recognized that the hydraulic conductivity is related to the grain-size distribution of granular porous media, it can be seen that this correlation is not easily established. In fact, the hydraulic conductivity is correlated to soil properties like pore size and particle size (grain size) distributions, and soil texture.
With the background above, the purpose of the present work was to present a novel laboratory method determining the hydraulic permeability of sand layer based on the capillary rise (CR) theory. In nature, this method was also based on the Darcy's law. The tests results showed that this method had a good accuracy and low requirement for the amount of test samples. The present method can provide an important basis for analyzing the heat transfer process of BHEs in a sand-based aquifer, and also be applied for other porous materials with the grain diameter of 0.1-0.6mm.

2. Theoretical modelling

In this study, the problem of a liquid (e.g. water) rising in a cylindrical capillary tube by the effect of capillary force is considered. Initially, the tube is filled with porous media (e.g. sand grains) with the height h_o and immersion depth Z_R.
In order to simplify the analysis, the following assumptions are made:
(i) Darcy's law is valid in the two monophasic regions;
(ii) The liquid is incompressible and immiscible;
(iii) There is no a noticeable pressure drop when the liquid contacts with the water reservoir.
The velocity of the wetting front v can be given by

$$v = \frac{dh}{dt} = \frac{K}{\phi} \frac{P_c - (\rho_1 - \rho_2)gh}{\mu_1(h + Z_R) + \mu_2(L - h - Z_R)} \tag{1}$$

where h is the CR height, K is the permeability, L is the length of the capillary tube, P_c is the macroscopic capillary pressure, ϕ is the porosity, μ_1 and μ_2 are the viscosity of water and air, and ρ_1 and ρ_2 are the density of water and air, respectively. Here, the limiting conditions are as follows: $L > h_o$ and $h_o \geq h + Z_R$.
If the variation of the capillary pressure during the rising process is neglected, the hydraulic equilibrium height h_e is calculated by

$$h_e = \frac{P_c}{(\rho_1 - \rho_2)g} \tag{2}$$

The microscopic velocity of fluid flowing through the porous media only by the gravity force is obtained by

$$v_g = \frac{K}{\phi} \frac{(\rho_1 - \rho_2)g}{\mu_1} \tag{3}$$

Based on Equation (1) to (3), the front velocity v can be rewritten as

$$v = \frac{V_g(h_e - h)}{h + Z_R + \mu_2 / \mu_1(L - Z_R - h)} \tag{4}$$

Notice that $\mu_2 << \mu_1$ for the case when water displaces air, Equation (4) can be simplified as

$$\frac{dh}{dt} = v_g \frac{(h_e - h)}{h + Z_R} \tag{5}$$

By means of the integration of Equation (5) with the initial condition $h = 0$ at $t = 0$, the time evolution of the wetting front's height h is calculated by

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$$t = \frac{h_e + Z_R}{v_g} \ln(\frac{h_e}{h_e - h}) - \frac{h}{v_g} \tag{6}$$

Equation (6) was also called as Washburn equation, which is frequently used to describe the CR phenomenon [11-12]. It can be seen from Equation (6) that once the equilibrium height h_e and the fluid velocity v_g are determined from the observation on the dynamic CR process, the hydraulic permeability of porous media can be estimated by Equation (3). The required porosity ϕ can be determined directly by laboratory tests.

3. Experimental investigations

3.1 Experimental setup

Figure 1 gives a simplified diagram of the experimental setup measuring the hydraulic permeability of porous media (e.g. sand layer) based on the CR method. In this experiment, sand samples were chosen by the screen sizer, and their grain diameters ranged from 0.1 to 0.6 mm. In order to reduce the effect of clay contents as possible, sand samples were cleaned using the distilled water, dried in an oven at 105 ^oC and then cooled at room temperature. The capillary rise occurred in an 80mm long capillary glass tube with the inner diameter of 11mm. The tube was installed on a support mount and its position could be adjustable if required. The tube was also labelled with a millimeter scale, and the minimum resolution was 1 mm.

Prior to the experiment, a stable water level in the reservoir was kept by the overhead tank. Once the bottom of the capillary tube contacted with water in the reservoir, the dynamic movement of the wetting front was observed and recorded by a digital camera. Each test lasted for 800-1000 s, depending on the initial sample conditions. After treating the images, the dynamic CR curve with time can be plotted.

