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A suggested analytical solution of oblique crack effect on the beam vibration

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Abstract
In this research development the derivation of general equation of motion for beam with crack effect presented in paper [1], to derivation the general equation of motion of beam with oblique crack effect. The derivation of equation of motion of beam included suggested analytical solution for effect of oblique in crack on the natural frequency of beam with added the effect oblique of crack in stiffness (EI) beam with calculated the equivalent stiffness, (EI), for a rectangular beam to involve an exponential function with depth, location and ordination of oblique crack effect, with solution of assuming equivalent stiffness beam (EI) by using of Fourier series method. The natural frequency of a cracked beam with simply supported beam is investigated analytically, with solution of general equation of motion of beam with oblique crack effect, and numerically by finite element method, with using of ANSYS program ver. 14, for different crack depth, location and crack orientation effect and the results are compared. The same beam materials studied in paper, [1], are study in this research as, low carbon steel, Alloys Aluminum, and Bronze materials with different beam length, depth and crack orientation. A comparison made between analytical results from theoretical solution of general equation of motion of beam with oblique crack effect with numerical solution by finite element method with using ANSYS results, where the biggest error percentage is about (1.8 %). The results of natural frequency of beam shows that the natural frequency of beam increasing with increasing the crack orientation, the effect of crack orientation decreasing with increase the orientation of crack.

Keywords: Vibration crack beam; Oblique crack effect; Health monitoring; Theoretical vibration oblique crack beam; Frequency of beam with crack orientation effect; Simply supported crack beam; Finite element method; Finite element crack beam.

1. Introduction

All structures are prone to damage, may be due to over-stressing in operation or due to extreme environmental conditions or due to any accidental event. Crack present in the component may grow during service and may result in the component failure once they grow beyond a critical limit. It is desirable to investigate the damage occurred in the structure at the early stage to protect the structure from possible catastrophic failures, [2].

Vibration principles are the inherent properties of the physical science applicable to all structures subjected to static or dynamic loads. All structures again due to their rigid nature develop some irregularities in their life span which leads to the development of crack. The problem on crack is the

basic problem of science of resistance of materials. Considering the crack as a significant form of such damage, its modelling is an important step in studying the behaviour of damaged structures, [3].
The purpose of the present work is drive the general equation of motion of beam with oblique crack effect, evaluated the natural frequency of beam with effect of crack orientation with different crack depth and location. The theoretical study included evaluated the equivalent stiffness of beam with oblique crack effect by using Fourier series method, and then, driving the general equation of natural frequency of beam with effect of orientation of oblique crack effect.

2. Literature review

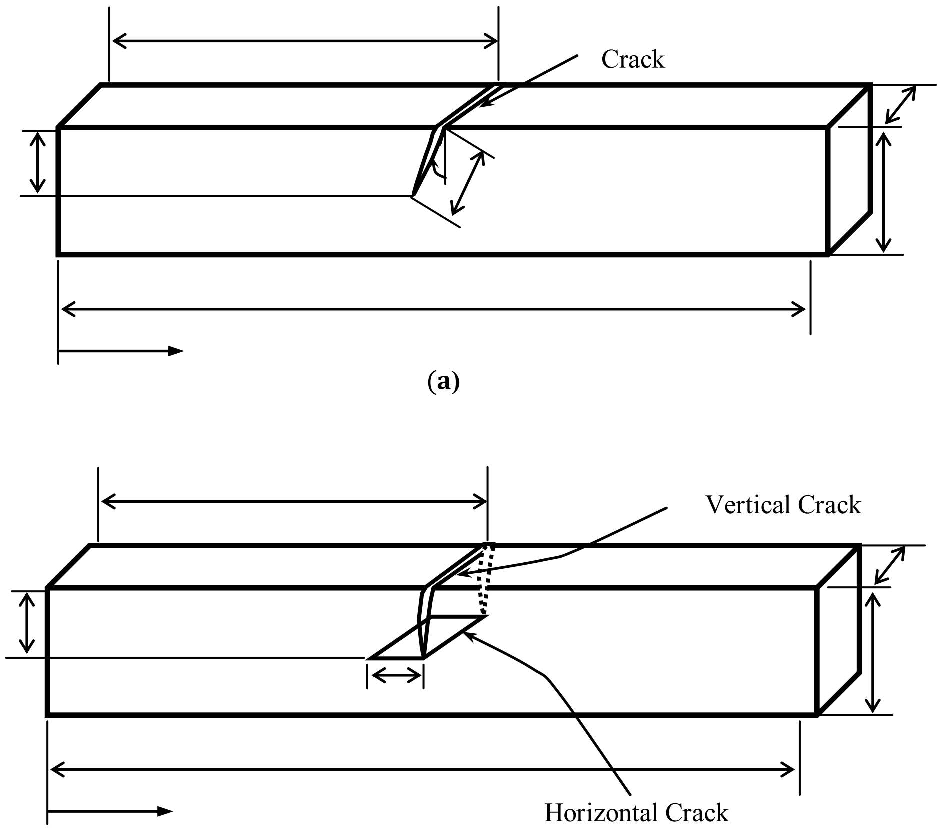
Many studies were performed to examine the vibration study of different types of crack beam, as, Muhannad Al-Waily [1] and [4], in this researches the natural frequency of a cracked beam with different supported, simply and clamped beam, is investigated analytically and numerically by finite element method with using of ANSYS program ver. 14, [1], and analytically and experimental, [4], with different crack depth and location effect and the results are compared. The analytical results of the effect of a crack in a continuous beam are calculated the equivalent stiffness, EI, for a rectangular beam to involve an exponential function with depth and location of crack effect, with solution of assuming equivalent stiffness beam (EI) by using of Fourier series method. And, the different beam materials studied with different beam length and depth and crack depth and location effect.
Kaushar H. Barad et al [2], during the last few decades, intense research on the detection of crack using the vibration based techniques has been done and various approaches have been developed by researchers. In the presented paper, detection of the crack presence on the surface of beam-type structural element using natural frequency is presented. First two natural frequencies of the cracked beam have been obtained experimentally and used for detection of crack location and size. Also, the effect of the crack location and the crack depth on the natural frequency is presented.
SachinS.Naik et al [5], the paper presented the full formulation for a crack model for analyzing the triply coupled free vibration of both Timoshenko (short) and Euler–Bernoulli (long) shaft beams based on compliance approach in the presence of a planar open edge crack in an arbitrary angular orientation with a reference direction. The compliance coefficients to account for the local flexibility due to the crack for both the beams have been obtained through the concept of strain energy release rate and crack tip stress field given in terms of the stress intensity factors. The variation of the coefficients with crack orientation is presented. Equations governing the free transverse and torsion vibrations are derived and solved.
Ashish K. Darpe [6], a finite element model of a rotor with slant crack is presented. Based on fracture mechanics, a new flexibility matrix for the slant crack is derived that accounts for the additional stress intensity factors due to orientation of the crack compared to the transverse crack. Comparison between rotor with slant and transverse crack is made with regard to the stiffness coefficients and coupled vibration response characteristics. Compared to transverse crack, the stiffness matrix for slant crack is more populated with additional cross coupled coefficients. The influence of angle of orientation of the slant crack on the stiffness values is also investigated.
Murat Kisa et al [7], this paper presented a novel numerical technique applicable to analyses the free vibration analysis of uniform and stepped cracked beams with circular cross section. In this approach in which the finite element and component mode synthesis methods are used together, the beam is detached into parts from the crack section. These substructures are joined by using the flexibility matrices taking into account the interaction forces derived by virtue of fracture mechanics theory as the inverse of the compliance matrix found with the appropriate stress intensity factors and strain energy release rate expressions.
H. Nahvi et al [8], in this paper, an analytical, as well as experimental approach to the crack detection in cantilever beams by vibration analysis is established. An experimental setup is designed in which a cracked cantilever beam is excited by a hammer and the response is obtained using an accelerometer attached to the beam. To avoid non-linearity, it is assumed that the crack is always open. To identify the crack, contours of the normalized frequency in terms of the normalized crack depth and location are plotted. The intersection of contours with the constant modal natural frequency planes is used to relate the crack location and depth.
Hai-Ping Lin [9], an analytical transfer matrix method is used to solve the direct and inverse problems of simply supported beams with an open crack. The crack is modeled as a rotational spring with sectional flexibility. By using the Timoshenko beam theory on two separate beams respectively and applying the compatibility requirements of the crack, the characteristic equation for this cracked system can be

obtained explicitly. This characteristic equation is a function of the eigenvalue (natural frequency), the location of the crack and its sectional flexibility. When any two natural frequencies in this cracked system are measured, the location and the sectional flexibility can be determined using the characteristic equation.

The objective of this paper is to study the effect of crack orientation, depth and position on the natural frequency of the beam by using of analytical solution of general equation of motion of beam with oblique crack effect and compared with numerical results evaluated by finite element method with using of Ansys Program Ver. 14. To achieve the above objectives, analytical solution is developed for dynamic analysis of beam with and without crack effect to evaluate the fundamental natural frequency of beam by using the analytical solution of general equation of motion of beam with oblique crack effect, by building a computer program for analytical solution using Fortran power station Ver. 4.0 program.

3. Mathematical model

Consider the beam with oblique crack, through the thickness of beam, shown in Figure 1a, having the following geometrical and material characteristics $(l, w, d, d_c, d_{v_c}, \theta_c, E, I(x), \rho)$, where; E-modulus of elasticity; and ρ-density of beam and other notations as shown in the figure. The beam is supposed to be loaded with a bending moment and to have a uniform transverse surface crack of depth a located at a given position x_c from the left edge of the beam.



(**a**)

(**b**)

Figure 1. Dimensions of Beam with oblique crack

Assuming the oblique crack, through the thickness of beam, is vertical crack with crack depth $(d_{v_c})$and horizontal crack with crack width (d_{h_c}),as shown in Figure 1b, divided the effect of oblique crack in vertical and horizontal direction of beam effect.

To studying the effect of oblique crack suggested the beam with vertical crack (d_{v_c})only and neglected the effect of horizontal crack effect, since the effect of horizontal crack less than the effect of vertical crack. And derivation the general equation of motion of beam with crack effect with using the equivalent stiffness of beam with crack effect, EI, for vertical crack depth only (d_{v_c}), as,

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$$d_{v_c} = d_c . \cos(\theta_c) \quad (1)$$

where, d_c is the oblique crack depth.
And, with using the general equation of beam vibration can be written as, [10],

$$\frac{\partial^2}{\partial x^2}\left[EI(x)\frac{\partial^2 w}{\partial x^2}(x,t)\right] + \rho A(x)\frac{\partial^2 w}{\partial t^2}(x,t) = 0 \quad (2)$$

The effect of a crack in a continuous beam and calculated the stiffness, EI, for a rectangular beam to involve an exponential function given by, [11]:

$$EI(x) = \frac{EI_0}{1+C\exp(-2\alpha|x-x_c|/d)} \quad (3)$$

where, $C = \frac{(I_0 - I_c)}{I_c}$, for, $I_0 = \frac{wd^3}{12}$ and $I_c = \frac{w(d-d_c)^3}{12}$, and, w, d are the width and depth of the beam, x is the position along the beam, and x_c the position of the crack and, α is a constant equal to (0.667), [11].
To adding the effect of the oblique crack suggested to changing the value of moment of inertia of crack plate (I_c)with moment of inertia of oblique crack (I_{obl_c}), as,

$$I_{obl_c} = \frac{w\left(d - d_{v_c}\right)^3}{12} \quad (4)$$

where, $\left(d_{v_c}\right)$ is depth of crack in vertical direction of beam defined as in Eq. 1.
And the value of C in Eq. 3, become

$$C = \frac{(I_0 - I_{obl\ c})}{I_{obl\ c}} \quad (5)$$

Then, by substation Eq. 5 into Eq. 3, get,

$$EI(x) = \left(\frac{EI_0 . I_{obl\ c}}{I_{obl\ c} + (I_0 - I_{obl\ c})\exp(-2\alpha|x-x_c|/d)}\right) \quad (6)$$

