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# Transient thermal behavior of a homogeneous composite micro-domain: The hyperbolic heat-conduction model

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**Abstract**
The transient thermal behavior of a homogeneous composite micro-domain described by the hyperbolic heat-conduction model with neglecting conduction in the fluid domain is investigated semi-analytically. The composite micro-domain consists of a matrix (fluid domain) and inserts (solid domain), each made of different material. The effect of different parameters that affect the local thermal equilibrium assumption under the effect of the hyperbolic heat conduction model is investigated.


**Keywords:** Composite; Heat conduction; Hyperbolic model; Thermal equilibrium assumption; Porous micro-channel.

## 1. Introduction

over the past two to three decades, the study of heat transfer in porous media has evolved as result of it is importance in the study of many engineering applications. Nield and Bejan [1] highlighted the new conceptual development and applications of convection in porous media. One of the main issues in the study of porous media is the assumption of local thermal equilibrium (LTE) were it is assumed that both the fluid and solid are in LTE, therefore only one energy equation is considered [1], limiting the results to certain special cases and applications. On other hand [2-7] several studies adopted the two-phase model where there are two energy equations for the solid and fluid domain. It is clear that there is a need to establish the conditions when the LTE can be used in the study of convection in porous media.
Numerous studies [8-12] investigated the validity of LTE assumption in porous media for different flow conditions and geometries. They established a group of dimensionless parameters that control the LTE assumption for different flow conditions in porous media, and derived the criteria necessary for LTE assumption. All the previous studies were described by the parabolic heat conduction models.
Recently Nnanna et al.[13] performed experimental study of non-Fourier thermal response in porous media, in this study a two equation model that uses non-Fourier (dual phase lag) to study the response of a porous medium subjected to a short time thermal disturbance is verified experimentally. Also, they showed that during a rapid transient even when the fluid and solid have the same temperature the Fourier conduction model failed to describe temperature filed.
Rapid transient is encountered in many applications that involves porous medium, such as laser synthesis and processing of thin-film deposition where in this application a heat source such as a laser and/or microwave of extremely short duration or very high frequency is used. In the present study the thermal equilibrium assumption in transient natural convection flow in porous channel as described by a hyperbolic heat-conduction model is investigated.

## 2. Analysis

Consider the problem of unsteady natural convection fluid flow into a parallel plate channel totally filled with porous media. The unsteadiness in the channel thermal behavior is due to a sudden change in the temperature of the channel wall. Referring to Figure 1,

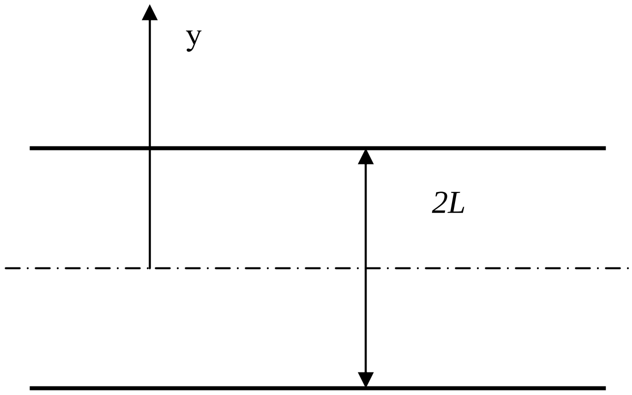


Figure 1. Schematic representation of the domain under consideration

The energy equations with the initial and boundary conditions for both the fluid and solid domains for the hyperbolic heat conduction model are given as:

$$\tau_f \frac{\partial^2 \theta_f}{\partial \eta^2} + \frac{\partial \theta_f}{\partial \eta} = Bi\left(\theta_s - \theta_f\right) + \tau_f \frac{\partial}{\partial \eta}\left(\theta_s - \theta_f\right) \tag{1}$$

$$\tau_s \frac{\partial^2 \theta_s}{\partial \eta^2} + \frac{\partial \theta_s}{\partial \eta} = \frac{K_R}{C_R}\frac{\partial^2 \theta_s}{\partial Y^2} - Bi\left(\theta_s - \theta_f\right) - Bi\,\tau_s \frac{\partial}{\partial \eta}\left(\theta_s - \theta_f\right) \tag{2}$$

where $C_R = \dfrac{(1-\varepsilon)\rho_s c_s}{\varepsilon \rho_f c_f}$ $Bi = \dfrac{h_v L}{k_f}$ $K_R = \dfrac{k_s}{k_f}$

