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Natural convection mass transfer hydromagnetic flow past an oscillating porous plate with heat source in a porous medium

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Abstract
This paper analyzes the effect of mass transfer on natural convection hydromagnetic flow of a viscous incompressible fluid through a porous medium past an oscillating porous plate in a porous medium with heat source. The governing equations of the flow field are solved analytically and the expressions for velocity and temperature of the flow field, skin friction τ and the heat flux in terms of Nusselts number N_u are obtained. The effects of the important flow parameters such as magnetic parameter M, permeability parameter K_p, Grashof number for heat and mass transfer G_r, G_c, Schmidt number S_c, heat source parameter S and the Prandtl number P_r on the velocity and temperature of the flow field are to be discussed with the help of figures. It is observed that a growing magnetic parameter M retards the magnitude of the velocity of the flow field at all points due to the action of the Lorentz force on the flow field. The heat source parameter S has an accelerating effect on the magnitude of the velocity of the flow field at all points. The effect of growing Grashof number for mass transfer G_c and the permeability parameter K_p is to enhance the velocity (absolute value) of the flow field at all points. An increase in Schmidt number S_c is to increase the magnitude of the velocity of the flow field at all points. A growing rarefaction parameter R enhances the magnitude of the velocity of the flow field at all points.

Keywords: Natural convection; Mass transfer; Hydromagnetic flow; Porous medium; Oscillating plate; Heat source.

1. Introduction

Hydromagnetic flow through a porous medium with heat and mass transfer is gathering momentum day by day in view of its possible applications to geophysical sciences, astrophysical sciences and also in industry. The study of fluctuating flow is important in paper industry and many other technological fields. In view of these applications several researchers have given much attention towards fluctuating flows of viscous incompressible fluids past an infinite plate.

The nature of vertical natural convection flow resulting from the combined buoyancy effects of thermal and mass diffusion effects was analyzed by Gebhart and Pera [1]. Georgantopoulos *et al*. [2] estimated the effect of free convection and mass transfer on the hydro-magnetic oscillatory flow past an infinite vertical porous plate. Hossain and Begum [3] discussed the effect of mass transfer and free convection on the flow past a vertical plate. Bejan and Khair [4] studied the heat and mass transfer effects by natural convection in a porous medium. Hossain and Begum [5] discussed the effect of mass transfer on the unsteady flow past an accelerated vertical porous plate with variable suction. Raptis and Perdikis [6] analyzed the oscillatory flow through a porous medium in presence of free convection. Sattar [7] reported the free and forced convection boundary layer flow through a porous medium with large suction, Chamkha [8] studied the hydromagnetic three-dimensional free convection flow on a vertical stretching surface with heat generation/absorption.

The effect of combined heat and mass transfer hydromagnetic flow by natural convection from a permeable surface embedded in a fluid saturated porous medium was analyzed by Chamkha and Khaled [9]. Nagraju *et al*. [10] discussed the simultaneous radiative and convective heat transfer in a variable porosity medium. The problem of heat and mass transfer in MHD flow of a viscous fluid past a vertical plate under oscillatory suction velocity was studied by Singh and his co-workers [11]. Hayat *et al*. [12] discussed the flow of a visco-elastic fluid on an oscillating plate. Jain and Gupta [13] have reported the unsteady hydromagnetic thermal boundary layer flow past an infinite porous surface in the slip flow regime. Singh and Gupta [14] studied the MHD free convective mass transfer flow of a viscous fluid through a porous medium bounded by an oscillating porous plate in slip flow regime.

Sharma and Singh [15] estimated the unsteady MHD free convective flow and heat transfer along a vertical porous plate with variable suction and internal heat generation. Das and his associates [16] studied the mass transfer effects on MHD flow and heat transfer past a vertical porous plate through a porous medium under oscillatory suction and heat source. Das *et al*. [17] reported the hydromagnetic convective flow past a vertical porous plate through a porous medium with suction and heat source. Natural convection unsteady magnetohydrodynamic mass transfer flow past an infinite vertical porous plate in presence of suction and heat sink was studied by Das and his team [18] Recently, Das and his co-workers [19] analyzed the hydromagnetic mixed convective mass diffusion boundary layer flow past an accelerated vertical porous plate through a porous medium with suction by finite difference scheme.