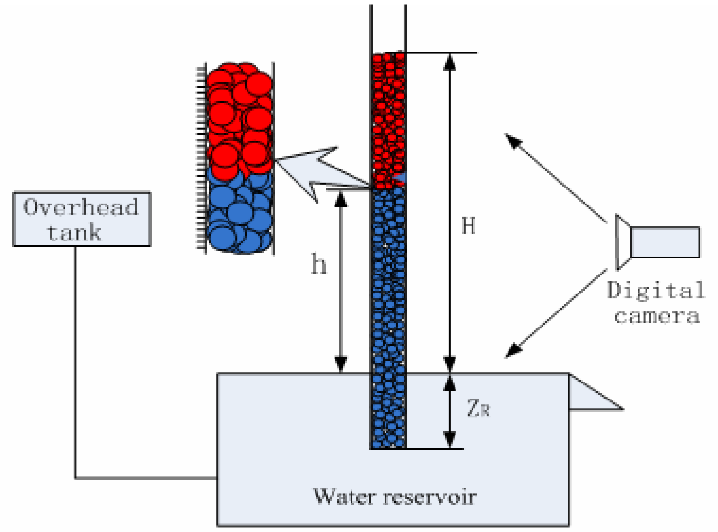

Figure 1. Diagram of the experimental setup measuring the hydraulic permeability of sand layer based on the CR method

3.2 Data treatment

The porosity ϕ was tested according to the national standard Soil Test Method (GB/T 50123-1999), and calculated by

$$\phi = 100 - \frac{100\rho_d}{1000G} \tag{7}$$

Based on the measurement of natural density ρ and moisture content w, the dry density ρ_d of sand grains was calculated by

$$\rho_d = \frac{\rho}{1 + 0.01w} \tag{8}$$

In this experiment, the dry density of sand samples ranged from 1600 to 1700 kg/m^3. Specific weight G was measured by a liquid densimeter. Usually, G ranges from 2.65 to 2.75 for most soil grains and was 2.67 on average in this experiment. For each sample, the maximum relative error of ϕ among three measurements was 0.5 %.
Figure 2. shows a typical dynamic CR process of dry sand with the grain diameter of 0.3-0.4mm. It can be seen that at the initial stage, there was a good agreement between the measured and prediction results by the Washburn equation of $h \sim t^{1/2}$. When the inflection point S was exceeded, however, the measured data began to deviate from the theoretical predictions. The main reason was that an abrupt interface between the wet and dry regions had been assumed in the Washburn's approach, where the wetted region was presumed to be completely saturated [13]. Due to the neglect of the dynamic saturation gradients, the steep saturation gradients near the front at early time decayed at later stages. Therefore, for the long-term CR process, the Washburn equation tended to be an increasing prediction error due to the failure of the assumptions in Equation (2). In spite of this, Equation (6) is valid at the early stage and then can be used to estimate the hydraulic permeability of porous media such as sand layer, glass beads, and other granular solids.

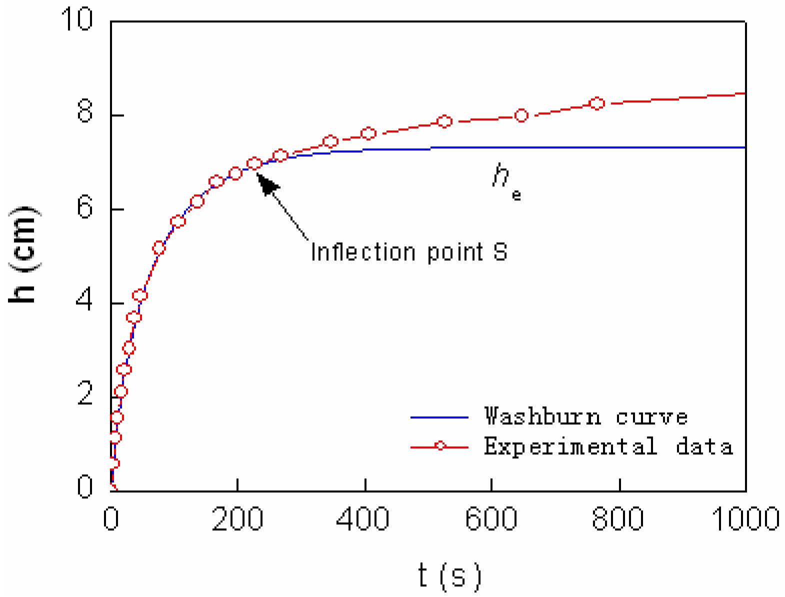

Figure 2. A typical $h-t$ curve of dry sand with the grain diameter of 0.3-0.4mm during the CR process

