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Effect of heat source on MHD free convection flow past an oscillating porous plate in the slip flow regime

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Abstract

This paper investigates the effect of heat source on free convective flow of a viscous incompressible electrically conducting fluid through a porous medium bounded by an oscillating porous plate in the slip flow regime in presence of a transverse magnetic field. The governing equations of the flow field are solved analytically and the expressions for velocity, temperature, skin friction τ and the heat flux in terms of Nusselts number N_u are obtained. The effects of the important flow parameters such as magnetic parameter M, permeability parameter K_p, Grashof number for heat transfer G_r, heat source parameter S and rarefaction parameter R on the velocity of the flow field are analyzed quantitatively with the help of figures.

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Keywords: MHD flow; Free convection; Porous medium; Oscillating plate; Slip flow; Heat source.

1. Introduction

The phenomenon of free convective flow in presence of heat source has been a subject of interest of many researchers because of its possible applications to geophysical sciences, astrophysical sciences and in cosmical studies. Such flows arise either due to unsteady motion of boundary or boundary temperature. The study of fluctuating flow finds application in paper industry and many other technological fields. Unsteady oscillatory free convective flows play an important role in chemical engineering, turbo-machinery and in aerospace technology. In view of these, much attention have been given by several researchers towards fluctuating flows.

Messiha [1] discussed the problem of laminar boundary layers in oscillatory flow along an infinite flat plate with variable suction. Singh *et al.* [2] analyzed the unsteady MHD free convective flow through a porous medium between two infinite vertical parallel oscillating porous plates with different amplitude. Devi and Jothimani [3] studied the heat transfer in unsteady MHD oscillatory flow. Deka *et al.* [4] showed the effect of free convection on MHD flow past an infinite vertical oscillating plate with constant heat flux. Taneja and Jain [5] solved the hydrodynamic flow in slip flow regime with time-dependent suction. Singh and his co-workers [6] analyzed the heat and mass transfer in MHD flow of a viscous fluid past a vertical plate under oscillatory suction velocity. Asghar *et al.* [7] studied the flow of a non-Newtonian fluid induced due to the oscillations of a porous plate. Hayat *et al.* [8] reported the flow of a visco-elastic fluid on an oscillating plate. Krishna *et al* [9] analyzed the hydromagnetic oscillatory flow of a second order Rivlin-Erickson fluid in a channel.

Ogulu and Prakash [10] solved numerically the oscillatory MHD slip flow along a porous vertical wall in a medium with variable suction in the presence of radiation employing finite difference scheme. Singh and Gupta [11] discussed the MHD free convective flow of a viscous fluid through a porous medium bounded by an oscillating porous plate in slip flow regime with mass transfer. Khandelwal and Jain [12] analyzed the unsteady MHD flow of stratified fluid through porous medium over a moving plate in slip flow regime. Ogulu and Abbey [13] had given a simulation of heat transfer on an oscillatory blood flow in an indented porous tube. Das and his associates [14] estimated the mass transfer effects on free convective MHD flow of a viscous fluid bounded by an oscillating porous plate in the slip flow regime with heat source. Sharma and Singh [15] reported the unsteady MHD free convective flow and heat transfer along a vertical porous plate with variable suction and internal heat generation. Das *et al.* [16] analyzed the magnetohydrodynamic unsteady flow of a viscous stratified fluid through a porous medium past a porous flat moving plate in the slip flow regime with heat source. Das and his co-workers [17] discussed the mass transfer effects on MHD flow and heat transfer past a vertical porous plate through a porous medium under oscillatory suction and heat source. Recently, Das and his team [18] investigated the hydromagnetic convective flow past a vertical porous plate through a porous medium with suction and heat source.

In the proposed study, we analyze the effect of heat source on the free convective flow of a viscous incompressible electrically conducting fluid through a porous medium bounded by an oscillating porous plate in the slip flow regime in presence of a transverse magnetic field. The expressions for velocity, temperature, skin friction τ and the heat flux in terms of Nusselts number N_u are obtained by solving analytically the governing equations of the flow field and the effects of the pertinent parameters such as magnetic parameter M, permeability parameter K_p, Grashof number for heat transfer G_r, heat source parameter S and rarefaction parameter R on the velocity of the flow field are discussed with the help of figures.