The initial and boundary conditions become:

$$\theta_s(0,Y) = \theta_f(0,Y) = 0 \quad \frac{\partial \theta_s(0,Y)}{\partial \eta} = \frac{\partial \theta_f(0,Y)}{\partial \eta} = 0$$
$$\theta_s(\eta,1) - 1 = Kn\frac{\Omega}{\Pr}Q(\eta,1) \quad \theta_f(\eta,1) - 1 = Kn\frac{\Omega}{\Pr}Q(\eta,1) \quad \frac{\partial \theta_s(\eta,0)}{\partial Y} = 0 \tag{3}$$

Equations (1-3) are solved using Laplace transformation technique. Now with the notation that $L\{\theta_s(\eta,Y)\} = W_s(S,Y)$ and $L\{\theta_f(\eta,Y)\} = W_f(S,Y)$, Laplace transformation of Eqs.(1-3) yields:

$$W_f = \frac{\left(\tau_f S + Bi\right)}{\left(\tau_f S^2 + \tau_f S + S + Bi\right)} W_s \tag{4}$$

$$\left(\frac{K_R}{C_R}\right)\frac{\partial^2 W_s}{\partial^2 Y} - \left(\tau_s S^2 + S + Bi + Bi\,\tau_s\right)W_s = -\left(Bi + Bi\,\tau_s\right)W_f \tag{5}$$

Also, the Laplace transformation of the boundary conditions is given as:

$$\left.\begin{aligned} & W_s(S,1)-\frac{1}{S}=-Kn\,\frac{\Omega}{\Pr}W_s'(S,1) \\ & \frac{\partial W_s}{\partial Y}(S,0)=0 \end{aligned}\right\} \tag{6}$$

According to the boundary conditions given in Eq. (6), Eqs. (4-5) are solved to give:

$$W_s = C\left(e^{HY}+e^{-HY}\right) \tag{7}$$

$$W_f = C\frac{\left(\tau_f S+Bi\right)\left(e^{HY}+e^{-HY}\right)}{\left(\tau_f S^2+\tau_f S+S+Bi\right)} \tag{8}$$

where $H^2=\left[\left(\tau_s S^2+S+Bi+Bi\tau_s\left(\frac{C_R}{K_R}\right)-\frac{\left(Bi+Bi\tau_s\right)\left(\frac{C_R}{K_R}\right)}{\left(\tau_f S^2+\tau_f S+S+Bi\right)\left(Bi+Bi\tau_f\right)}\right]$ and $C=\frac{1/S}{\left[\left(e^{H}+e^{-H}\right)+Kn\frac{\Omega}{\Pr}\left(e^{H}-e^{-H}\right)\right]}$

Equations (7-8) are inverted using a computer program based on Riemann-sum approximation [14] as:

$$\theta(\eta,Y)\cong\frac{e^{\gamma\eta}}{\eta}\left[\frac{1}{2}W(\gamma,Y)+\mathrm{Re}\sum_{n=1}^{N}W\left(\gamma+\frac{in\pi}{\eta},Y\right)(-1)^n\right] \tag{9}$$

where Re refers to the "real part of" and $i=\sqrt{-1}$ is the imaginary number, N is the number of terms used in Riemann-sum approximation and $\gamma$ is the real part of the Bromwich contour that is used in inverting Laplace transforms. The Riemann-sum approximation for the Laplace inversion involves a single summation for numerical process. Its accuracy depends on the value of $\gamma$ and the truncation error dictated by N.