The mass for the beam can be calculated by,

$$\rho A(x) = \rho * w * d = \rho A \quad (7)$$

For vibration analysis of the beam having a crack with a finite length, relation Eq. 6 can be expanded as a sum of sine and cosine functions in the domain $0 \le x \le L$ by Fourier series, as,

$$EI = A_o + \sum_{n=1}^{\infty} A_n \cos\frac{2n\pi x}{L} + \sum_{n=1}^{\infty} B_n \sin\frac{2n\pi x}{L} \quad (8)$$

where, $A_o, A_n,$ and B_n are Fourier series constant can be evaluated as, [12],

$$A_o = \frac{1}{L}\int_0^L EI(x)dx = \frac{1}{L}\int_0^L \frac{EI_0 . I_{obl_c}}{I_{obl_c} + \left(I_0 - I_{obl_c}\right)\exp(-2\alpha|x-x_c|/d)} dx$$

$$A_n = \frac{2}{L}\int_0^L EI(x)\cos\frac{2n\pi x}{L}dx = \frac{2}{L}\int_0^L \frac{EI_0 . I_{obl_c}}{I_{obl_c} + \left(I_0 - I_{obl_c}\right)\exp(-2\alpha|x-x_c|/d)} \cos\frac{2n\pi x}{L}dx$$

$$B_n = \frac{2}{L}\int_0^L EI(x)\sin\frac{2n\pi x}{L}dx = \frac{2}{L}\int_0^L \frac{(EI_0 . I_{obl\ c})}{I_{obl\ c} + (I_0 - I_{obl\ c})\exp(-2\alpha|x-x_c|/d)}\sin\frac{2n\pi x}{L}dx \quad (9)$$

By integral Eq. 9 by x, using Simpson's rule integration method [13], get the Fourier series constant, as,

$$\int_{x_i}^{x_f} f(x)dx = \frac{1}{3}\left(\frac{x_f - x_i}{\overline{m}_d}\right)\left[f(x_i) + 4\sum_{s=1,3,5,\dots}^{\overline{m}_d-1} f(x_s) + 2\sum_{s=2,4,6,\dots}^{\overline{m}_d-2} f(x_s) + f(x_f)\right] \quad (10)$$

where, $x_i = 0$ and $x_f = L$, and $\overline{m}_d$ is the subdivisions of interval $[x_i, x_f]$, usually even number. And, $x_s = x_i + \left(\frac{x_f - x_i}{\overline{m}_d}\right) s$

Then,

$$A_o = \frac{1}{3L}\left(\frac{x_f - x_i}{\overline{m}_d}\right)\left(\begin{array}{c} \frac{EI_0 . I_{obl_c}}{I_{obl_c} + \left(I_0 - I_{obl_c}\right)\exp(-2\alpha|x_i - x_c|/d)} + \\ 4\sum_{s=1,3,5,\dots}^{\overline{m}_d-1} \frac{EI_0 . I_{obl_c}}{I_{obl_c} + \left(I_0 - I_{obl_c}\right)\exp(-2\alpha|x_s - x_c|/d)} + \\ 2\sum_{s=2,4,6,\dots}^{\overline{m}_d-2} \frac{EI_0 . I_{obl_c}}{I_{obl_c} + \left(I_0 - I_{obl_c}\right)\exp(-2\alpha|x_s - x_c|/d)} + \\ \frac{EI_0 . I_{obl_c}}{I_{obl_c} + \left(I_0 - I_{obl_c}\right)\exp\left(-2\alpha\left|x_f - x_c\right|/d\right)} \end{array}\right)$$

$$A_n = \frac{2}{3L}\left(\frac{x_f - x_i}{\overline{m}_d}\right)\left(\begin{array}{c} \frac{EI_0 . I_{obl_c}}{I_{obl_c} + \left(I_0 - I_{obl_c}\right)\exp(-2\alpha|x_i - x_c|/d)}\cos\frac{2n\,\pi\,x_i}{L} + \\ 4\sum_{s=1,3,5,\dots}^{\overline{m}_d-1} \frac{EI_0 . I_{obl_c}}{I_{obl_c} + \left(I_0 - I_{obl_c}\right)\exp(-2\alpha|x_s - x_c|/d)}\cos\frac{2n\,\pi\,x_s}{L} + \\ 2\sum_{s=2,4,6,\dots}^{\overline{m}_d-2} \frac{EI_0 . I_{obl_c}}{I_{obl_c} + \left(I_0 - I_{obl_c}\right)\exp(-2\alpha|x_s - x_c|/d)}\cos\frac{2n\,\pi\,x_s}{L} + \\ \frac{EI_0 . I_{obl_c}}{I_{obl_c} + \left(I_0 - I_{obl_c}\right)\exp\left(-2\alpha\left|x_f - x_c\right|/d\right)}\cos\frac{2n\,\pi\,x_f}{L} \end{array}\right)$$

$$B_n = \frac{2}{3L}\left(\frac{x_f - x_i}{\overline{m}_d}\right)\left(\begin{array}{c} \frac{EI_0 . I_{obl_c}}{I_{obl_c} + \left(I_0 - I_{obl_c}\right)\exp(-2\alpha|x_i - x_c|/d)}\sin\frac{2n\,\pi\,x_i}{L} + \\ 4\sum_{s=1,3,5,\dots}^{\overline{m}_d-1} \frac{EI_0 . I_{obl_c}}{I_{obl_c} + \left(I_0 - I_{obl_c}\right)\exp(-2\alpha|x_s - x_c|/d)}\sin\frac{2n\,\pi\,x_s}{L} + \\ 2\sum_{s=2,4,6,\dots}^{\overline{m}_d-2} \frac{EI_0 . I_{obl_c}}{I_{obl_c} + \left(I_0 - I_{obl_c}\right)\exp(-2\alpha|x_s - x_c|/d)}\sin\frac{2n\,\pi\,x_s}{L} + \\ \frac{EI_0 . I_{obl_c}}{I_{obl_c} + \left(I_0 - I_{obl_c}\right)\exp\left(-2\alpha\left|x_f - x_c\right|/d\right)}\sin\frac{2n\,\pi\,x_f}{L} \end{array}\right) \quad (11)$$

Then, substation Eqs. 7 and 8 in to Eq. 2, get,

$$\frac{\partial^2}{\partial x^2}\left[\left(A_o + \sum_{n=1}^{\infty} A_n \cos\frac{2\,n\,\pi\,x}{L} + \sum_{n=1}^{\infty} B_n \sin\frac{2\,n\,\pi\,x}{L}\right)\frac{\partial^2 w}{\partial x^2}(x,t)\right] + \rho A \frac{\partial^2 w}{\partial t^2}(x,t) = 0 \quad (12)$$

Then, by differential Eq. 12, get,

$$\left[\begin{array}{l} \frac{\partial^4 w}{\partial x^4}(x,t)\left(A_o + \sum_{n=1}^{\infty} A_n \cos\frac{2\,n\,\pi\,x}{L} + \sum_{n=1}^{\infty} B_n \sin\frac{2\,n\,\pi\,x}{L}\right) - \\ \frac{\partial^3 w}{\partial x^3}(x,t)\left(\frac{4\,n\,\pi}{L}\right)\left(\sum_{n=1}^{\infty} A_n \sin\frac{2\,n\,\pi\,x}{L} - \sum_{n=1}^{\infty} B_n \cos\frac{2\,n\,\pi\,x}{L}\right) - \\ \frac{\partial^2 w}{\partial x^2}(x,t)\left(\frac{2\,n\,\pi}{L}\right)^2\left(\sum_{n=1}^{\infty} A_n \cos\frac{2\,n\,\pi\,x}{L} + \sum_{n=1}^{\infty} B_n \sin\frac{2\,n\,\pi\,x}{L}\right) \end{array}\right] + \rho A \frac{\partial^2 w}{\partial t^2}(x,t) \quad (13)$$

Assuming the effect of crack small on the deflection of beam, then can be assuming the behaviours of beam with crack same the behaviours of beam without crack, as, [10], for simply supported beam,

$$w(x) = \overline{A}_{\overline{n}} \sin(\beta_{\overline{n}} x) \tag{14}$$

For, $\beta_1 l = \pi, \beta_2 l = 2\pi, \beta_3 l = 3\pi, \beta_4 l = 4\pi, \ldots$
And, the general behavior of beam as a faction of x and time, as,

$$w(x,t) = \overline{A}_{\overline{n}} \sin(\beta_{\overline{n}} x) \sin(\omega t) \tag{15}$$

Then, by substation Eq. 15 in to Eq. 13, get the general equation of motion for beam with crack effect as,

$$\begin{bmatrix} (\beta_{\overline{n}})^4 \sin(\beta_{\overline{n}} x)\left(A_o + \sum_{n=1}^{\infty} A_n \cos\frac{2\,n\,\pi\,x}{L} + \sum_{n=1}^{\infty} B_n \sin\frac{2\,n\,\pi\,x}{L}\right) + \\ (\beta_{\overline{n}})^3 \cos(\beta_{\overline{n}} x)\left(\frac{4\,n\,\pi}{L}\right)\left(\sum_{n=1}^{\infty} A_n \sin\frac{2\,n\,\pi\,x}{L} - \sum_{n=1}^{\infty} B_n \cos\frac{2\,n\,\pi\,x}{L}\right) + \\ (\beta_{\overline{n}})^2 \sin(\beta_{\overline{n}} x)\left(\frac{2\,n\,\pi}{L}\right)^2 \left(\sum_{n=1}^{\infty} A_n \cos\frac{2\,n\,\pi\,x}{L} + \sum_{n=1}^{\infty} B_n \sin\frac{2\,n\,\pi\,x}{L}\right) \end{bmatrix} - \omega^2 \sin(\beta_{\overline{n}} x)\rho A = 0 \tag{16}$$

By pre multiplying Eq. 16 by $\sin(\beta_{\overline{n}} x)$ and integral with x for $0 \le x \le l$, get, the natural frequency of beam with crack,

$$\omega^2 = \frac{\int_0^l \begin{bmatrix} (\beta_{\overline{n}})^4 \left(\sin(\beta_{\overline{n}} x)\right)^2 \left(A_o + \sum_{n=1}^{\infty} A_n \cos\frac{2\,n\,\pi\,x}{L} + \sum_{n=1}^{\infty} B_n \sin\frac{2\,n\,\pi\,x}{L}\right) + \\ (\beta_{\overline{n}})^3 \sin(2\,\beta_{\overline{n}} x)\left(\frac{2\,n\,\pi}{L}\right)\left(\sum_{n=1}^{\infty} A_n \sin\frac{2\,n\,\pi\,x}{L} - \sum_{n=1}^{\infty} B_n \cos\frac{2\,n\,\pi\,x}{L}\right) + \\ (\beta_{\overline{n}})^2 \left(\sin(\beta_{\overline{n}} x)\right)^2 \left(\frac{2\,n\,\pi}{L}\right)^2 \left(\sum_{n=1}^{\infty} A_n \cos\frac{2\,n\,\pi\,x}{L} + \sum_{n=1}^{\infty} B_n \sin\frac{2\,n\,\pi\,x}{L}\right) \end{bmatrix} dx}{\int_0^l \left(\sin(\beta_{\overline{n}} x)\right)^2 \rho A\, dx} \tag{17}$$