For the case shown in Figure 2, it can be seen that the hydraulic equilibrium height h_e ranged from 6 to 8 cm. In order to capture an accurate value of h_e, the steepest descent method (SDM) was used based on the software MATLAB 6.0. After calculation, h_e and v_g were determined as 7.14 cm and 0.1365 cm/s, respectively. Thus, the hydraulic permeability K was obtained as 5.48×10^{-7} cm^2. Compared with the test result (6.55×10^{-7} cm^2) by the conventional constant pressure head method, the relative error was 16.3 %. This indicated that the present method had enough accuracy for practical engineering applications.

4. Results and discussion

Figure 3 compares the hydraulic permeability under different diameters of sand grains. It can be seen that as the grain diameter increased, the hydraulic permeability tended to be growing. For example, for fine sand (0.1-0.25 mm) and medium sand (0.25-0.5 mm), the corresponding average hydraulic permeability was measured as 0.16×10^{-6} cm^2 and 0.55×10^{-6} cm^2, respectively. When the grain diameter exceeded the value of 0.5 mm (e.g. coarse sand of 0.58 mm), the hydraulic permeability reached 1.5×10^{-6} cm^2, which was almost ten times higher than that of fine sand.
In order to further check the effectiveness of the present method, the measured data were compared with the prediction results by three semi-empirical models. It was found that compared with Slichter's model (1898)[14], the present method showed a relative error of over 30 %, which was mainly caused by the incompleteness of the early empirical model. Compared with Kozeny-Carman's model (1927), the present method had a small relative error within the range of 0.35 mm, and when the grain diameter was

over 0.4 mm, there tended to be a large relative error. Further, compared with Chapuis's model (2004) and Lei's model (2006), the present method had a relative error of 17.2 % and 8.1 %, respectively. Therefore, the present method determining the hydraulic permeability of sand layer has a good measuring accuracy.

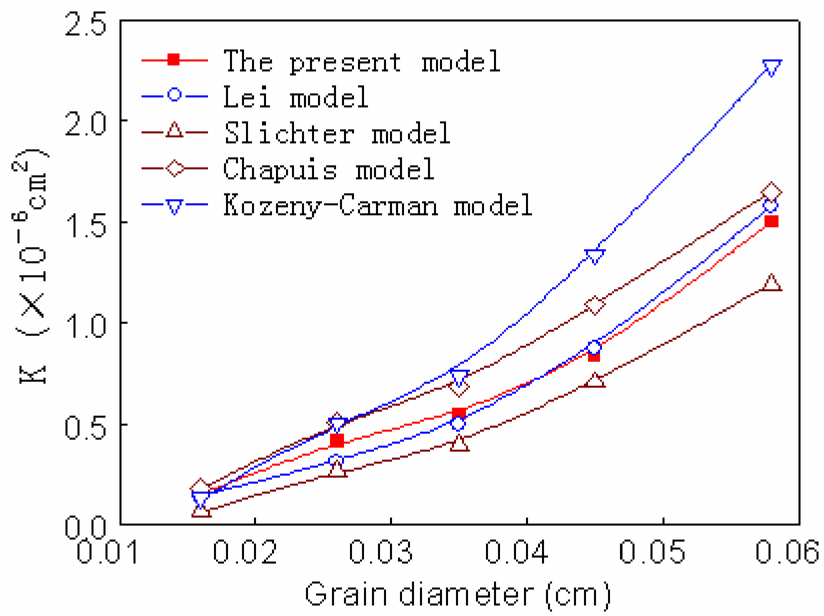

Figure 3. Variations of the hydraulic permeability with the diameter of sand grains

Figure 4 plots the relationship between the capillary equilibrium height and the reciprocal of the grain diameter. It can be seen that there was a good linear correlation, which was in an agreement with Lago's predictions [15]. In theory, if neglecting the effects of the temperature on the viscosity and surface tension of the fluid, $h_e \cong mD^{-1}$ when $h_e >> D$, where the coefficient m depends strongly on the contact angle θ. Generally, m increases with the increasing contact angle. For example, according to Lago's result, m was 2.587 and 2.818 when $\theta = 0$ and $\theta = 15.4°$, respectively. In this experiment, θ was measured as 40°, and the corresponding m was 3.47 after regression.