## 3. Results and discussion

The effect of different parameters on the validity of the thermal equilibrium assumption in transient natural convection flow in porous channel as described by a hyperbolic heat-conduction model is investigated in Figures 2-6 for the case (neglecting conduction in the fluid domain)

Figure 2 shows the transient behavior of the fluid and solid temperatures at different $K_R$ with neglecting the conduction in the fluid domain. As shown, the difference between the fluid and solid temperatures increases as $K_R$ decreases, which implies that as $K_R$ increases the thermal resistance of the solid domain decreases or the thermal resistance of the fluid increases. The effect of total thermal capacity ratio $C_R$ on the transient behavior of the fluid and solid temperatures is shown in Figure 3. It is clear that the difference between the fluid and solid temperatures increases as the value of $C_R$ decreases. The transverse conduction in the fluid domain is neglected which implies that the effect of the thermal disturbance is carried into the channel directly through the solid domain and then the solid domain transfer it to the fluid domain through the volumetric convective heat transfer coefficient.

Figure 4 shows the effect of Biot number on the transient fluid and solid temperatures with neglecting conduction in the fluid domain. It is obvious from these figures that the difference decreases as Biot number increases. This implies that the effect of Bi number on the temperature difference is insignificant at large values of Bi. This is justified, since the time required for both fluid and solid domain to attain the same temperature is inversely proportional to q, where q is the convective heat transfer between the fluid and solid domain. The transient behavior of the difference between the fluid and solid temperatures at different $\tau_f$ and $\tau_s$ is shown in Figure 5 with neglecting conduction in the fluid domain. It is clear from this figure that the difference increases as $\tau_f$ and $\tau_s$ decrease. Effect of Knudsen number Kn on the transient fluid and solid temperatures is shown in Figure 6.

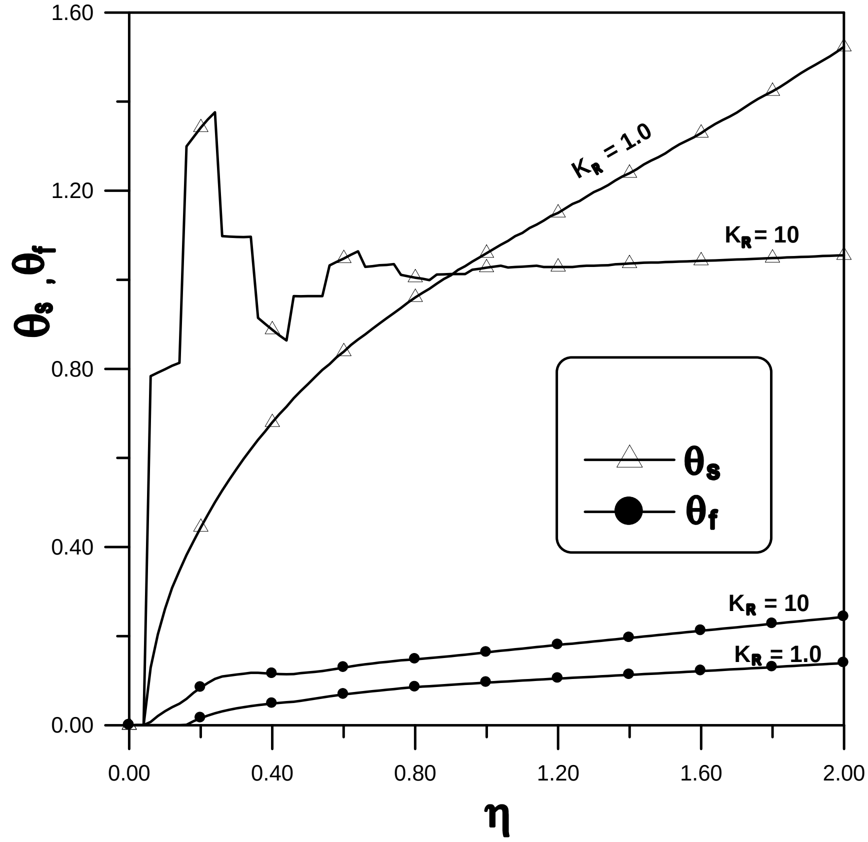


Figure 2. Transient behavior of the fluid and solid temperature at different $K_R$