The study reported herein analyzes the effect of mass transfer on natural convection hydromagnetic flow of a viscous incompressible fluid in a porous medium past an oscillating porous plate with heat source. The governing equations of the flow field are solved analytically and the expressions for velocity and temperature of the flow field, skin friction τ and the heat flux in terms of Nusselts number N_u are obtained. The effects of the important flow parameters such as magnetic parameter M, porosity parameter K_p, Grashof number for heat and mass transfer G_r, G_c, Schmidt number S_c, heat source parameter S and the Prandtl number P_r on the flow field are to be discussed with the help of figures.

2. Formulation of the problem

Consider the natural convection mass transfer flow of a viscous incompressible fluid past an oscillating porous plate with heat source in a porous medium in presence of a transverse magnetic field B_0. Let u and v be the velocity components in x- and y- directions respectively. All the physical variables are functions of y and t only. The Reynolds number is assumed to be very small and the induced magnetic field due to the flow is neglected with respect to the applied magnetic field and the pressure in the flow field is assumed to be constant. If v_0 denotes the suction/injection velocity at the plate, the equation of continuity is

$$\frac{\partial v}{\partial y} = 0 \tag{1}$$

Under the condition $y = 0$, $v = -v_0$ everywhere.

Now the governing boundary layer equations of the flow field in non-dimensional form are

$$\frac{\partial u}{\partial t} - v_0 \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) + g\beta^*(C - C_\infty) - \frac{\nu}{K_0} u - \frac{\sigma B_0^2}{\rho} u \tag{2}$$

$$\frac{\partial T}{\partial t} - v_0 \frac{\partial T}{\partial y} = k \frac{\partial^2 T}{\partial y^2} - S(T - T_\infty) \tag{3}$$

$$\frac{\partial C}{\partial t} - v_0 \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2}, \tag{4}$$

where g is the acceleration due to gravity , ν is the kinematic viscosity, k is the thermal diffusivity, K_0 is the permeability coefficient, β is the volumetric coefficient of expansion for heat transfer, β^* is the volumetric coefficient of expansion for mass transfer, ρ is the density , σ is the electrical conductivity of the fluid , T is the temperature, T_∞ is the temperature of the fluid far away from the plate, C is the concentration, C_∞ is the concentration of the fluid far away from the plate and D is the molecular diffusivity.

Now the first order velocity slip boundary conditions of the problem when the plate executes linear harmonic oscillations in its own plane are given by

$$u = U_0 e^{i\omega t} + L_1 \frac{\partial u}{\partial y}, \quad T = T_w, \quad C = C_w \quad \text{at } y = 0,$$

$$u \to 0, \; T \to T_\infty, \; C \to C_\infty \quad \text{as} \quad y \to \infty \tag{5}$$

where $L_1 = \frac{(2-m)}{m} L$ and $L = \mu \left(\frac{\pi}{2 p \rho} \right)^{\frac{1}{2}}$ is the mean free path and m is the Maxwell's reflection coefficient.

We now introduce the following non-dimensional quantities

$$y^* = U_0 \frac{y}{\nu}, \quad u^* = \frac{u}{U_0}, \quad T = \frac{T - T_\infty}{T_w - T_\infty}, \quad C = \frac{C - C_\infty}{C_w - C_\infty}, \quad t^* = U_0^2 \frac{t}{\nu}, \; v_0^* = \frac{V_0}{U_0}, \; \omega^* = \frac{\nu\omega}{U_0^2},$$

$S^* = \frac{\nu S}{U_0^2}$ (Heat source parameter), $R = U_0 \frac{L_1}{\nu}$ (Rarefaction parameter),

$M = \frac{B_0}{U_0} \left(\frac{\nu\sigma}{\rho} \right)^{\frac{1}{2}}$ (Hartmann number/ magnetic parameter), $P_r = \frac{\nu}{k}$ (Prandtl number),

$K_p = \frac{K_0 U_0^2}{\nu^2}$ (Permeability parameter), $G_r = \nu g \beta \frac{(T_w - T_\infty)}{U_0^3}$ (Grashof number for heat transfer),

$G_c = \nu g \beta^* \frac{(C_w - C_\infty)}{U_0^3}$ (Grashof number for mass transfer),

$S_c = \frac{\nu}{D}$ (Schmidt number). (6)