2. Formulation of the problem

Consider the free convective flow of a viscous incompressible electrically conducting fluid bounded by an oscillating porous plate through a porous medium in the slip flow regime with heat source in presence of a transverse magnetic field B_0. Let u and v be the velocity components in x- and y- directions respectively. All the physical variables are functions of y and t only. The Reynolds number is assumed to be very small and the induced magnetic field due to the flow is neglected with respect to the applied magnetic field and the pressure in the flow field is assumed to be constant. If v_0 denotes the suction/injection velocity at the plate, the equation of continuity is

$$\frac{\partial v}{\partial y} = 0 \tag{1}$$

Under the condition $y = 0$, $v = -v_0$ everywhere.

Now the governing boundary layer equations of the flow field in non-dimensional form are

$$\frac{\partial u}{\partial t} - v_0 \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + g\beta\left(T - T_\infty\right) - \frac{\nu}{K_0} u - \frac{\sigma B_0^2}{\rho} u \tag{2}$$

$$\frac{\partial T}{\partial t} - v_0 \frac{\partial T}{\partial y} = k \frac{\partial^2 T}{\partial y^2} - S(T - T_\infty) \tag{3}$$

where ν is the kinematic viscosity, k is the thermal diffusivity, K_0 is the permeability coefficient, β is the volumetric coefficient of expansion for heat transfer, ρ is the density , σ is the electrical conductivity of the fluid , g is the acceleration due to gravity , T is the temperature, T_∞ is the temperature of the fluid far away from the plate.

Now the first order velocity slip boundary conditions of the problem when the plate executes linear harmonic oscillations in its own plane are given by

$$u = U_0 e^{i\omega t} + L_1 \frac{\partial u}{\partial y}, \quad T = T_w, \quad \text{at } y = 0,$$

$$u \to 0,\ T \to T_\infty,\ \text{as } y \to \infty \tag{4}$$

where $L_1 = \frac{(2-m)}{m} L$ and $L = \mu\left(\frac{\pi}{2p\rho}\right)^{\frac{1}{2}}$ is the mean free path and m is the Maxwell's reflexion coefficient.

We now introduce the following non-dimensional quantities

$$y^* = U_0 \frac{y}{\nu},\ u^* = \frac{u}{U_0},\ T^* = \frac{T - T_\infty}{T_w - T_\infty},\ t^* = U_0^2 \frac{t}{\nu},\ v_0^* = \frac{v_0}{U_0},\ \omega^* = \frac{\nu\omega}{U_0^2},\ S^* = \frac{\nu S}{U_0^2},\ R = U_0 \frac{L_1}{\nu},$$

$$M = \frac{B_0}{U_0}\left(\frac{\nu\sigma}{\rho}\right)^{\frac{1}{2}}, P_r = \frac{\nu}{k},\ K_p = \frac{K_0 U_0^2}{\nu^2}, G_r = \nu g \beta \frac{(T_w - T_\infty)}{U_0^3} \tag{5}$$

where S is the heat source parameter, R is the rarefaction parameter, M is the Hartmann number/magnetic parameter, P_r is the Prandtl number, K_p is the permeability parameter and G_r is the Grashof number for heat transfer.

Introducing the non-dimensional parameters (5) mentioned above in equations (2) and (3) and dropping the asterisks, the governing equations now reduce to the following non-dimensional forms:

$$\frac{\partial u}{\partial t} - v_0 \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + G_r T - \left(M^2 + \frac{1}{K_p}\right) u \tag{6}$$

$$\frac{\partial T}{\partial t} - v_0 \frac{\partial T}{\partial y} = \frac{1}{P_r} \frac{\partial^2 T}{\partial y^2} - ST \tag{7}$$

The boundary conditions now reduce to

$$u = e^{i\omega t} + R \frac{\partial u}{\partial y}, \quad T = 1, \quad \text{at } y = 0,$$

$$u \to 0,\ T \to 0,\ \text{as } y \to \infty \tag{8}$$

3. Method of solution

For solving equations (6) and (7), we assume the following for the velocity and temperature of the flow field.