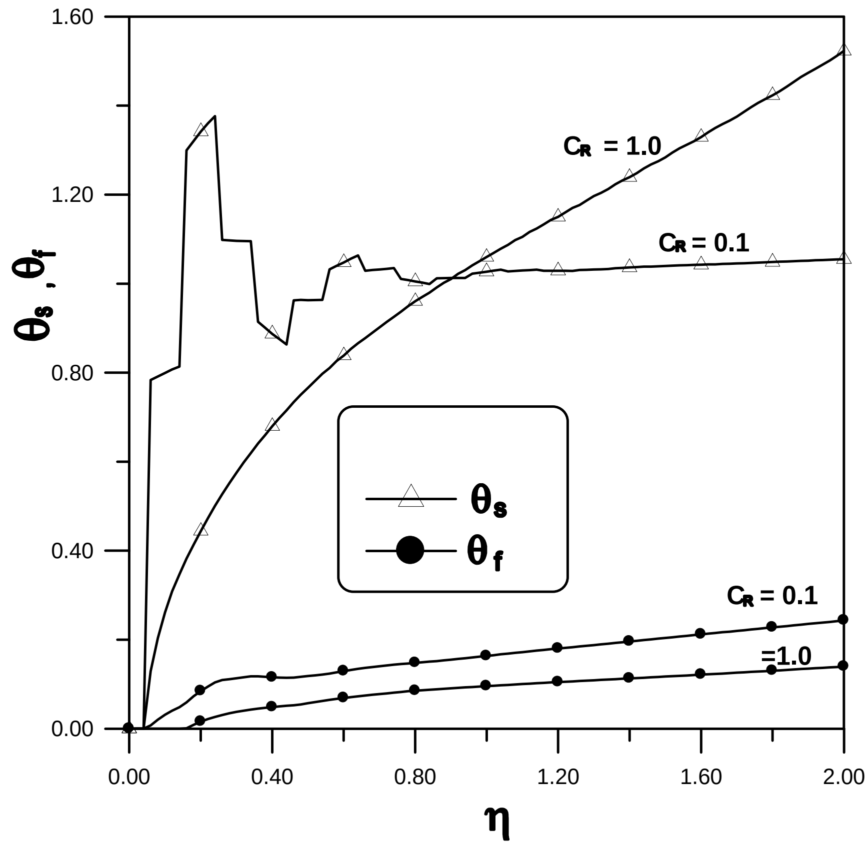


Figure 3. Transient behavior of the fluid and solid temperature at different $C_R$

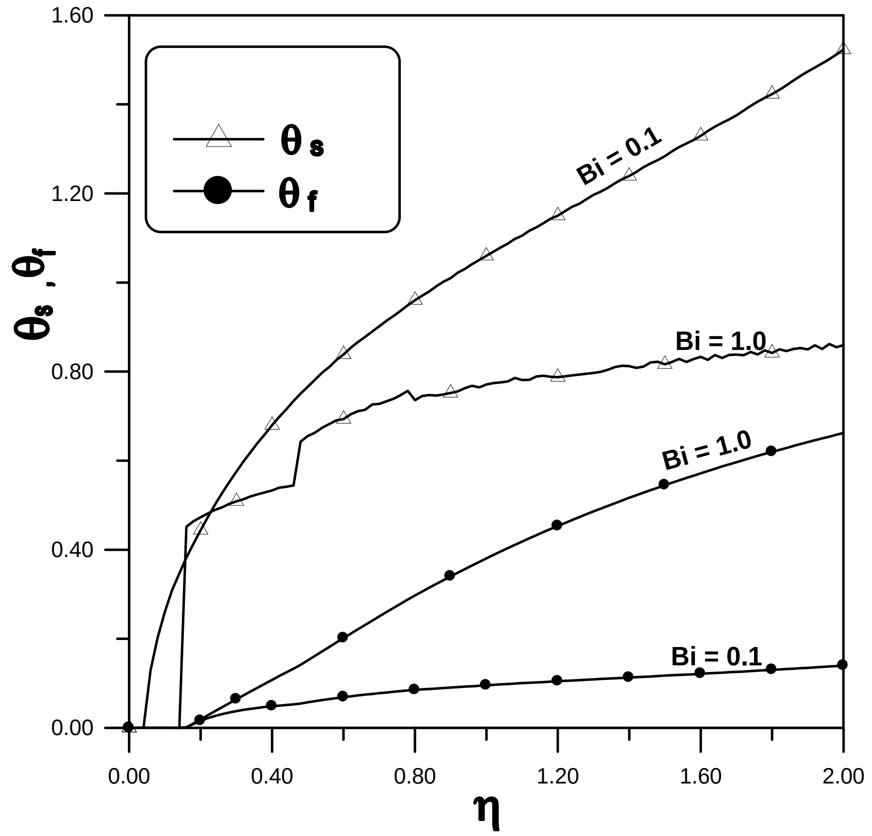


Figure 4. Transient behavior of the fluid and solid temperature at different $B_i$