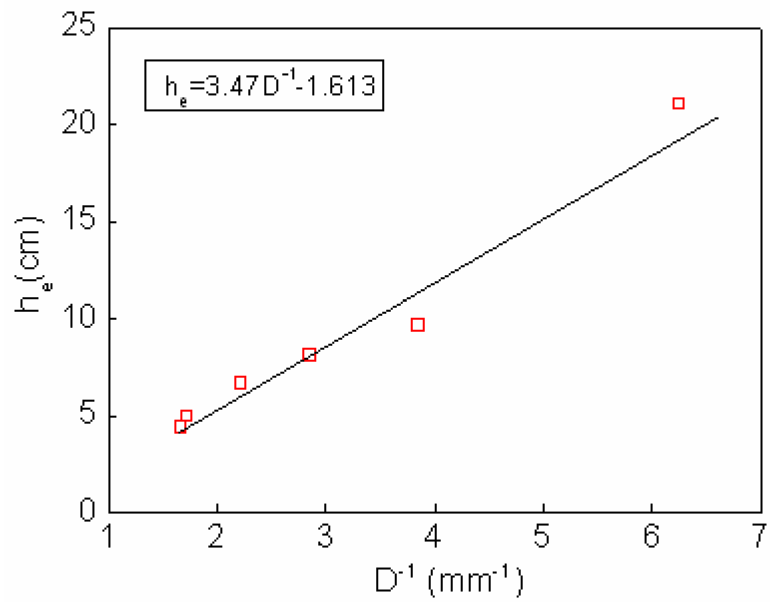

Figure 4. Variations of the capillary equilibrium height as a function of the grain diameter

In this experiment, it was found that within the range of 20-40 °C, the effect of the fluid temperature on the measured results can be neglected. At the same time, it was verified that the filling height h_o of sand samples and the immersion depth Z_R of the capillary tube at the initial state may have a great impact on the test accuracy. Figure 5 shows the hydraulic permeability and equilibrium time of sand layer under different initial filling heights, where the grain diameter of sand samples ranged from 0.5 to 0.65 mm. It can be seen that when the initial filling height of sand samples was lower than 20 cm, the inhibition effect of the gas among the pores on the capillary pressure during the CR process tended to be increasing, thereby reducing the hydraulic permeability and prolonging the final equilibrium time. As the initial filling height h_o increased gradually, the two results above reached steady. In the present experimental conditions, it was suggested that the filling height of sand samples should be higher than 30 cm.

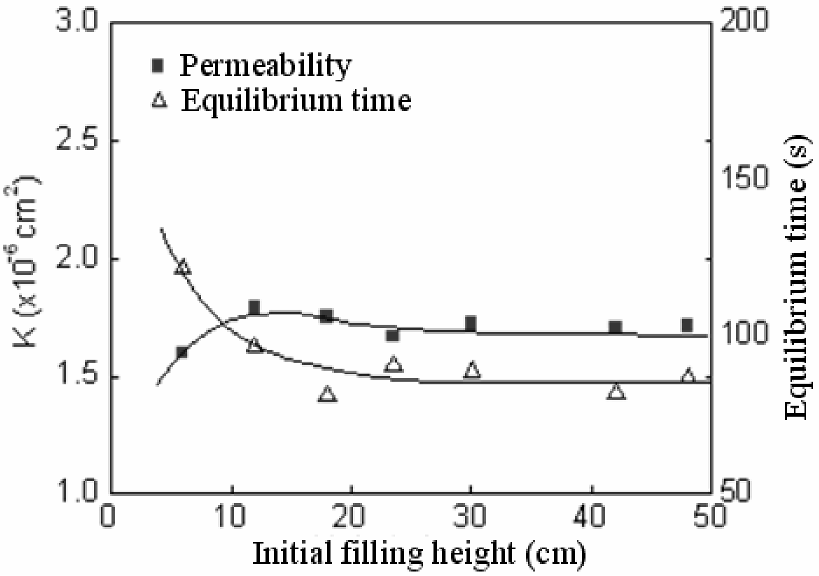

Figure 5. Hydraulic permeability and equilibrium time under different initial filling heights of sand