$$u = u_0 + u_1 e^{i\omega t} \tag{9}$$

$$T = T_0 + T_1 e^{i\omega t} \tag{10}$$

Using equations (9) and (10) in equations (6) and (7) and separating the harmonic and non-harmonic terms, we get

$$u_0'' + v_0 u_0' - \left(M^2 + \frac{1}{K_p}\right) u_0 = -G_r T_0 \tag{11}$$

$$u_1'' + v_0 u_1' - \left(M^2 + \frac{1}{K_p} + i\omega\right) u_1 = -G_r T_1 \tag{12}$$

$$T_0'' + P_r v_0 T_0' + S P_r T_0 = 0 \tag{13}$$

$$T_1''+P_r v_0 T_1'+(S-i\omega)P_r T_1=0 \tag{14}$$

The corresponding boundary conditions are

$$u_0=R\frac{\partial u_0}{\partial y},\quad u_1=1+R\frac{\partial u_1}{\partial y},\ T_0=1,\ T_1=0,\quad \text{at } y=0,$$

$$u_0\to 0,\ u_1\to 0,\ T_0\to 0,\ T_1\to 0,\ \text{as } y\to\infty \tag{15}$$

Solving equations (11)-(14) under boundary conditions (15), we get the following solutions for velocity, temperature and the concentration distributions of the flow field.

$$T_0=e^{m_1 y} \tag{16}$$

$$T_1=0 \tag{17}$$

$$u_0=A_1 e^{-m_2 y}-A_2 e^{m_1 y} \tag{18}$$

$$u_1=A_3 e^{m_4 y} \tag{19}$$

where $m_1=\frac{1}{2}\left[-P_r v_0-\sqrt{P_r^2 v_0^2-4SP_r}\right]$, $m_2=\frac{1}{2}\left[P_r v_0+\sqrt{P_r^2 v_0^2-4P_r(S-i\omega)}\right]$,

$m_3=\frac{1}{2}\left[-P_r v_0+\sqrt{P_r^2 v_0^2-4P_r(S-i\omega)}\right]$, $m_4=-\frac{1}{2}\left[-v_0+\sqrt{v_0^2+4\left(M^2+\frac{1}{K_p}+i\omega\right)}\right]$,

$$A_1=\frac{1}{(Rm_2+1)}\left[A_2(1-m_1)\right],\ A_2=\frac{G_r}{(m_1+m_2)(m_1-m_3)},\ A_3=\frac{1}{(1-Rm_4)} \tag{20}$$

Using equations (16)-(19) in equations (9) and (10), the solutions for velocity and temperature of the flow field are given by

$$u=A_1 e^{-m_2 y}-A_2 e^{m_1 y}+A_3 e^{m_4 y+i\omega t} \tag{21}$$

$$T=e^{m_1 y} \tag{22}$$

Skin friction

Using equation (21), the skin friction at the wall is given by

$$\tau=\left(\frac{\partial u}{\partial y}\right)_{y=0}=-m_2 A_1-m_1 A_2+m_4 A_3 e^{i\omega t} \tag{23}$$

Heat flux

The rate of heat transfer i. e. the heat flux at the wall in terms of Nusselt number is given by

$$N_u=\left(\frac{\partial T}{\partial y}\right)_{y=0}=m_1 \tag{24}$$

4. Discussions and results

The effect of heat source on free convective flow of a viscous incompressible electrically conducting fluid through a porous medium bounded by an oscillating porous plate in the slip flow regime in presence of a transverse magnetic field has been considered. The effects of the important flow parameters such as magnetic parameter M, permeability parameter K_p, rarefaction parameter R and the heat source parameter S on the velocity profiles of the flow field have been discussed with the help of Figures 1-3.

The velocity of the flow field is observed to change substantially with the variation of the flow parameters. The major flow parameters affecting the velocity of the flow field are magnetic parameter M, heat source parameter S, rarefaction parameter R and permeability parameter K_p. The effects of these parameters on the velocity field are depicted in Figures 1-3.