By integral Eq. 17 by x, using Simpson's rule integration method [13], get the natural frequency of beam with crack effect, as,

$$\int_{x_i}^{x_f} f(x)dx = \frac{1}{3}\left(\frac{x_f - x_i}{\overline{m}_d}\right)\left[f(x_i) + 4\sum_{s=1,3,5,\ldots}^{\overline{m}_d - 1} f(x_s) + 2\sum_{s=2,4,6,\ldots}^{\overline{m}_d - 2} f(x_s) + f(x_f)\right] \tag{18}$$

where, $x_i = 0$ and $x_f = L$, and $\overline{m}_d$ is the subdivisions of interval $[x_i, x_f]$, usually even number.
And, $x_s = x_i + \left(\frac{x_f - x_i}{\overline{m}_d}\right) s$
Then,

$$\omega^2 = \frac{\frac{1}{3}\left(\frac{x_f - x_i}{\overline{m}_d}\right)\left(\begin{matrix} \begin{bmatrix} (\beta_{\overline{n}})^4 \left(\sin(\beta_{\overline{n}} x_i)\right)^2 \left(A_o + \sum_{n=1}^{\infty} A_n \cos\frac{2\,n\,\pi\,x_i}{L} + \sum_{n=1}^{\infty} B_n \sin\frac{2\,n\,\pi\,x_i}{L}\right) + \\ (\beta_{\overline{n}})^3 \sin(2\,\beta_{\overline{n}} x_i)\left(\frac{2\,n\,\pi}{L}\right)\left(\sum_{n=1}^{\infty} A_n \sin\frac{2\,n\,\pi\,x_i}{L} - \sum_{n=1}^{\infty} B_n \cos\frac{2\,n\,\pi\,x_i}{L}\right) + \\ (\beta_{\overline{n}})^2 \left(\sin(\beta_{\overline{n}} x_i)\right)^2 \left(\frac{2\,n\,\pi}{L}\right)^2 \left(\sum_{n=1}^{\infty} A_n \cos\frac{2\,n\,\pi\,x_i}{L} + \sum_{n=1}^{\infty} B_n \sin\frac{2\,n\,\pi\,x_i}{L}\right) \end{bmatrix} + \\ 4\sum_{s=1,3,5,\ldots}^{\overline{m}_d - 1} \begin{bmatrix} (\beta_{\overline{n}})^4 \left(\sin(\beta_{\overline{n}} x_s)\right)^2 \left(A_o + \sum_{n=1}^{\infty} A_n \cos\frac{2\,n\,\pi\,x_s}{L} + \sum_{n=1}^{\infty} B_n \sin\frac{2\,n\,\pi\,x_s}{L}\right) + \\ (\beta_{\overline{n}})^3 \sin(2\,\beta_{\overline{n}} x_s)\left(\frac{2\,n\,\pi}{L}\right)\left(\sum_{n=1}^{\infty} A_n \sin\frac{2\,n\,\pi\,x_s}{L} - \sum_{n=1}^{\infty} B_n \cos\frac{2\,n\,\pi\,x_s}{L}\right) + \\ (\beta_{\overline{n}})^2 \left(\sin(\beta_{\overline{n}} x_s)\right)^2 \left(\frac{2\,n\,\pi}{L}\right)^2 \left(\sum_{n=1}^{\infty} A_n \cos\frac{2\,n\,\pi\,x_s}{L} + \sum_{n=1}^{\infty} B_n \sin\frac{2\,n\,\pi\,x_s}{L}\right) \end{bmatrix} + \\ 2\sum_{s=2,4,6,\ldots}^{\overline{m}_d - 2} \begin{bmatrix} (\beta_{\overline{n}})^4 \left(\sin(\beta_{\overline{n}} x_s)\right)^2 \left(A_o + \sum_{n=1}^{\infty} A_n \cos\frac{2\,n\,\pi\,x_s}{L} + \sum_{n=1}^{\infty} B_n \sin\frac{2\,n\,\pi\,x_s}{L}\right) + \\ (\beta_{\overline{n}})^3 \sin(2\,\beta_{\overline{n}} x_s)\left(\frac{2\,n\,\pi}{L}\right)\left(\sum_{n=1}^{\infty} A_n \sin\frac{2\,n\,\pi\,x_s}{L} - \sum_{n=1}^{\infty} B_n \cos\frac{2\,n\,\pi\,x_s}{L}\right) + \\ (\beta_{\overline{n}})^2 \left(\sin(\beta_{\overline{n}} x_s)\right)^2 \left(\frac{2\,n\,\pi}{L}\right)^2 \left(\sum_{n=1}^{\infty} A_n \cos\frac{2\,n\,\pi\,x_s}{L} + \sum_{n=1}^{\infty} B_n \sin\frac{2\,n\,\pi\,x_s}{L}\right) \end{bmatrix} + \\ \begin{bmatrix} (\beta_{\overline{n}})^4 \left(\sin(\beta_{\overline{n}} x_f)\right)^2 \left(A_o + \sum_{n=1}^{\infty} A_n \cos\frac{2\,n\,\pi\,x_f}{L} + \sum_{n=1}^{\infty} B_n \sin\frac{2\,n\,\pi\,x_f}{L}\right) + \\ (\beta_{\overline{n}})^3 \sin(2\,\beta_{\overline{n}} x_f)\left(\frac{2\,n\,\pi}{L}\right)\left(\sum_{n=1}^{\infty} A_n \sin\frac{2\,n\,\pi\,x_f}{L} - \sum_{n=1}^{\infty} B_n \cos\frac{2\,n\,\pi\,x_f}{L}\right) + \\ (\beta_{\overline{n}})^2 \left(\sin(\beta_{\overline{n}} x_f)\right)^2 \left(\frac{2\,n\,\pi}{L}\right)^2 \left(\sum_{n=1}^{\infty} A_n \cos\frac{2\,n\,\pi\,x_f}{L} + \sum_{n=1}^{\infty} B_n \sin\frac{2\,n\,\pi\,x_f}{L}\right) \end{bmatrix} \end{matrix} \right)}{\frac{1}{3}\left(\frac{x_f - x_i}{\overline{m}_d}\right)\left(\left(\sin(\beta_{\overline{n}} x_i)\right)^2 \rho A + 4\sum_{s=1,3,5,\ldots}^{\overline{m}_d - 1}\left(\sin(\beta_{\overline{n}} x_s)\right)^2 \rho A + 2\sum_{s=2,4,6,\ldots}^{\overline{m}_d - 2}\left(\sin(\beta_{\overline{n}} x_s)\right)^2 \rho A + \left(\sin(\beta_{\overline{n}} x_f)\right)^2 \rho A\right)} \tag{19}$$

By using of the building a computer program for analytical solution using Fortran power station 4.0 program, as shown in the flow chart in Figure 2, can be get the results of Eq. 19 to evaluated natural frequency of beam with oblique crack effect. The building program requirement input data of beam as, the beam dimensions (length, depth and width of beam) and the mechanical properties of beam (modulus of elasticity and density of beam). And, the output of program are the natural frequency of simply supported beam with effect of crack orientation through the thickness of beam with various beam materials (low carbon steel, Alloys Aluminum, and Bronze materials) and different crack size and location effect. And then, the theoretical results evaluated with building program are compare with numerical results evaluated with finite element method, with using Ansys program Ver. 14.

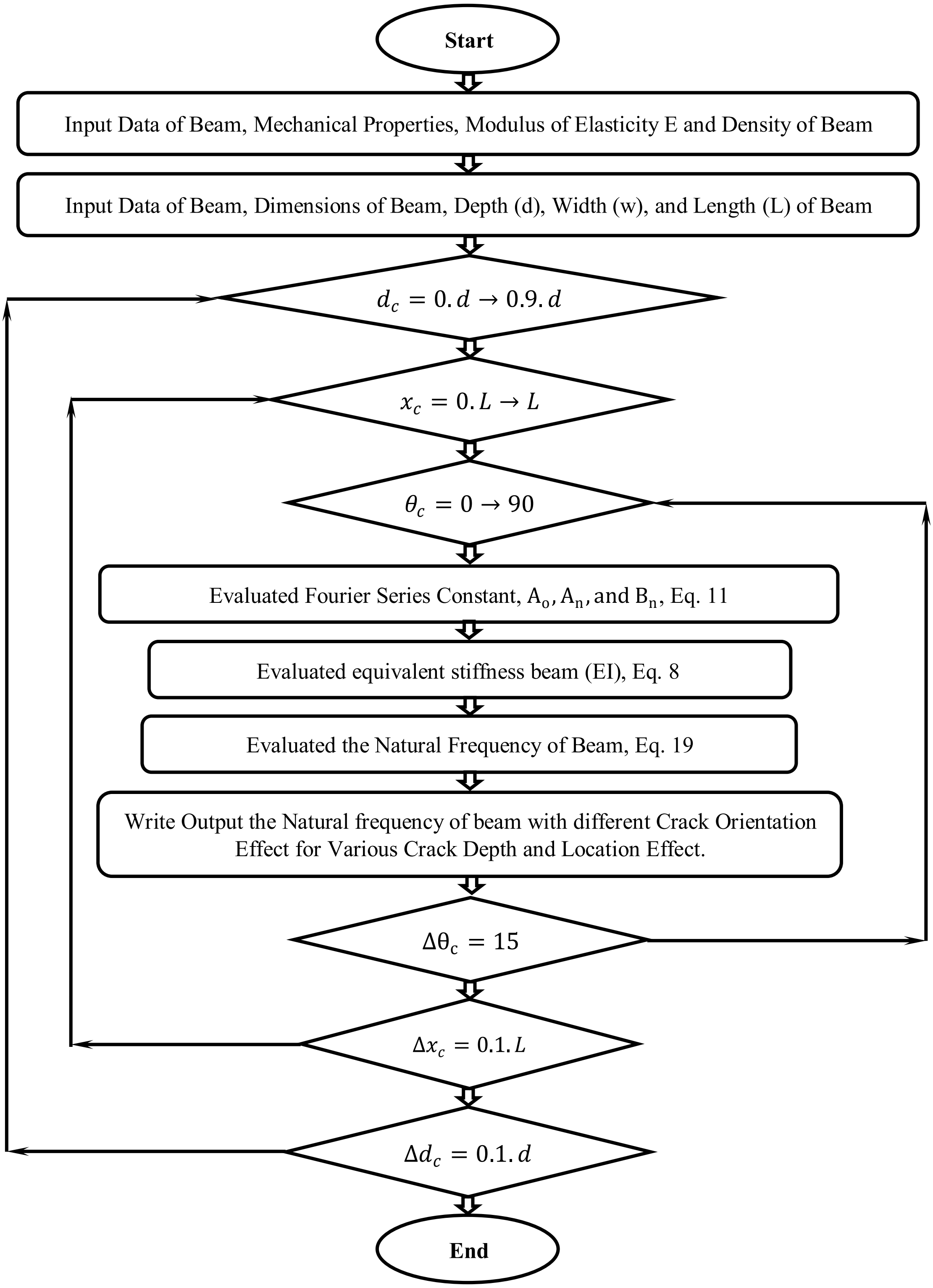
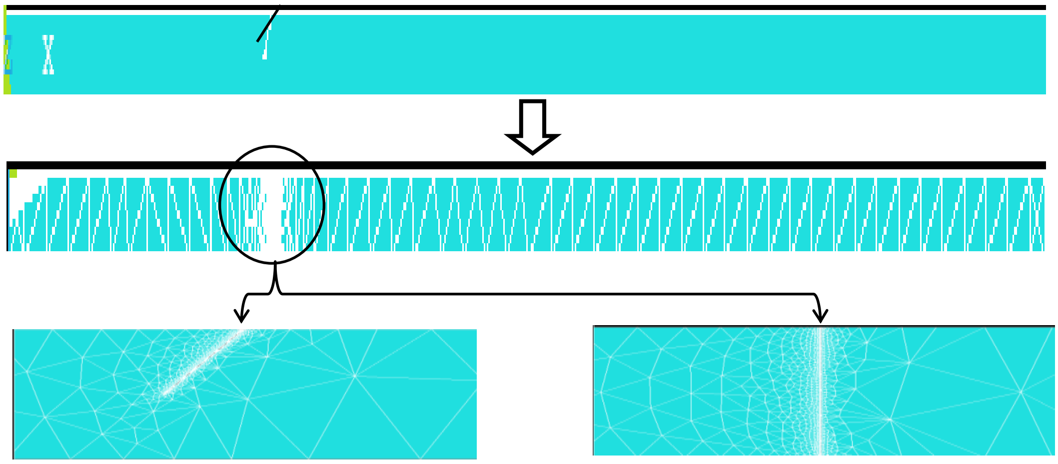