Introducing the non-dimensional parameters mentioned above (6) in equations (2)-(4) and dropping the asterisks, the governing equations now reduce to the following non-dimensional forms:

$$\frac{\partial u}{\partial t} - v_0 \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + G_r T + G_c C - \left(M^2 + \frac{1}{K_p} \right) u \tag{7}$$

$$\frac{\partial T}{\partial t} - v_0 \frac{\partial T}{\partial y} = \frac{1}{P_r} \frac{\partial^2 T}{\partial y^2} - ST \tag{8}$$

$$\frac{\partial C}{\partial t} - v_0 \frac{\partial C}{\partial y} = \frac{1}{S_c} \frac{\partial^2 C}{\partial y^2} \qquad (9)$$

The boundary conditions now reduce to

$$u = e^{i\omega t} + R\frac{\partial u}{\partial y}, \quad T = 1, \quad C = 1 \quad \text{at } y = 0,$$
$$u \to 0, \; T \to 0, \; C \to 0 \quad \text{as } y \to \infty. \qquad (10)$$

3. Method of solution

For solving equations (7)-(9), we assume the following for the velocity, temperature and concentration distribution of the flow field.

$$u = u_0 + u_1 e^{i\omega t}, \qquad (11)$$

$$T = T_0 + T_1 e^{i\omega t}, \qquad (12)$$

$$C = C_0 + C_1 e^{i\omega t}. \qquad (13)$$

Using equations (11)-(13) in equations (7)-(9) and separating the harmonic and non-harmonic terms, we get

$$u_0'' + v_0 u_0' - \left(M^2 + \frac{1}{K_p} \right) u_0 = -G_r T_0 - G_c C_0, \qquad (14)$$

$$u_1'' + v_0 u_1' - \left(M^2 + \frac{1}{K_p} + i\omega \right) u_1 = -G_r T_1 - G_c C_1, \qquad (15)$$

$$T_0'' + P_r v_0 T_0' + S P_r T_0 = 0, \qquad (16)$$

$$T_1'' + P_r v_0 T_1' + (S - i\omega) P_r T_1 = 0, \qquad (17)$$

$$C_0'' + S_c v_0 C_0' = 0, \qquad (18)$$

$$C_1'' + S_c v_0 C_1' - i\omega C_1 = 0. \qquad (19)$$

The corresponding boundary conditions are

$$u_0 = R\frac{\partial u_0}{\partial y}, \quad u_1 = 1 + R\frac{\partial u_1}{\partial y}, \; T_0 = 1, \; T_1 = 0, \; C_0 = 1, \; C_1 = 0 \quad \text{at } y = 0,$$
$$u_0 \to 0, \; u_1 \to 0, \; T_0 \to 0, \; T_1 \to 0, \; C_0 \to 0, \; C_1 \to 0 \quad \text{as } y \to \infty \qquad (20)$$

Solving equations (14)-(19) under boundary conditions (20), we get the following solutions for velocity, temperature and the concentration distributions of the flow field.

$$T_0 = e^{\lambda 1 y}, \qquad (21)$$

$$T_1 = 0, \qquad (22)$$

$C_0 = e^{-Sc v_0 y}$, (23)

$C_1 = 0$, (24)

$u_0 = A_1 e^{-\lambda_2 y} - A_2 e^{\lambda_1 y} - A_3 e^{-S_c v_0 y}$, (25)

$u_1 = A_4 e^{\lambda_4 y}$, (26)

where $\lambda_1 = \frac{1}{2}\left[-P_r v_0 - \sqrt{P_r^2 v_0^2 - 4SP_r}\right]$, $\lambda_2 = \frac{1}{2}\left[P_r v_0 + \sqrt{P_r^2 v_0^2 - 4P_r(S - i\omega)}\right]$,

$$\lambda_3 = \frac{1}{2}\left[-P_r v_0 + \sqrt{P_r^2 v_0^2 - 4P_r(S - i\omega)}\right],\ \lambda_4 = -\frac{1}{2}\left[-v_0 + \sqrt{v_0^2 + 4\left(M^2 + \frac{1}{K_p} + i\omega\right)}\right],$$

$$A_1 = \frac{1}{(R\lambda_2 + 1)}\left[A_2(1-\lambda_1) - A_3(1 - S_c v_0)\right],\ A_2 = \frac{G_r}{(\lambda_1 + \lambda_2)(\lambda_1 - \lambda_3)}$$

$$A_3 = \frac{G_c}{(\lambda_2 - S_c v_0)(\lambda_3 + S_c v_0)},\quad A_4 = \frac{1}{(1 - R\lambda_4)}. \tag{27}$$