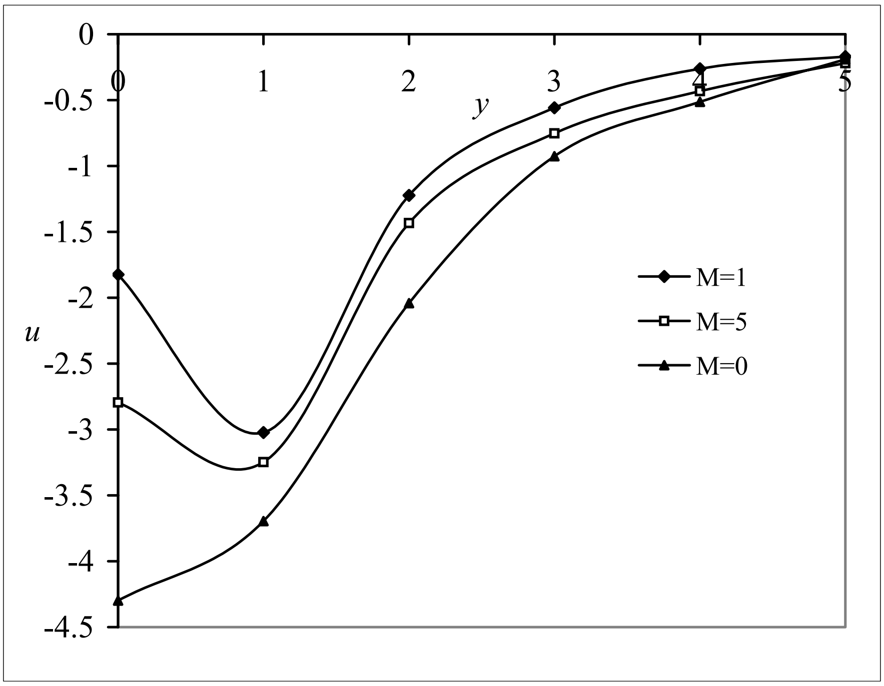

Figure 1. Velocity profiles against y for different values of M with G_r=5, K_p=1, S=0.5, P_r =0.71, v_0 =2, $\omega t = \pi/2$ and $\omega = 2$

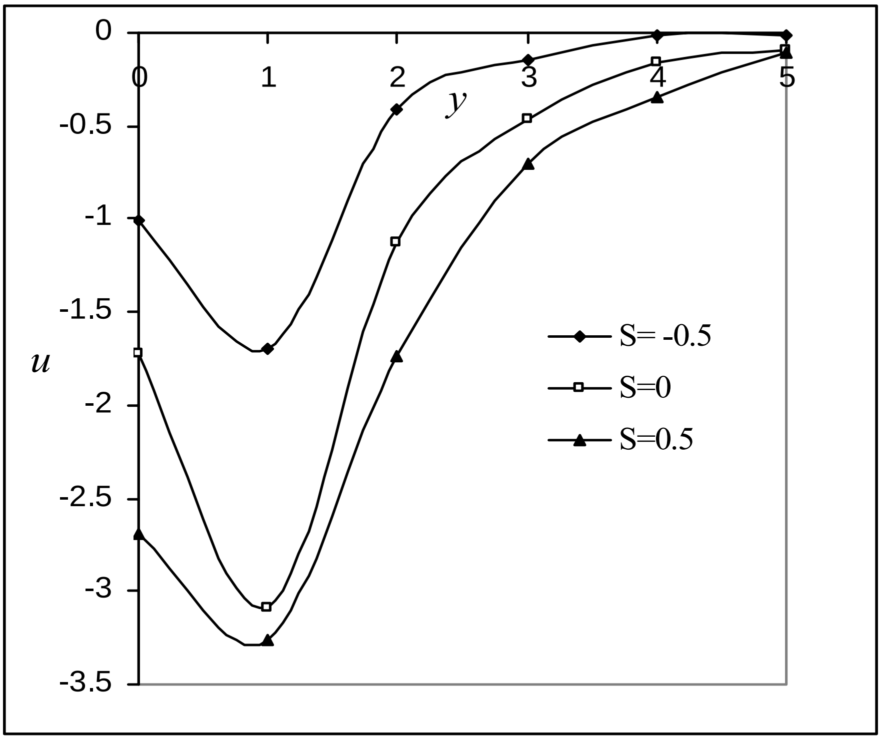

Figure 2. Velocity profiles against y for different values of S with M=1, G_r=5, K_p=1, P_r =0.71, v_0 =2, $\omega t = \pi/2$ and $\omega = 2$