Figure 2. Flow chart of fortran computer program, for evaluating the natural frequency of beam with crack orientation effect with different crack depth and location effect

4. Numerical model

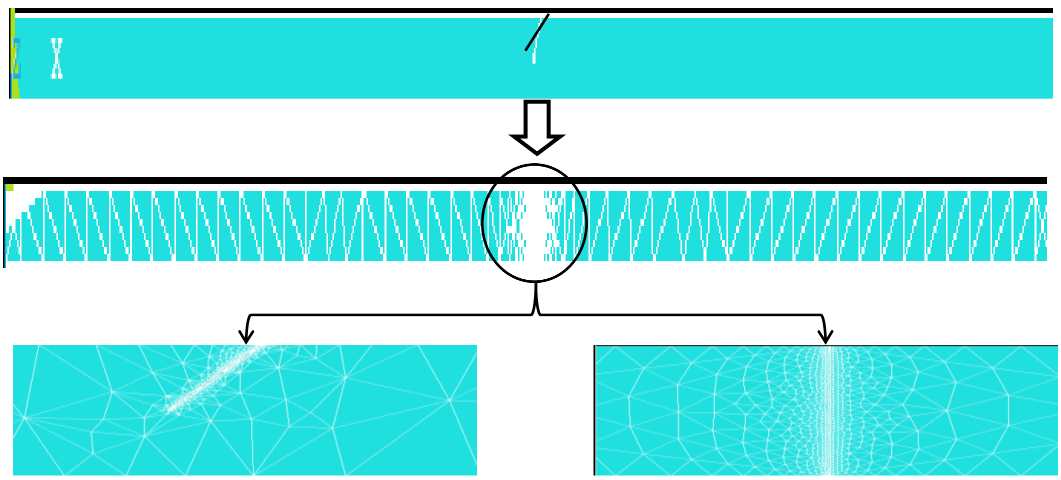
The numerical study of simply supported beam with oblique crack effect studied, included evaluated the natural frequency of beam with effect of crack orientation for different crack depth and location effect by using finite element methods with using Ansys program Ver. 14, and the numerical results are compare with the theoretical results.

The finite element method, by using Ansys program, using the three dimensional model were built and the element (Solid Tet 10 node 187) were used. A sample of meshed beam is shown in Figure 3. for different oblique crack location.



Side View of Mash in Crack Location | Top View of Mash in Crack Location

a. Beam with Side Crack Location



Side View of Mash in Crack Location | Top View of Mash in Crack Location

b. Beam with Middle Crack Location

Figure 3. Mash of beam with different crack location

5. Results and discussion

The vibration results of beam with oblique crack effect includes the evaluation of the natural frequency of different beam materials, with oblique crack effect of beam, included the effect of crack orientation, crack size, crack location, of simply supported beam.

Where The mechanical properties of beam studded, [14], are,

- Low Carbon Steel beam,
  E=207 GPa, G=80 GPa, ρ=7800 kg/m^3, ν=0.3
- Alloys Aluminum beam,
  E=69 GPa, G=26 GPa, ρ=2770 kg/m^3, ν=0.33
- Bronze beam,
  E=115 GPa, G=45 GPa, ρ=7650 kg/m^3, ν=0.28

And dimensions beam,

- Width of beam, $w = 0.015$ m
- Depth of beam, $d = 0.025$ m
- Length of beam, $L = 1$ m

The method studied to evaluate the natural frequency of beam with oblique crack effect are, theoretical study and numerical study, by using ANSYS Program Version 14. The theoretical and numerical work includes the study of the crack orientation effect of beam with different crack size (depth of crack), different crack position, different beam dimensions (depth and length of beam) and simply supported boundary conditions of beam.

The theoretical results compare with numerical results and shown the effect of the crack orientation of different beam materials (low carbon steel, alloys Aluminum and bronze beam) with different crack depth and location effect, as,

Figures 4 and5, shown theoretical and numerical results of natural of simply supported beam made of low carbon steel materials with crack orientation effect for different crack depth and crack location effect,($d_c = 10\%d, 30\%d, 50\%d, 70\%d, 90\%d$), ($x_c = 10\%L, 25\%L, 40\%L, 50\%L$), respectively.

Figures 6 and 7, shown theoretical and numerical results of natural of simply supported beam made of alloys aluminum materials with crack orientation effect for different crack depth and crack location effect ($d_c = 10\%d, 30\%d, 50\%d, 70\%d, 90\%d$), ($x_c = 10\%L, 25\%L, 40\%L, 50\%L$), respectively.

Figures 8 and 9, shown theoretical and numerical results of natural of simply supported beam made of bronze materials with crack orientation effect for different crack depth and different crack location effect ($d_c = 10\%d, 30\%d, 50\%d, 70\%d, 90\%d$), ($x_c = 10\%L, 25\%L, 40\%L, 50\%L$), respectively.

The theoretical results evaluated from solution of general equation of motion of beam, with crack effect are compared with those obtained numerically by using the ANSYS Program (Version 14) for each parameters effect studied as shown in Figures 4 to 9, shows a good approximation where the biggest error percentage is about (1.5 %).

The Figures 4 to 9. Shows that the natural frequency of beam increasing with increasing the crack orientation, since the vertical depth effect of crack decreasing with increases the crack angle, and then, the stiffness of beam increasing with decreases of vertical crack depth effect. The, can be see the effect of crack orientation cases decreasing the effect of crack through decreasing of the vertical crack depth effect.

And, to shown the effect of crack depth on the natural frequency of simply supported beam, Figures 10 and 11 study the natural frequency of different materials simply supported beam with crack location (x_c=25%L, 50%L) and different crack depth with various crack orientation effect. The effect of crack size as depth of crack for different beam materials, with different crack orientation, shows that the natural frequency decreasing with increasing crack depth for different crack orientation, this is because the changing in stiffness beam.

Also, the effect of crack location on the natural frequency of simply supported beam studding in Figures 12 and 13 for different beam materials with crack depth (d_c=30%d, 70%d) and various oblique crack orientation and location effect. The effect of crack location on the natural frequency of different beam materials with various crack orientation effect shown that the natural frequency of beam decreases when the crack near the middle location of beam more than other location of crack.

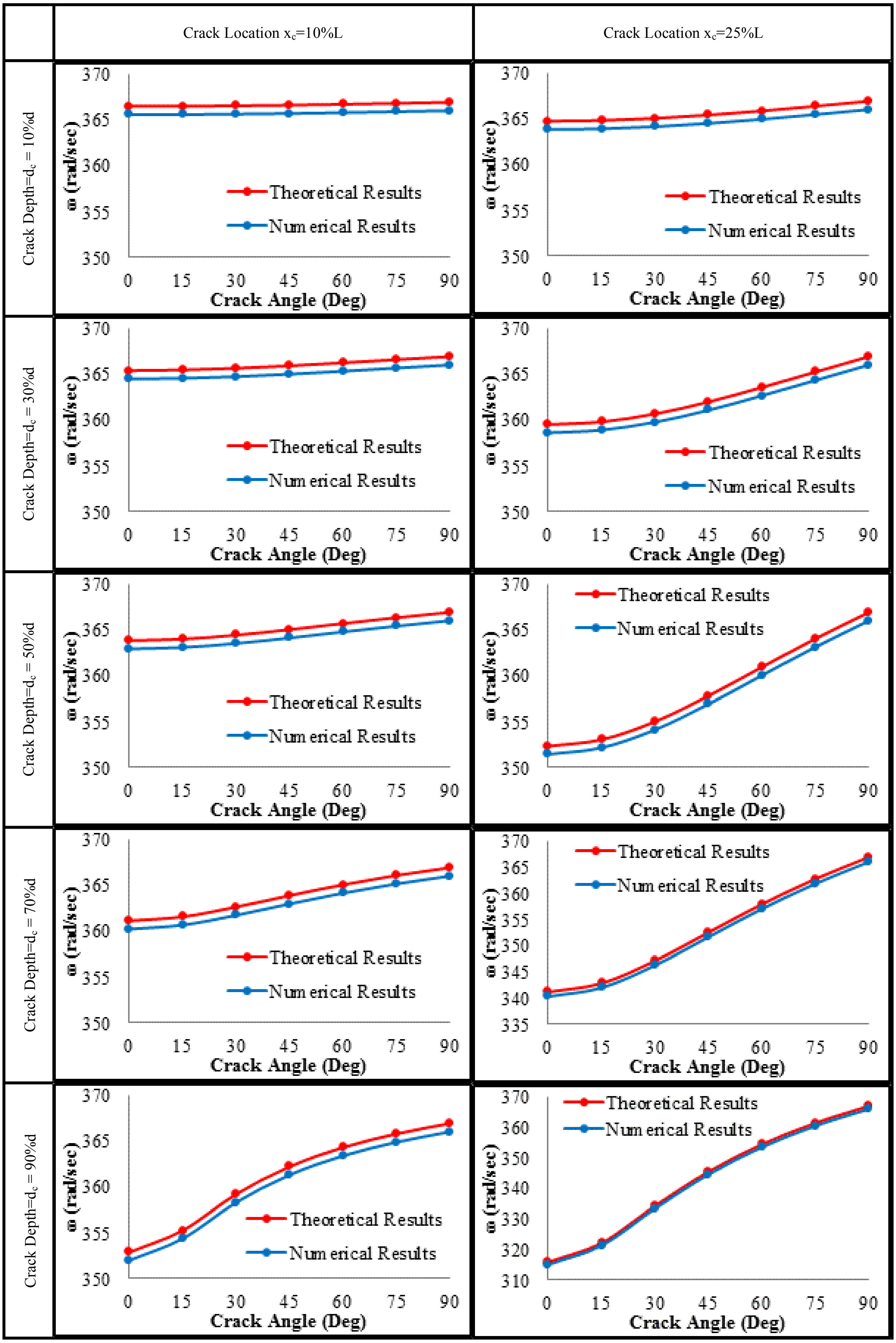


Figure 4. Theoretical and numerical natural frequency for low carbon steel materials beam with various crack angle and different crack depth effect, with crack location (x_c=10%L and x_c=25%L)