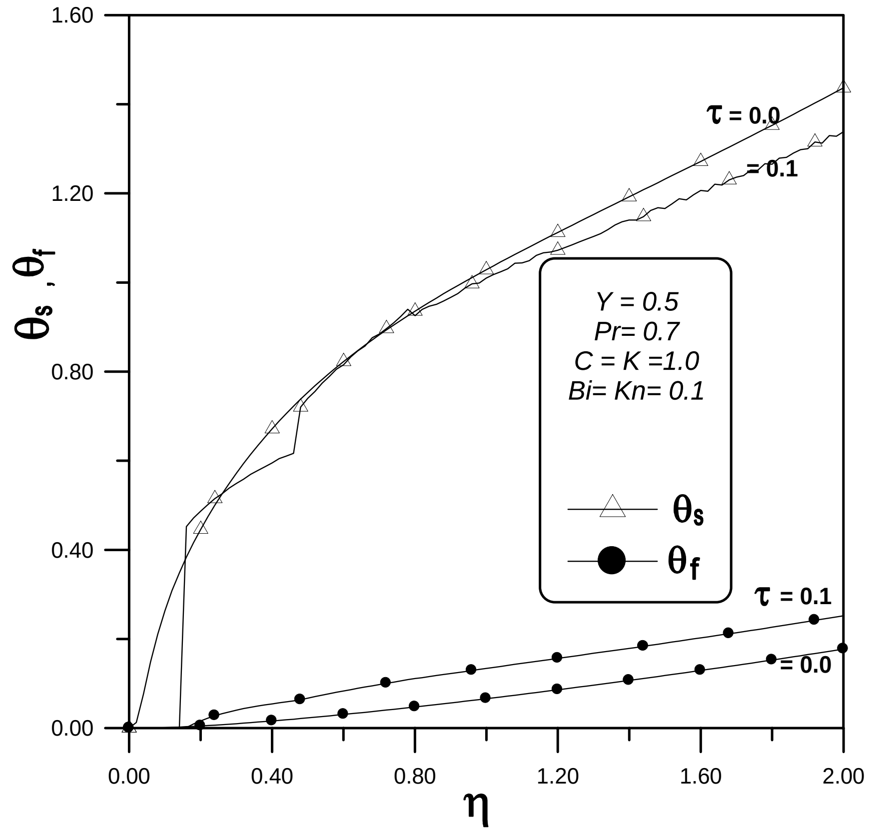


Figure 5. Transient behavior of the fluid and solid temperature at different $\tau$

 