Figure 1 elucidates the effect of magnetic parameter *M* on the velocity profiles of the flow field. Curve with *M* =0 corresponds to the case of non-MHD flow. A comparison of the curves of the figure shows that a growing magnetic parameter *M* leads to accelerate the magnitude of velocity of the flow field at all points. Further, the velocity profiles assume maximum value in non-MHD flow than that in case of MHD flow. The effect of growing heat source parameter *S* on the velocity of the flow field is presented in Figure 2. It is observed that the heat source parameter leads to accelerate the magnitude of the velocity of the flow field at all points. Curve with *S*=0 corresponds to absence of heat source. Figure 3 discusses the effects of rarefaction parameter *R* and the permeability parameter K_p on the velocity profiles of the flow field. The rarefaction parameter *R* enhances the magnitude of the velocity of the flow field at all points. A growing permeability parameter K_p increases the magnitude of the velocity of the flow field at all points.

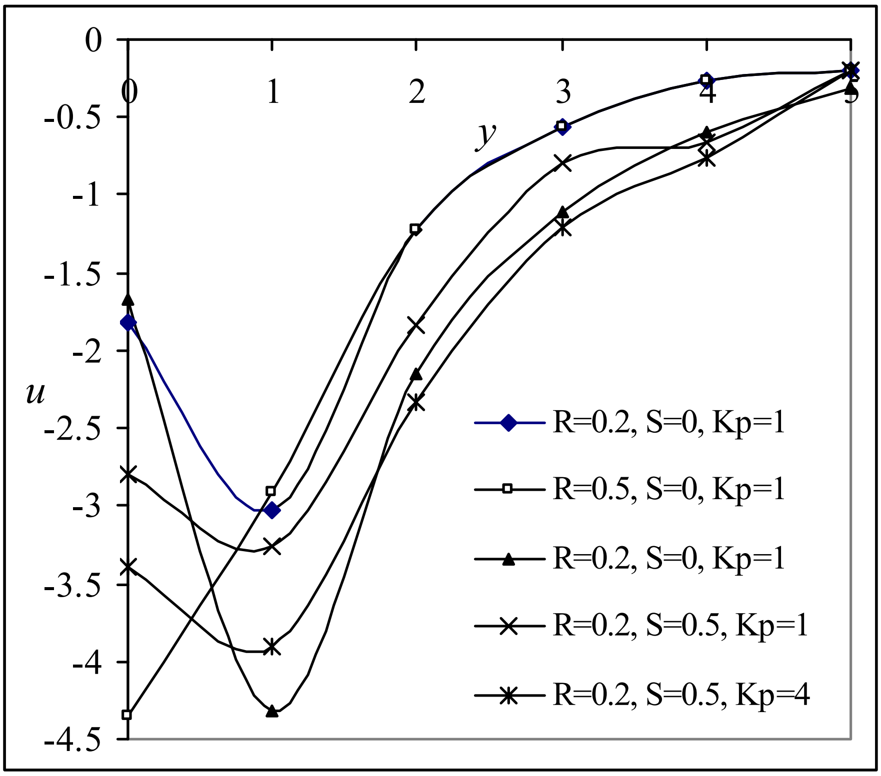

Figure 3. Velocity profiles against y for different values of R, S, K_p with M=1, G_r=5, P_r =0.71, v_0 =2, ωt = π/2 and ω = 2

5. Conclusion

We conclude below some of the results of physical interest from the above study.

1. A growing magnetic parameter *M* leads to accelerate the magnitude of velocity of the flow field at all points.
2. The effect of growing heat source parameter *S* leads to accelerate the magnitude of the velocity of the flow field at all points.
3. The rarefaction parameter *R* enhances the magnitude of the velocity of the flow field at all points.
4. A growing permeability parameter K_p increases the magnitude of the velocity of the flow field at all points.

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