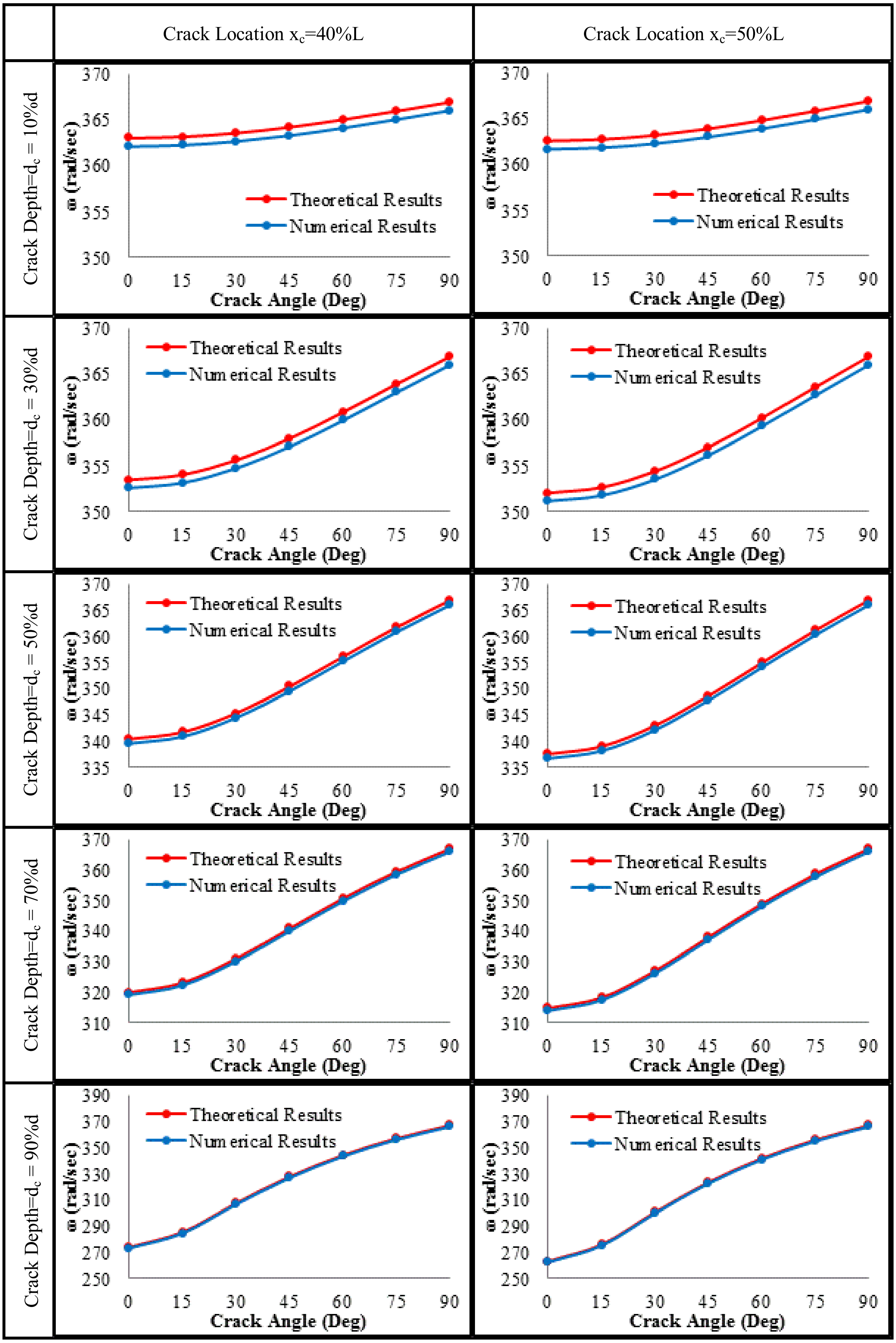


Figure 5. Theoretical and numerical natural frequency for low carbon steel materials beam with various crack angle and different crack depth effect, with crack location (x_c=40%L and x_c=50%L)

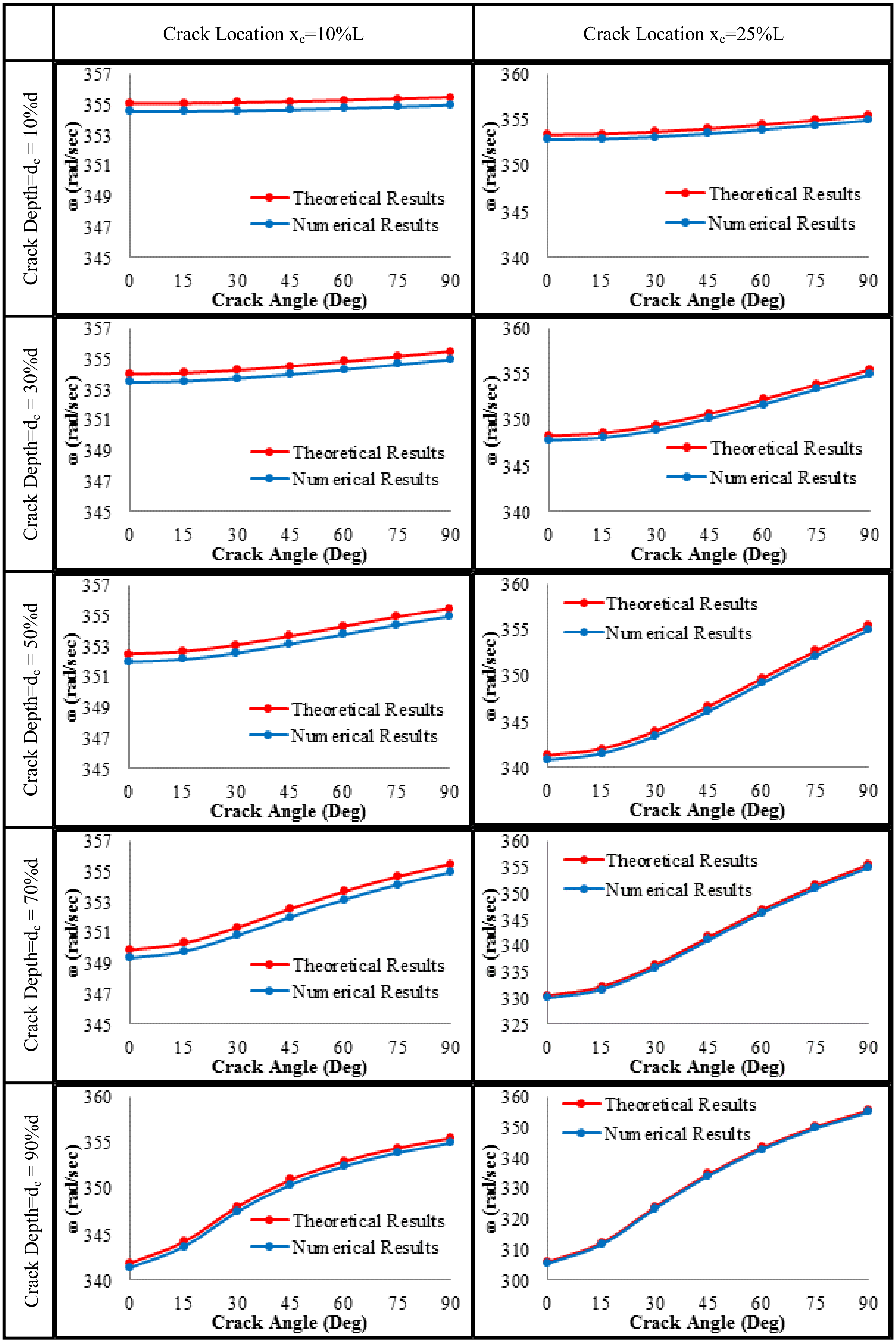


Figure 6. Theoretical and numerical natural frequency for alloys aluminum materials beam with various crack angle and different crack depth effect, with crack location (x_c=10%L and x_c=25%L)

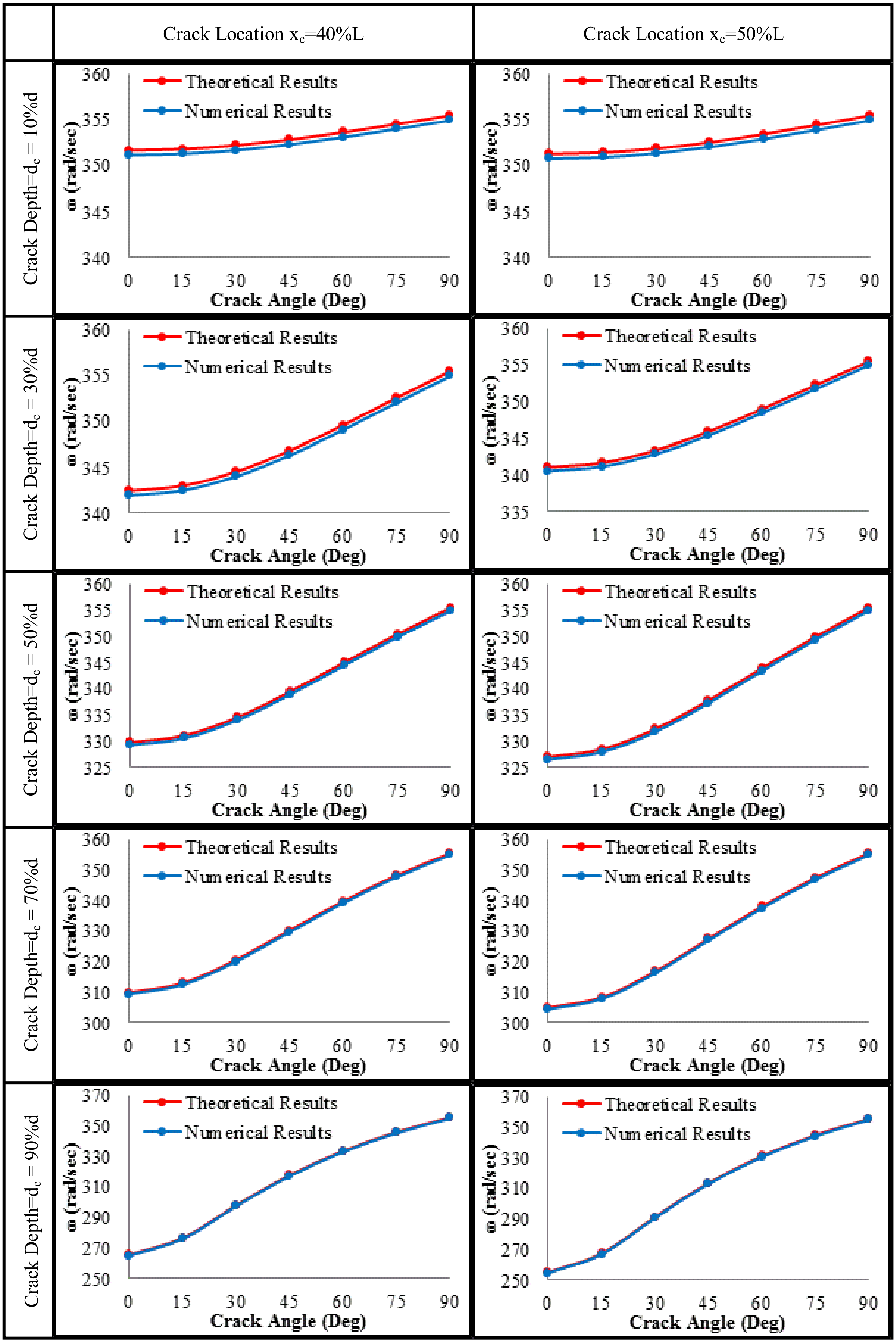


Figure 7. Theoretical and numerical natural frequency for alloys aluminum materials beam with various crack angle and different crack depth effect, with crack location (x_c=40%L and x_c=50%L)

