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Free vibration analysis of stiffened cylinder shell

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Abstract

In this paper, the effect of different types of stiffeners on the cylindrical shell structure is investigated. The dynamic properties (natural frequencies and damping ratio) were computed for each finite element models.

Finite element models for the different steel cylindrical stiffened shell have been created by considering helix angle, numbers, locations and height of stiffeners with a constant mass which is (4 kg), and a cantilever supported structures are used.

An experimental test has been done to check the validity of the stiffened shell model. The results obtained are the natural frequency and damping ratio. A comparison between the natural frequency and finite element result is made with an error (18.43%) is found.

Modal analysis is performed to each finite element models to extract the values of the natural frequencies. The model of (helix angle = 67.5°, height = 3.125cm, internal and eight stiffeners) have the highest value of the natural frequency when compared with the other models.

Finally, a comparison between experimental work of M. Bagheri and A.A. Jafari [1] and numerical part of the current paper has been occurred with a small percentage error between them.

Keywords: Modal analysis; Natural frequencies; Damping ratio; Finite element models; Stiffeners.

1. Introduction

The use of stiffened structural elements in most branches of structural engineering began in the nineteenth century with the application of flat or curved steel plates for hull of ships and subsequently with the development of steel bridges and aircraft structures. The stiffened form provides higher stiffness and carrying capacity for a given structural weight. Though the stiffened shell proved very efficient in cost and material economy, its analysis, however posed a formidable challenge. For this reason, the analysis of stiffened shell has attracted many research workers [2].

The stiffened shell was studied by several researchers in recent years. This paper includes several literatures to show the techniques used within this area,

M. Bagheri and A.A. Jafari [1], investigated modal analysis of cylindrical shells with circumferential stiffeners, i.e., rings with non-uniform stiffener eccentricity and unequal stiffener arrangement using analytical and practical procedures. The method applied in analytical solution is the Ritz method, while stiffeners are preserved as discrete elements. The effects of non-uniformity of stiffener distribution on natural frequencies are measured for free–free boundary conditions. Results show that, at constant stiffener mass, significant increments in natural frequencies can be attained using non-uniform stiffener distribution. In practical work, modal testing is performed to obtain modal parameters, including natural frequencies, mode shapes, and damping ratio in each mode. Analytical results are compared with practical ones, showing good agreement between them.

J. E. Jam et al. [3], Investigated the modal analysis of grid stiffened circular cylindrical shells based on the first Love's approximation theory using Galerkin method. Full free edges are considered for boundary conditions. An equivalent stiffness model (ESM) is used to develop the analytical solution of the grid stiffened circular cylindrical shell. The effect of helical stiffeners alignment and some of the geometric parameters of the structure have been shown. The accuracy of the analysis has been examined by comparing results with finite element method. Based on comparisons of the last method, it is concluded that the present method is more suitable, more operative and more accurate.

Meixia Chen et al. [4] developed Wave Based Method (WBM) which can be recognized as a semi-analytical and semi-numerical method to analyze the free vibration characteristics of ring stiffened cylindrical shells with intermediate large frame ribs for arbitrary boundary conditions. Boundary conditions and continuity conditions between different substructures are used to form the final matrix whose size is much smaller than the matrix formed in finite element method. Numerical calculations of WBM model show good agreement with the results calculated by finite element method.

In this paper, dynamic characteristic of stiffened cylindrical shell with a constant mass (4-kg) are investigated which are based on the variation in the stiffeners parameters such as; helix angle, location, height and number of stiffeners with uniform distribution on cylinder. The models are built by using finite element approach via (ANSYS program), where the natural frequency of the structures are obtained.

2. Theoretical considerations (Natural frequency)

The natural frequency (ω_i) of the vibration is important to give an idea about the oscillation of the system with time, stiffness to weight ratios for different modes of oscillations, the free vibration (modal analysis) is used to determine the basic dynamic characteristic (vibration characteristic) of structures, which are the natural frequencies, and mode shapes (normal modes). The natural frequencies and mode shapes are important parameters in the design of a structure under dynamic loading conditions. They are also needed if it's required doing dynamic analysis such as frequency, transient and spectrum analysis. To determine the natural frequencies of a structure, the governing differential equation of motion for the free vibration problem (no external applied loads) and undamped case is assumed [5].