Figure 6 shows the hydraulic permeability and equilibrium time of sand layer under different initial immersion depths, where the grain diameter of sand samples ranged from 0.3 to 0.4 mm. It can be seen that when the immersion depth Z_R was less than 10 cm, there tended to be a lower hydraulic permeability. This phenomenon can be explained by the fact that the fluid velocity flowing through the porous media increases as Z_R decreases, thereby shortening the time reaching the equilibrium state and then enlarging the data treatment error to a certain extent. Such a situation also occurs for sand samples with a large grain diameter, where the rapid capillary movement reduces the equilibrium time. For a fixed h_o, the equilibrium time increased quickly with the increasing immersion depth, but the hydraulic permeability tended to be steady. In the present experimental conditions, it was suggested that the immersion depth of the capillary tube should range from 12 to 16 cm.

We also found an interesting experimental fact that for sand sample with the same grain diameter, the value of $v_g h_e$ tended to be a constant. As shown in Figure 7, the value of $v_g h_e$ increased with a positive exponent as the grain diameter increased. In fact, if assuming that $Z_R << h_e$, Equation (6) can be converted into a limit form as

$$h = 2v_g h_e t^{1/2} \tag{9}$$

As discussed above, the early CR process follows: $h \sim t^{1/2}$. By comparison, the slope ($\sim v_g h_e$) of Equation (9) can be treated as an approximate constant. This fact is useful for the parameter estimation of v_g and h_e, and finally improving the test accuracy. It should be noted that, except for sand samples, the present method can also be applied for other porous materials with the grain diameter of 0.1-0.6 mm. For porous media with smaller grain diameters, the adaptability of the present method needs more experimental validations, which are not discussed in this paper.

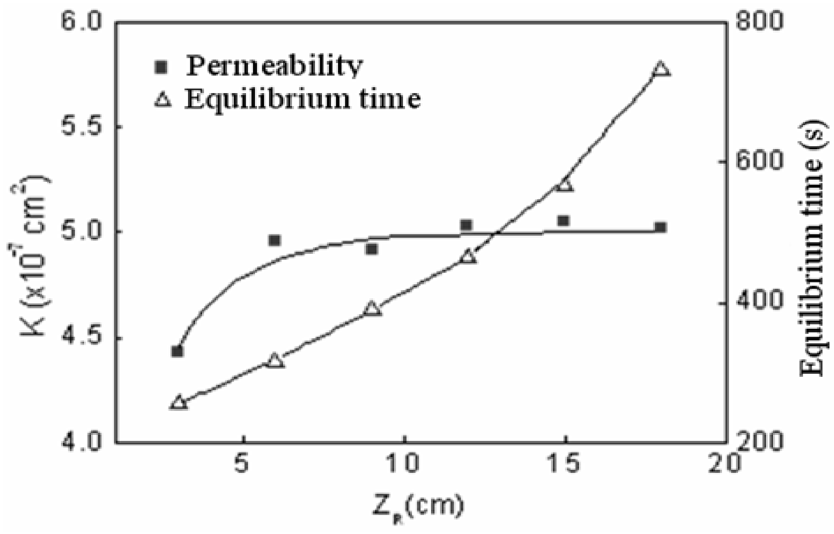

Figure 6. Hydraulic permeability and equilibrium time under different immersion depths