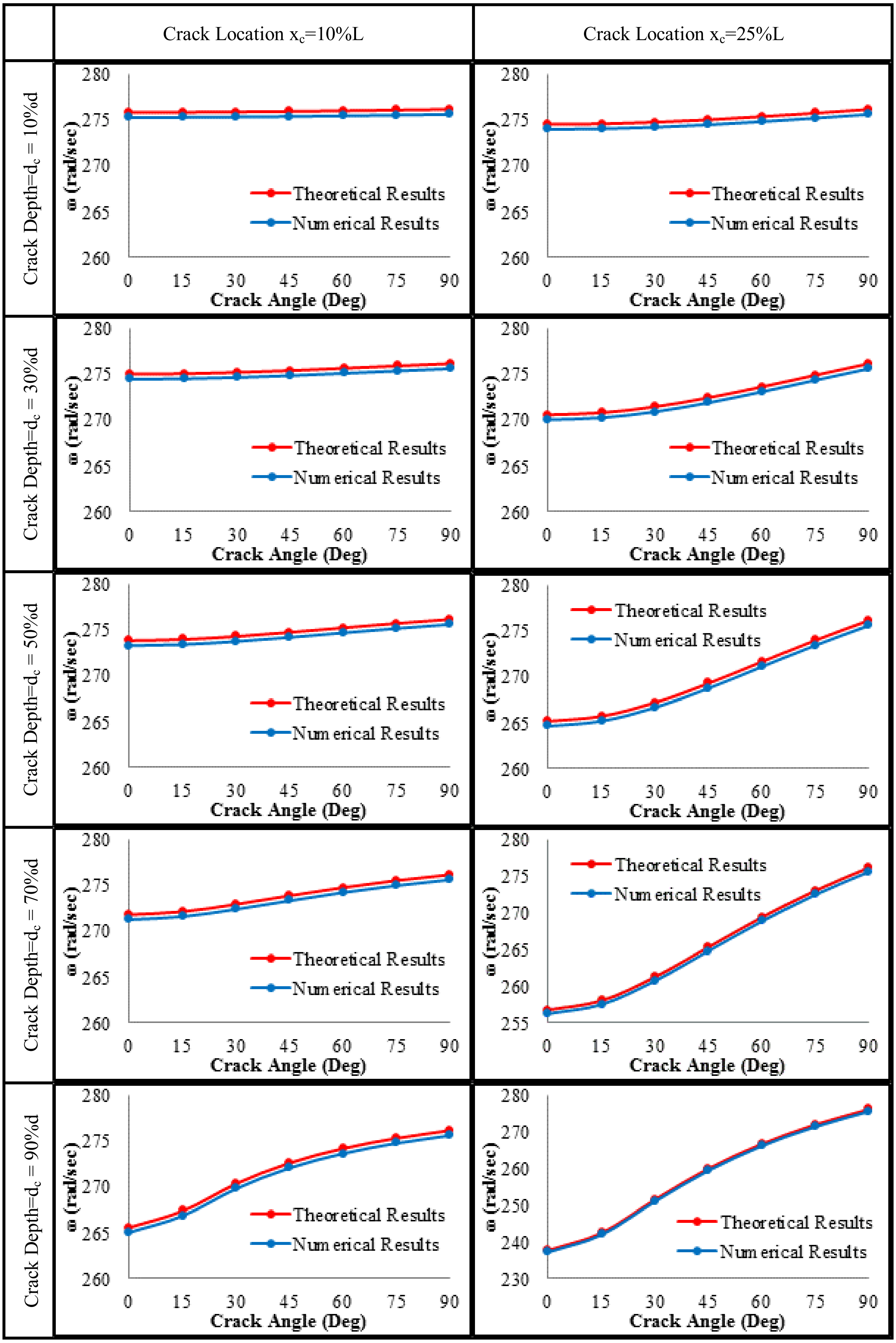


Figure 8. Theoretical and numerical natural frequency for bronze materials beam with various crack angle and different crack depth effect, with crack location (x_c=10%L and x_c=25%L)

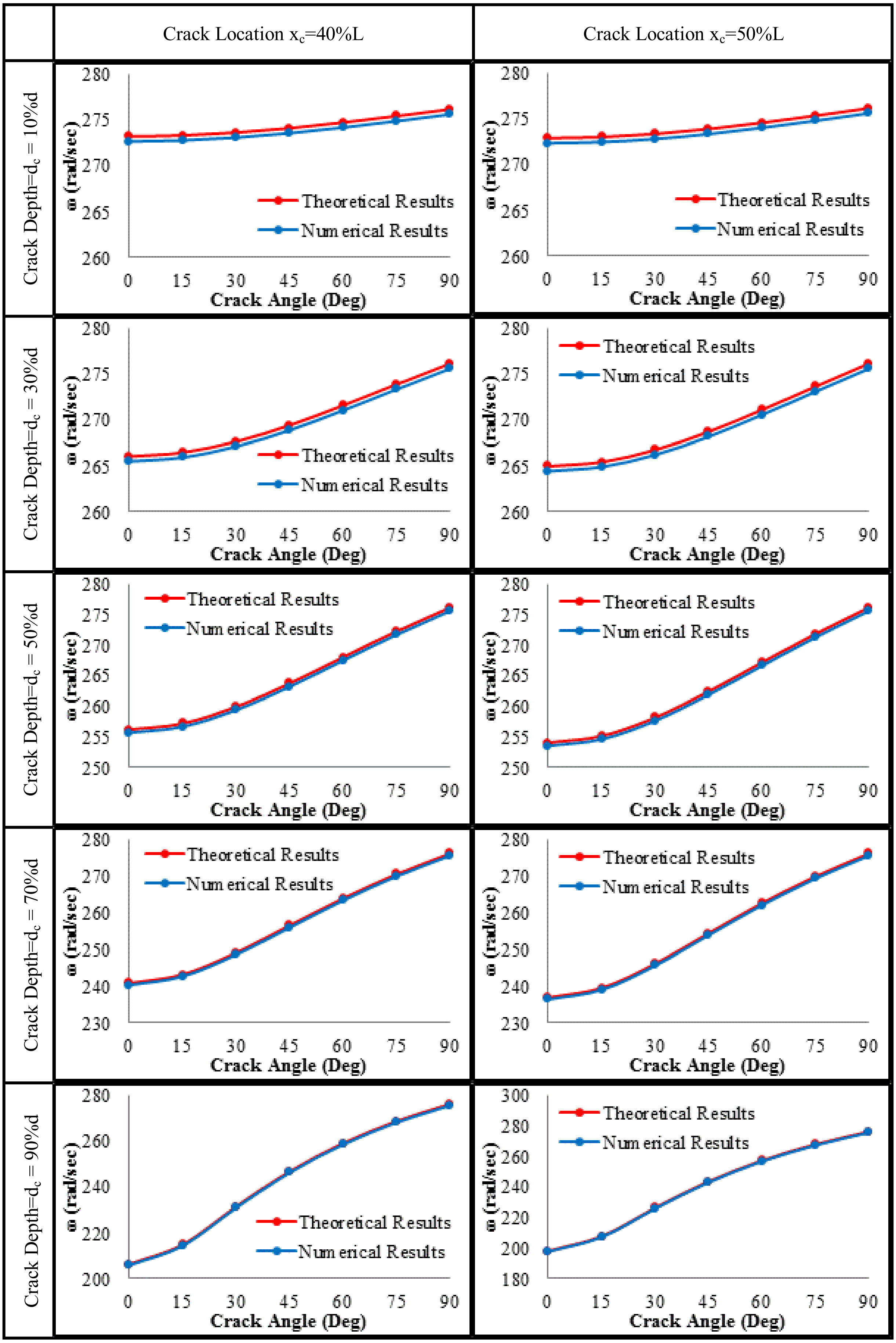
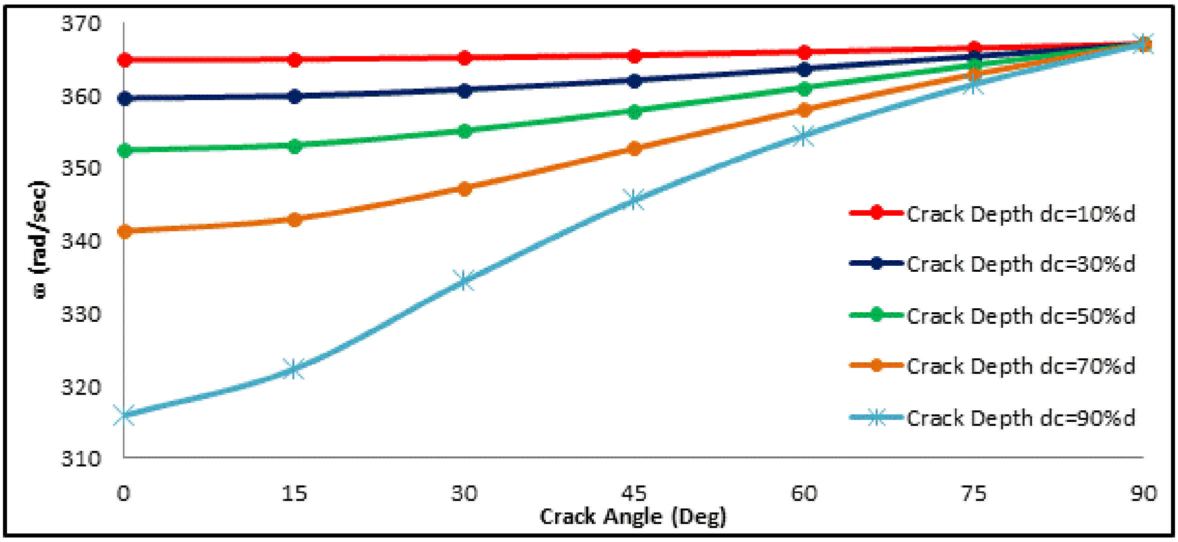
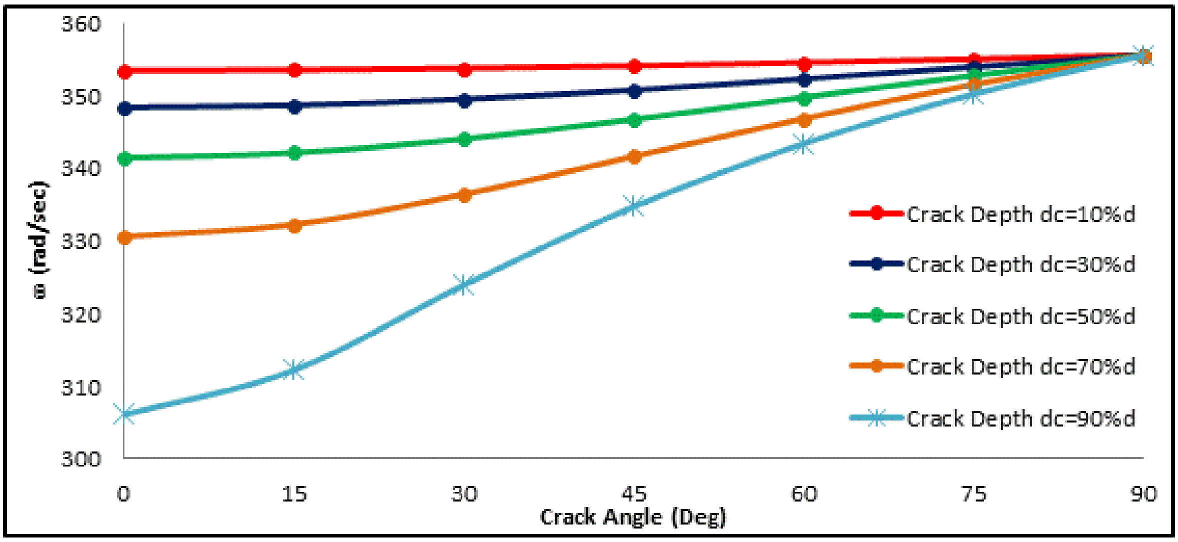