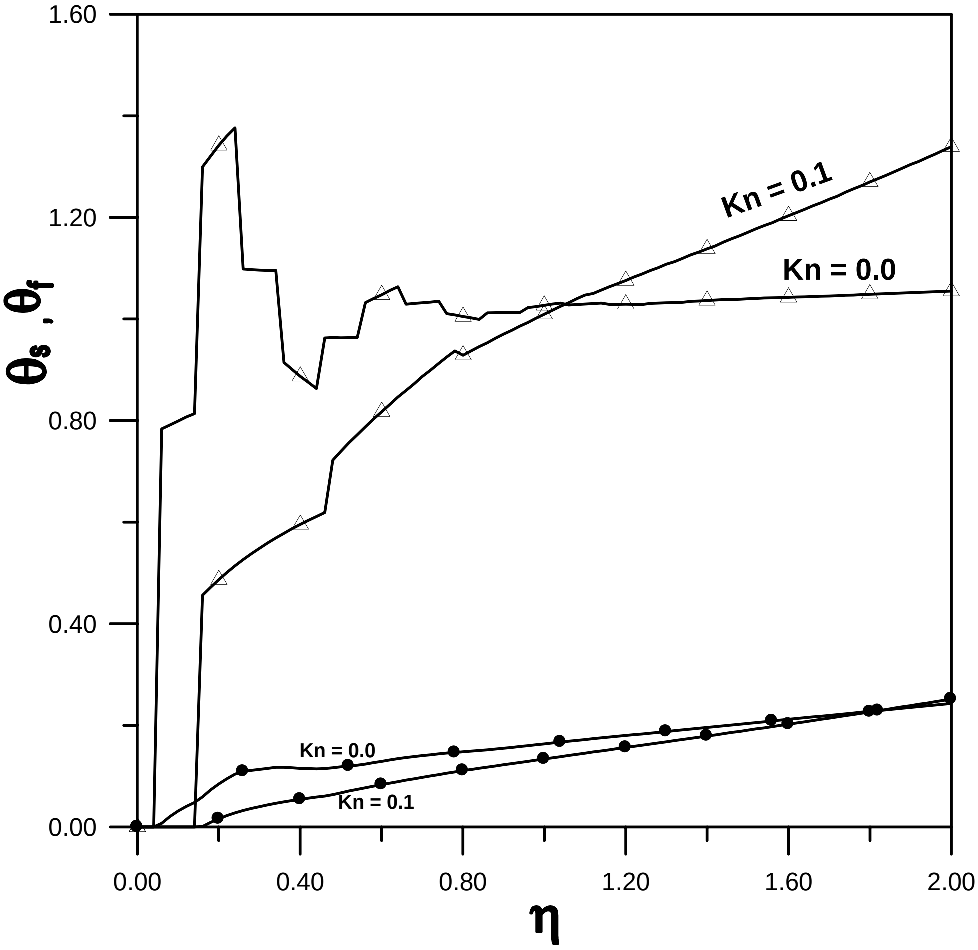


Figure 6. Transient behavior of the fluid and solid temperature at different Kn

## 4. Conclusions

Thermal equilibrium assumption in transient natural convection flow in porous channel as described by a hyperbolic heat-conduction model is investigated with neglecting the conduction in the fluid domain. It is found that the volumetric Biot number, thermal conductivity ratio, phase lag in heat flux, Knudsen number and total thermal capacity ratio have the most significant effect on the local thermal equilibrium assumption. The local thermal equilibrium assumption is secured for large values of Biot number, Knudsen number and thermal conductivity ratio and small values of total thermal capacity ratio, phase lag in heat flux.

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**Nomenclature**

| | | | |
|---|---|---|---|
| Bi | Biot number, $\frac{h_v L}{k_f}$ | ε | porosity |
| C | specific heat capacity, $J/kg\,K$ | Ω | $=\frac{2-\sigma_T}{\sigma_T}\left(\frac{2\gamma}{\gamma+1}\right)$ |
| $C_R$ | total thermal capacity ratio, $\frac{(1-\varepsilon)\rho_s c_s}{\varepsilon\rho_f c_f}$ | $\gamma$ | specific heat ratio |
| $h_v$ | volumetric heat transfer coefficient, $W/m^2\,K$ | $\lambda$ | mean free path, m |
| k | thermal conductivity, $W/m\,K$ | $\nu$ | kinematic viscosity, $m^2/s$ |
| $K_R$ | thermal conductivity ratio, $\frac{k_s}{k_f}$ | $\rho$ | density, $\frac{kg}{m^3}$ |
| Kn | Knudsen number (= $\lambda/L$ ) | $\sigma_T$ | thermal accommodation coefficient |
| 2L | slab thickness, m | $\sigma_v$ | Tangential-momentum accommodationcoefficient |
| S | Laplacian domain | θ | dimensionless temperature, $\frac{T-T_\infty}{T_\infty}$ |
| t | time, s | $\tilde{\tau}$ | phase lag in heat flux, s |
| $t_o$ | reference time, s | $\tau$ | dimensionless phase lag in heat flux, $\frac{\bar{\tau}}{t_o}$ |
| T | temperature, K | | |
| $T_w$ | wall temperature, K | *Subscripts* | |
| y | axial coordinate, m | f | fluid domain |
| $Y$ | dimensionless axial coordinate, $\frac{y}{L}$ | s | solid domain |
| *Greek symbols* | | w | wall |
| η | dimensionless time, $\frac{t}{t_o}$ | | |

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