Using equations (21)-(26) in equations (11)-(13), the solutions for velocity, temperature and concentration distribution of the flow field are given by

$$u = A_1 e^{-\lambda_2 y} - A_2 e^{\lambda_1 y} - A_3 e^{-S_c v_0 y} + A_4 e^{\lambda_4 y + i\omega t}, \tag{28}$$

$$T = e^{\lambda_1 y}, \tag{29}$$

$$C = e^{-S_c v_0 y}. \tag{30}$$

Skin friction

The skin friction at the wall is given by

$$\tau = \left(\frac{\partial u}{\partial y}\right)_{y=0} = -\lambda_2 A_1 - \lambda_1 A_2 + S_c v_0 A_3 + \lambda_4 A_4 e^{i\omega t}. \tag{31}$$

Heat flux

The rate of heat transfer or the heat flux at the wall in terms of Nusselts number is given by

$$N_u = \left(\frac{\partial T}{\partial y}\right)_{y=0} = \lambda_1. \tag{32}$$

4. Discussions and results

The effect of mass transfer on natural convection flow of a viscous incompressible electrically conducting fluid through a porous medium past an oscillating porous plate in with heat source in presence of a transverse magnetic field has been considered. The effects of the important flow parameters such as magnetic parameter M, heat source parameter S, Grashof number for mass transfer G_c, permeability parameter K_p, Schmidt number S_c on the velocity of the flow field have been discussed with the help of Figures 1-4.

4.1 Velocity field (u)

The velocity field suffers a change in magnitude with the variation of the flow parameters. The flow parameters responsible for this change in the velocity field are magnetic parameter *M*, heat source parameter *S*, Grashof number for mass transfer G_c, permeability parameter K_p, Schmidt number S_c. These variations in the velocity field are depicted in Figures 1-4.

4.2 Effect of magnetic parameter M

Figure 1 depicts the effect of magnetic parameter *M* on the velocity field for three different values of the magnetic parameter (M= 0, 3, 5). In the figure curve with M= 0 corresponds to the non-MHD flow. Comparing the curves of the figure, it is seen that the magnetic parameter decelerates the magnitude of the velocity of the flow field at all points due to the action of Lorentz force on the flow field.

4.3 Effect of heat source parameter S

The heat source parameter *S* plays a drastic role on the behaviour of the velocity field. The variations in the velocity field due to heat source parameter S is shown in Figure 2. In the figure curve with S=0 corresponds to the absence of heat source and the curves with S=0.3 and S=-0.3 correspond to the presence of heat source and heat sink in the flow field. A close observation on the curves of the Figure 2 shows that the heat source parameter increases the magnitude of the velocity at all points of the flow field.

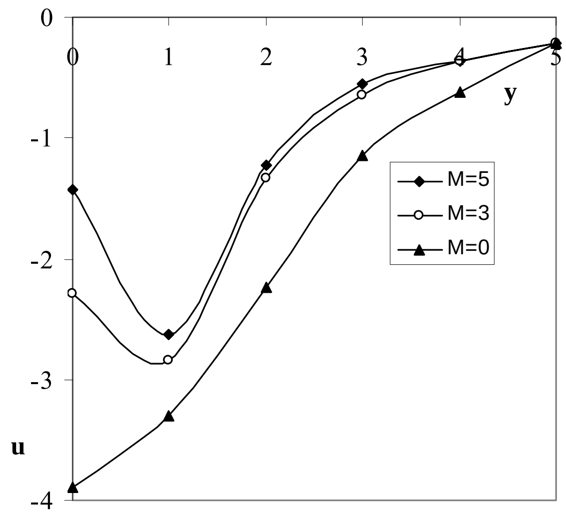

Figure 1. Velocity profiles against *y* for different values of *M* with *R*=0.3, G_r=3, G_c=3, S_c=0.22, K_p=2, P_r=0.71, *S*=0.5, v_0=2, $\omega t=\pi/2$, ω=2

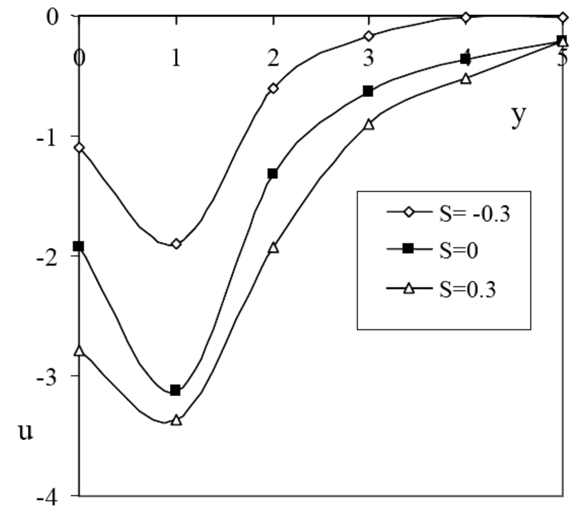

Figure 2. Velocity profiles against *y* for different values of S with *M*=2, *R*=0.3, G_r=3, G_c=3, S_c=0.22, K_p=2, P_r=0.71, v_0=2, $\omega t=\pi/2$, ω=2