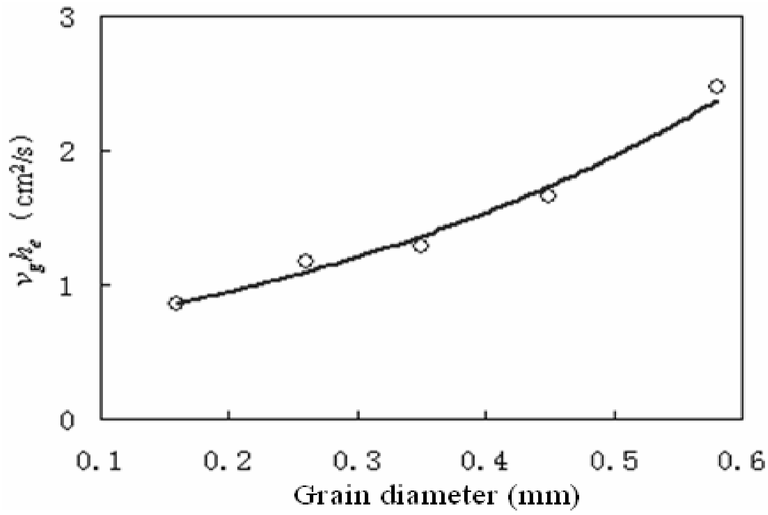

Figure 7. Values of $v_g h_e$ under different grain diameters

5. Conclusion

The permeability is a major parameter describing the hydraulic characteristics of porous media and especially plays an important role in the modelling of the heat transfer of BHEs in an aquifer for geothermal applications. This paper presented a novel laboratory method determining the hydraulic permeability of sand layer using the CR theory. In this method, based on the observation on the dynamic CR curve, the equilibrium height and fluid velocity were determined by the steepest descent method to obtain the hydraulic permeability. Further, an experimental setup was built to measure the hydraulic permeability of screened dry sand with different grain diameters. From the experimental results discussed above, the following conclusions can be obtained:

(i) The present measured results were in a good agreement with the predictions by the conventional test method and typical semi-empirical models. The present method can also be applied for other porous materials with the grain diameter of 0.1-0.6 mm.

(ii) The filling height of sand samples and the immersion depth of the capillary tube at the initial state had a great impact on the test accuracy. It is suggested that the filling height of sand samples should be higher than 30 cm and the immersion depth of the capillary tube should range from 12 to 16 cm.

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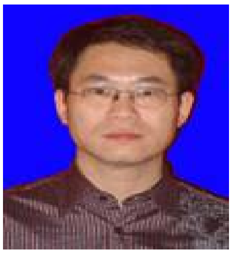
References

[1] Sanner B,Burkhard W, Current status of ground source heat pumps and underground thermal energy storage in Europe, Geothermics, 2003,32 (4–6),579–588.
[2] Chiasson A, Rees J, Spitler J, A preliminary assessment of the effects of groundwater flow on closed-loop ground-source heat pump systems, ASHRAE Transactions,2000, 106 (1), 380–393.
[3] Wang H, Qi C, Du H, Thermal performance of borehole heat exchanger under groundwater flow: A case study from Baoding, Energy and Buildings, 2009, 41(12), 1368-1373.
[4] Hazen A,Some physical properties of sands and gravels, with Special Reference to their Use in Filtration. 24th Annual Report of Massachusetts State Board of Health, 1892, 34,539-556.
[5] Shepherd RG, Correlations of permeability and grain size, Groundwater,1989,27(5),663-638.
[6] Uma KO, Egboka BC, Onuoha KM,New statistical grain-size method for evaluating the hydraulic conductivity of sandy aquifers, Journal of Hydrology,1989,108,367-386.
[7] Alyamani MS, Sen Z, Determination of hydraulic conductivity from complete grain-size distribution curves, Ground Water,1993, 31(4),551-555.
[8] Carman PC, Flow of Gases through Porous Media. Butterworths Scientific Publications, London, 1956.
[9] Chapuis RP, Predicting the saturated hydraulic conductivity of sand and gravel using effective diameter and void ratio, 2001, Canadian Geotechnical Journal, 2004, 41(5),787-795.
[10] Lei S, Hao J, An Z, Experimental study on air based and water based permeability of screened washed sands,Journal of Engineering Thermophysics,2006,27(2),105-109.
[11] Washburn E, The dynamics of capillary flow, Physics Review, 1921,17(3),273-283.
[12] Ergun S, Fluid flow through packed columns, Chemical Engineering Progress, 1952, 48(2), 89-94.
[13] Lockington DA, Parlange JY, A new equation for macroscopic description of capillary rise in porous media, Journal of Colloid and Interface Science, 2004, 278(2), 404-409.
[14] Slichter CS, Theoretical investigations of the motion of ground waters, 19th Annual Report United States Geological Survey, Washington, 1898, 295-384.
[15] Lago M, Mariela A, Capillary rise in porous media, Physica A, 2001, 289(1-2), 1-17.



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