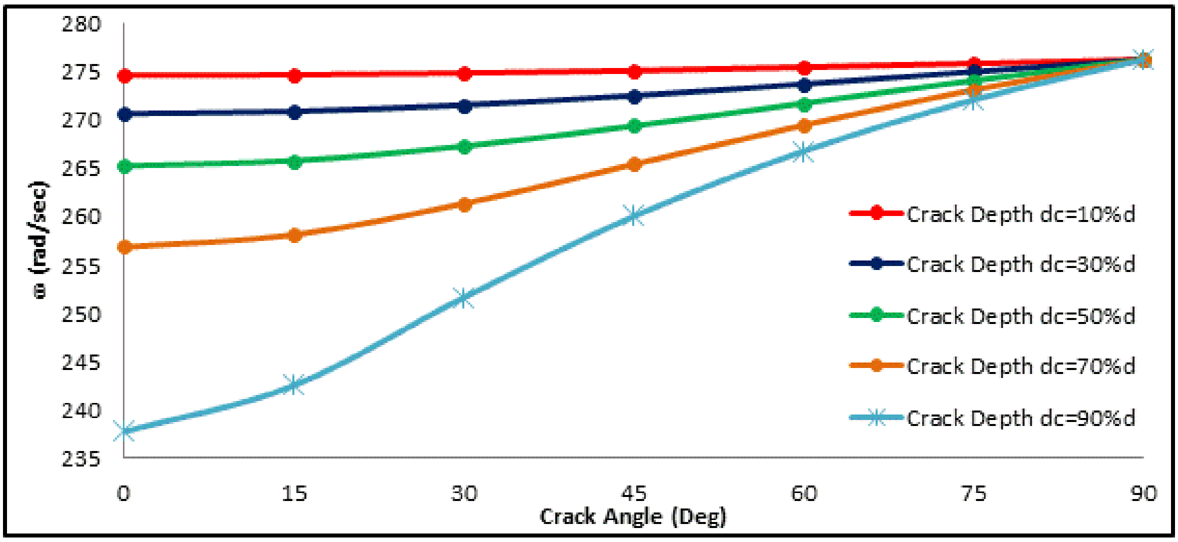
Figure 9. Theoretical and numerical natural frequency for bronze materials beam with various crack angle and different crack depth effect, with crack location (x_c=40%L and x_c=50%L)



(a) Low carbon steel beam

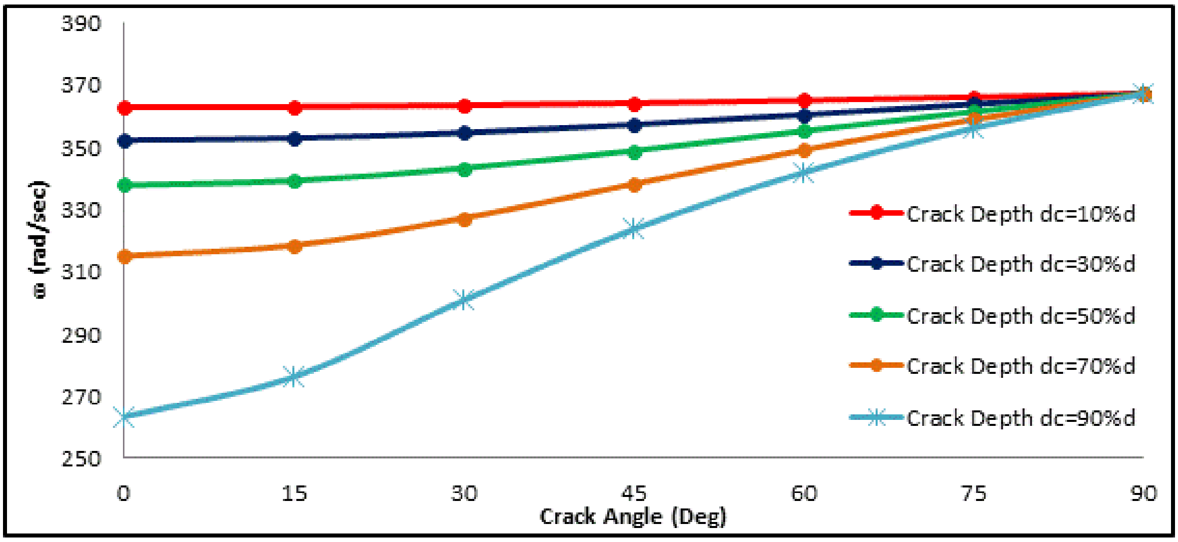


(b) Alloys aluminum beam

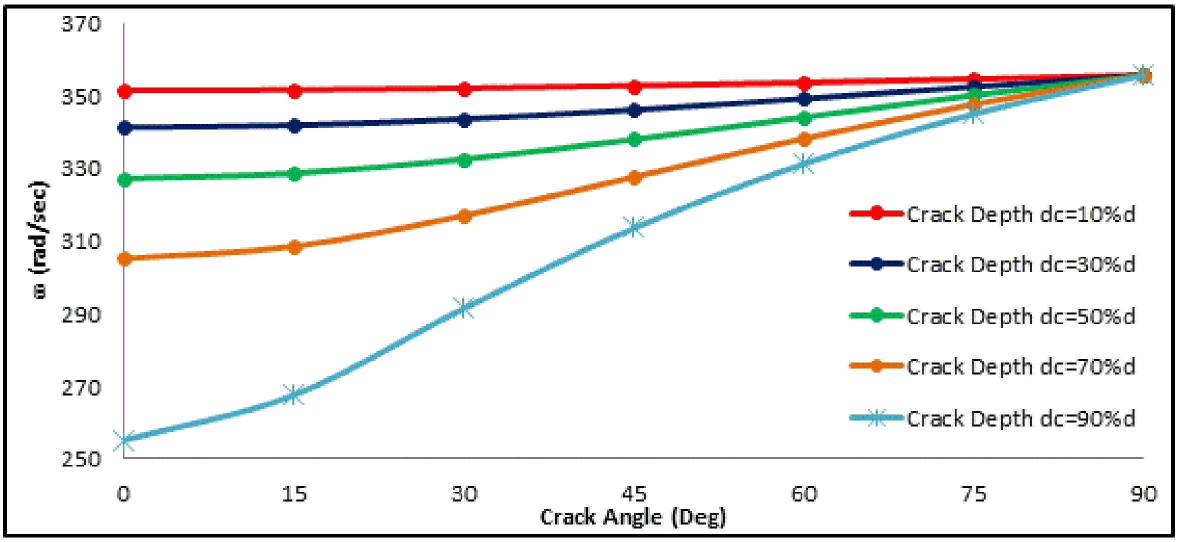


(c) Bronze beam materials

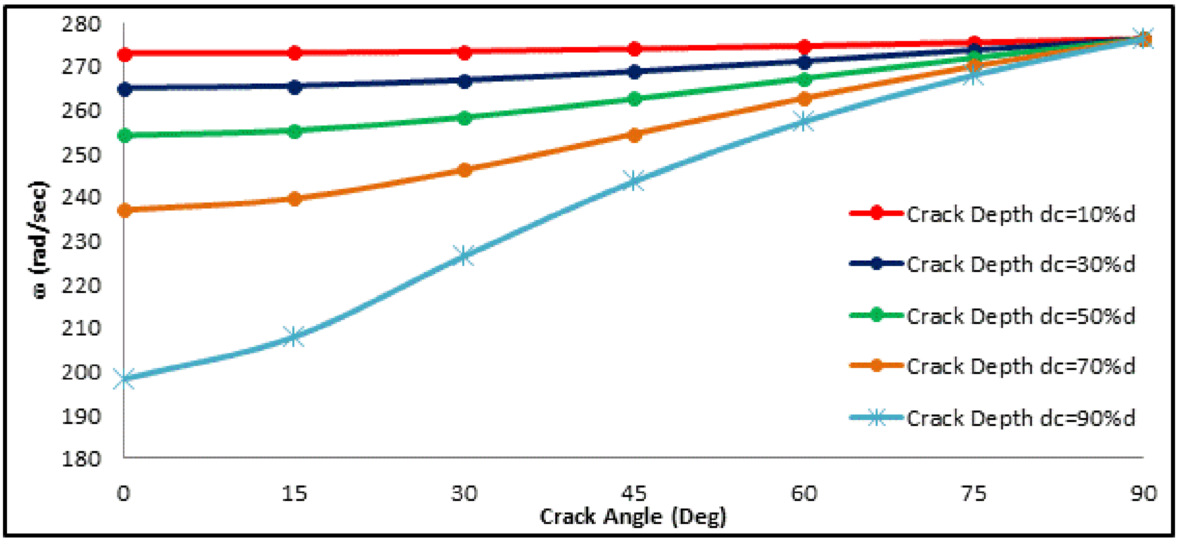
Figure 10. Theoretical natural frequency results for different simply supported beam materials with various crack angle effect for different crack depth effect and location of crack (xc=25%L)