4.4 Effect of Grashof number G_c, permeability parameter K_p and rarefaction parameter R

The effects of rarefaction parameter *R*, Grashof number for mass transfer G_c and the permeability parameter K_p on the velocity of the flow field are depicted in Figure 3. A comparative study of the curves of Figure 3 shows that the effect of the above parameters is to enhance the magnitude of the velocity at all points of the flow field.

4.5 Effect of Schmidt number S_c

The presence of foreign mass in the flow field influences the velocity of the field to an appreciable extent. These effects have been shown in Figure 4. In the figure curve with S_c= 0 refers to the absence of foreign mass in the flow field. A growing S_c (heavier diffusing species) is seen to enhance the magnitude of the velocity of the flow field at all points.

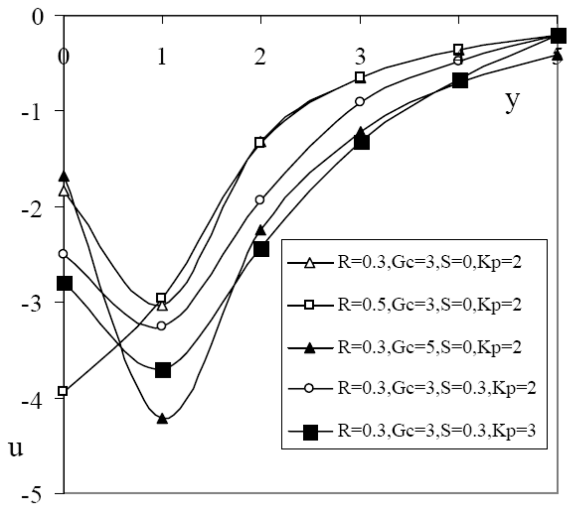

Figure 3. Velocity profiles against y for different values of G_c, K_p and R with M=2, G_r=3, P_r=0.71, S_c=0.22, v_0=2, ωt=π/2, ω=2

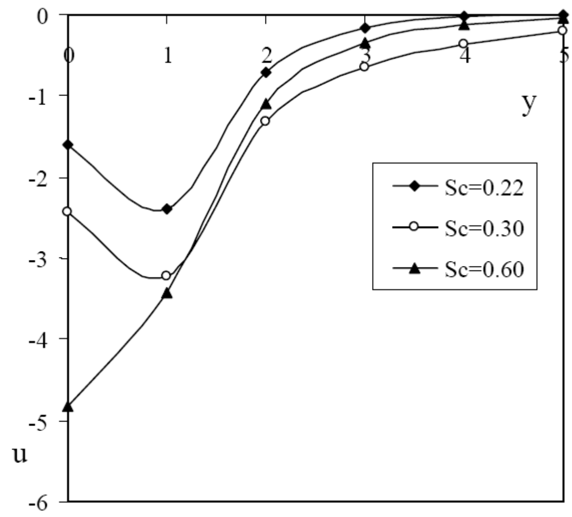

Figure 4. Velocity profiles against y for different values of S_c with M =2, R=0.3, G_r=3, G_c=3, K_p=2, P_r=0.71, S=0, v_0=2, ωt=π/2, ω=2

5. Conclusion

The above analysis points out the following interesting results of physical interest on the velocity of the flow field.

1. A growing magnetic parameter M retards the magnitude of the velocity of the flow field at all points due to the action of the Lorentz force acting on the flow field.
2. The heat source parameter S has an accelerating effect on the magnitude of the velocity of the flow field at all points.
3. The effect of growing Grashof number for mass transfer G_c and the permeability parameter K_p is to enhance the velocity (absolute value) of the flow field at all points.
4. An increase in Schmidt number S_c is to increase the magnitude of the velocity of the flow field at all points.
5. A growing rarefaction parameter R enhances the magnitude of the velocity of the flow field at all points.

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