(a) Low carbon steel beam

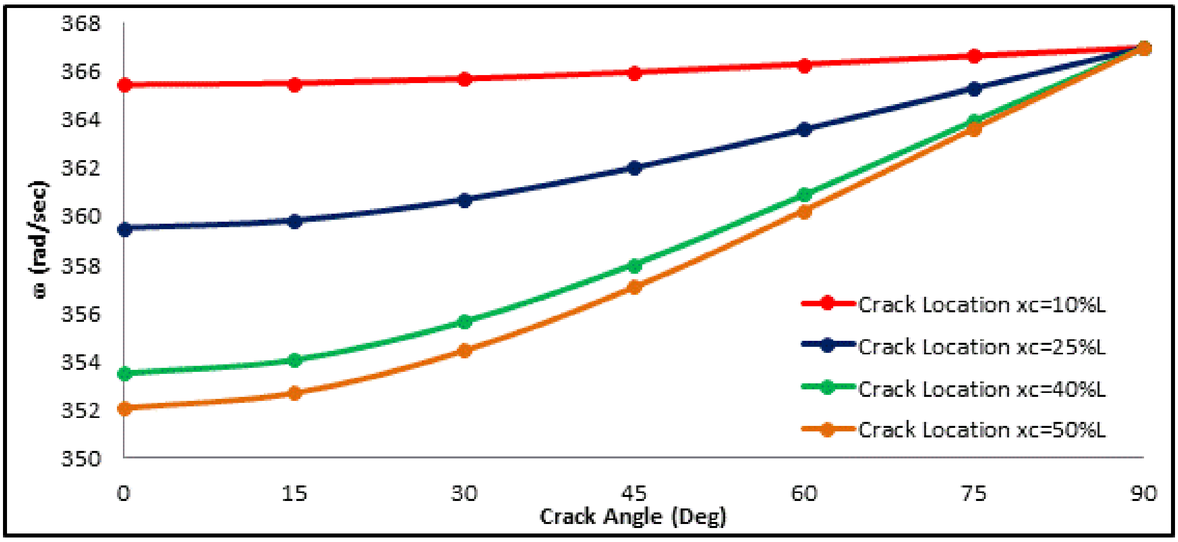


(b) Alloys aluminum beam

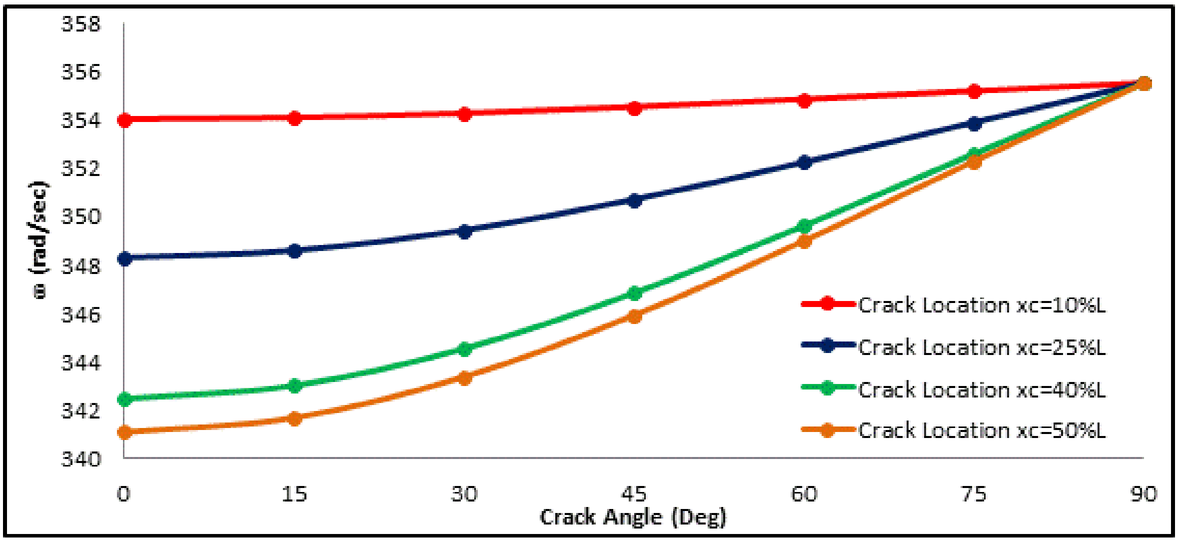


(c) Bronze beam materials

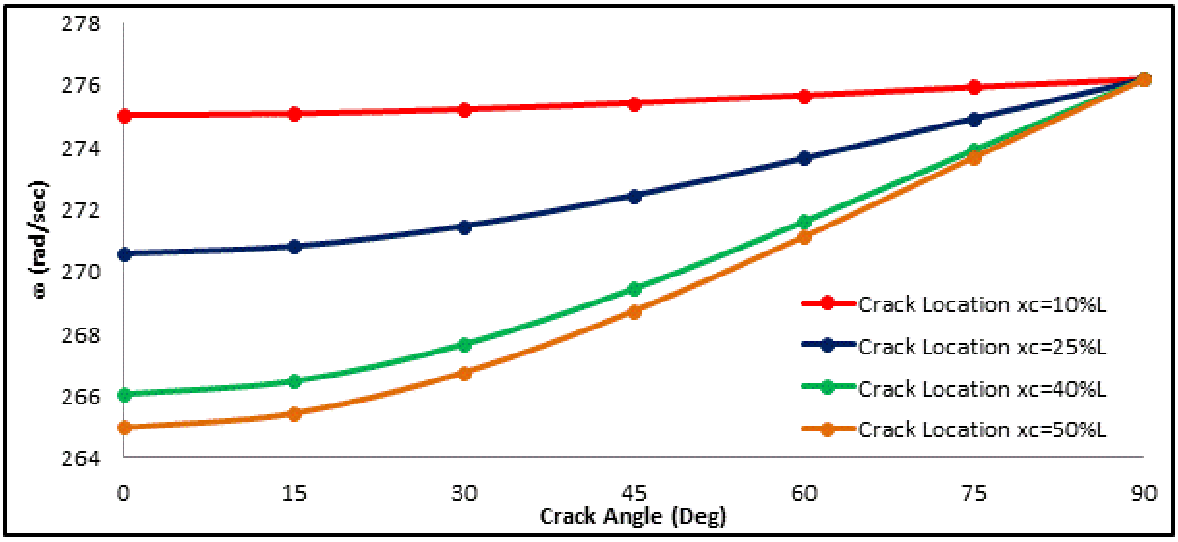
Figure 11. Theoretical natural frequency results for different simply supported beam materials with various crack angle effect for different crack depth effect and location of crack (x_c=50%L)



(a) Low carbon steel beam

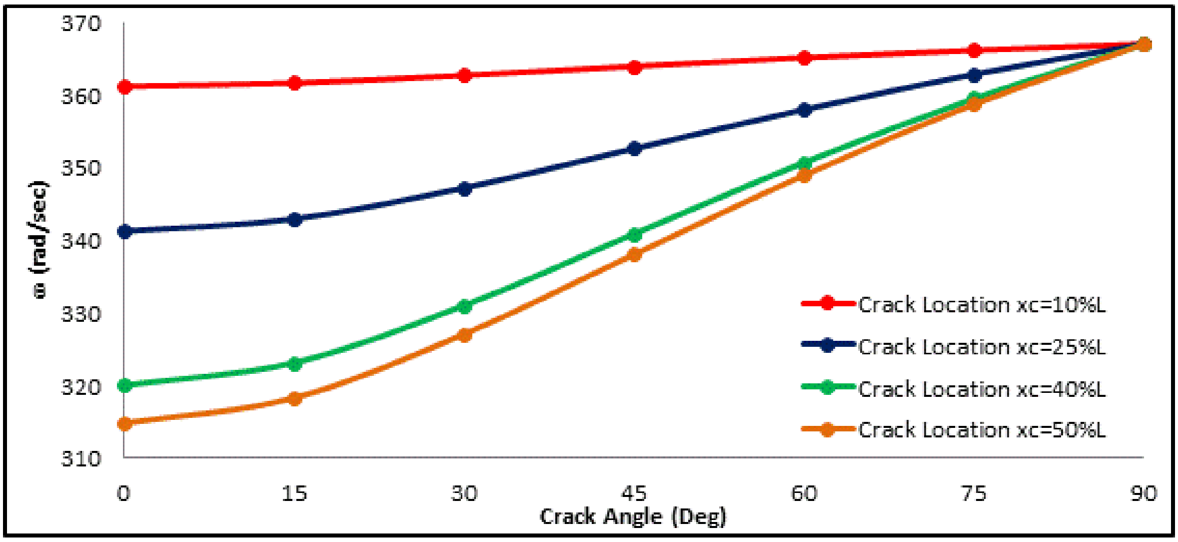


(b) Alloys aluminum beam

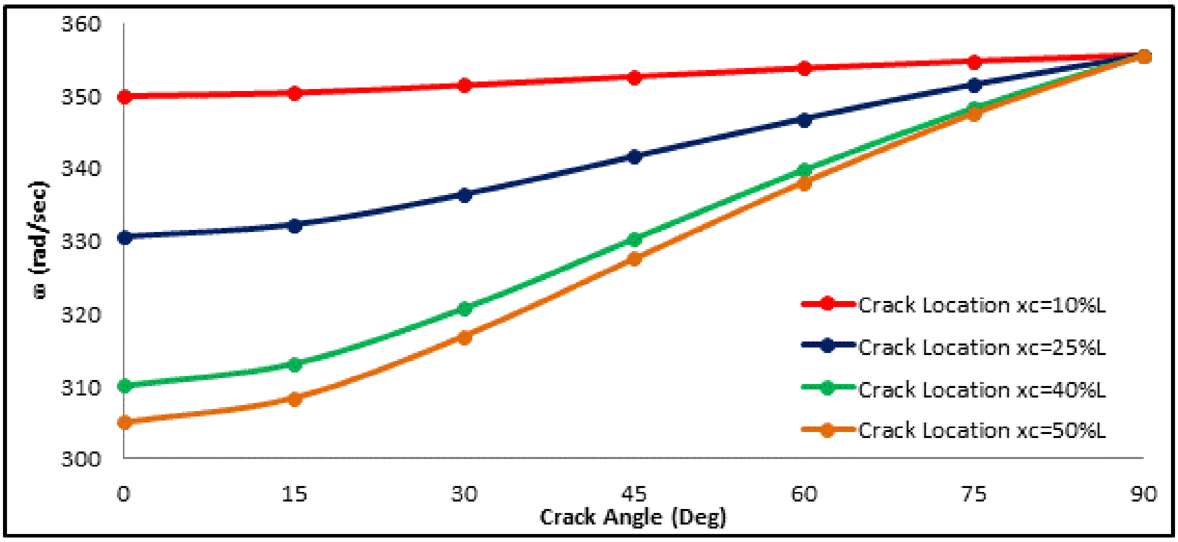


(c) Bronze beam materials

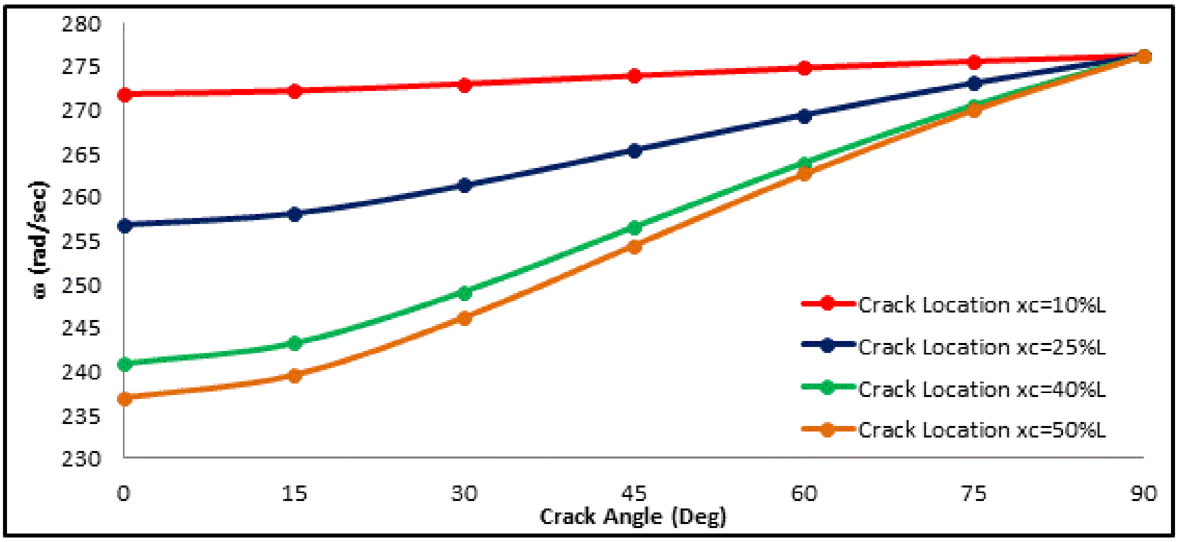
Figure 12. Theoretical natural frequency results for different simply supported beam materials with various crack angle effect for different crack location effect and depth of crack (d_c=30%d)



(a) Low carbon steel beam



(b) Alloys aluminum beam



(c) Bronze beam materials

Figure 13. Theoretical natural frequency results for different simply supported beam materials with various crack angle effect for different crack location effect and depth of crack (d_c=70%d)

6. Conclusion

Some concluding observations from the investigation of analytical and numerical study of natural frequency of beam with crack orientation effect, for different crack depth and location effect, are given below,

1. The suggested analytical solution is a powerful tool for natural frequency of beam with crack orientation, depth and location analysis study of simply supported beam with different materials beam., by solution the general differential equations of motion of beam with oblique crack.
2. A comparison made between analytical results from solution of general equation of motion of beam with crack effect with numerical results by finite elements method, with using Ansys program, shows a good approximation.
3. The crack causes, as expected, a decrease in the stiffness of beam, and then, decreases the natural frequencies of flexural vibrations of the beam.
4. The orientation of crack through the depth of beam causes divided the crack to two effect, vertical and horizontal effect. And then, since the vertical depth decreasing with increasing the crack orientation, then, the natural frequency of beam decreasing with increasing the crack angle.
5. The crack in the beam has an effect on the stiffness of the beam, this will affect the frequency of the beam, so with increasing of the crack depth this will cause a decreasing the natural frequency of the beam.
6. The position of crack in the beam near the middle of the beam has more effect on the stiffness and natural frequency of beam from the other positions.

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