If there are no applied actions, the undamped equations of motions are written in homogenous form as:

$$[M]\ddot{\bar{\delta}} + [K]\bar{\delta} = 0 \tag{1}$$

Equation (1) has a known solution that may be stated as follow:

$$\bar{\delta}_i = \bar{\phi}_i Sin(\omega_i t + \alpha_i), \qquad (i = 1,2,.........n) \tag{2}$$

where, n is the number of degrees of freedom.

In this harmonic expression, $\bar{\phi}_i$ is a vector of nodal amplitudes (the mode shape) for the ith mode of vibration. The symbol ω_i represents the angular (natural) frequency of mode i, and α_i denotes the phase angle. By differentiating equation (2) twice with respect to time t, it could be found that,

$$\ddot{\bar{\delta}}_i = -\omega_i^2 \bar{\phi}_i Sin(\omega_i t + \alpha_i) \tag{3}$$

Substituting equations (2) and (3) into equation (1) allows cancellation of the term $Sin(\omega_i t + \alpha_i)$ which leaves,

$$([K] - \omega_i^2 [M])\bar{\emptyset}_i = 0 \tag{4}$$

This manipulation has the effect of separating the variable time from those of space, leaving a set of n homogeneous algebraic equations.

Equation (4) has the form of the algebraic eigenvectors problem. From the theory of homogenous equations, nontrivial solutions exist only if the determinate of the coefficient matrix is equal to zero. Thus,

$$[\mathrm{K}] - \omega_{\mathrm{i}}^{2}[\mathrm{M}] = 0 \tag{5}$$

Expansion of this determinate yield a polynomial of order n called the characteristic equation. The n roots ω_i^2 of this polynomial are the characteristic values, or (eigenvalues). Substitution of these roots (one at a time) into the homogenous equations (4) produces the characteristic vectors, or (eigenvectors $\bar{\phi}_i$), within arbitrary constants. Alternatively, each eigenvector may be found as any column of the adjoint matrix $\left[H_i^a\right]$ of the characteristic matrix $\left[H_i\right]$, obtained from equation (4) as follow:

$$\left[H_i\right]\bar{\phi}_i = [0] \tag{6}$$

where,

$$[\mathrm{K}] - \omega_{\mathrm{i}}^{2}[\mathrm{M}] = [\mathrm{H}_{\mathrm{i}}] \tag{7}$$

The methods implied by equations (5), (6) and (7) are conductive to hand calculations for problems having a small number of degrees of freedom. However, a structure with a large number of freedoms (as in the present study) must be handled by a computer subprogram (or subroutine) for calculating eigenvalues and eigenvectors. Various schemes have been developed for a computer analysis to solve the eigenvalue problems for a complex structure, such as, the inverse iteration, Jacobian, subspace iteration, Lanczos iteration, etc., [6]. In the present work, subspace method is adopted to calculate the eigenvalues of the system.

3. Creation finite element model

In this work, FEM with the aid of ANSYS software is used as a numerical tool. The finite element method is a numerical procedure for analyzing structure and continually. Usually, the problem addressed is too complicated to be solved satisfactorily by classical analytical methods. The problem may concern stress analysis, heat conduction or any of several other areas. The finite element procedure produces many simultaneous algebraic equations, which generated and solved on a digital computer [7].

3.1 Element types

The ANSYS element library contains different element types. Each element type has a unique number and a prefix that identifies the element category: Beam, Link, Pipe, Solid, Shell, etc. The element type determines, among other structural members, [8]:

1. The degree-of-freedom set (which in turn implies the discipline-structural, thermal, magnetic, electric, quadrilateral, brick, etc.)
2. Whether the element lies in two-dimensional or three-dimensional space.

In this thesis, two types of elements are used to build the stiffened cylindrical shell model. The first one is SHELL281 element to model the cylinder body and SHELL181 element to simulate the stiffeners.

3.2 Geometrical and material properties of stiffened shell

The cylindrical shell has a stiffened shell of different helix angles, thickness, height and number of stiffeners. The heights of stiffeners have variable values of (1cm to 6.25cm). In addition, the numbers of stiffeners are ranged from 4 to 32 stiffeners. This modeling is started with random stiffener dimension and modified to keep the mass of cylindrical stiffened shell with constant value of 4-kg.

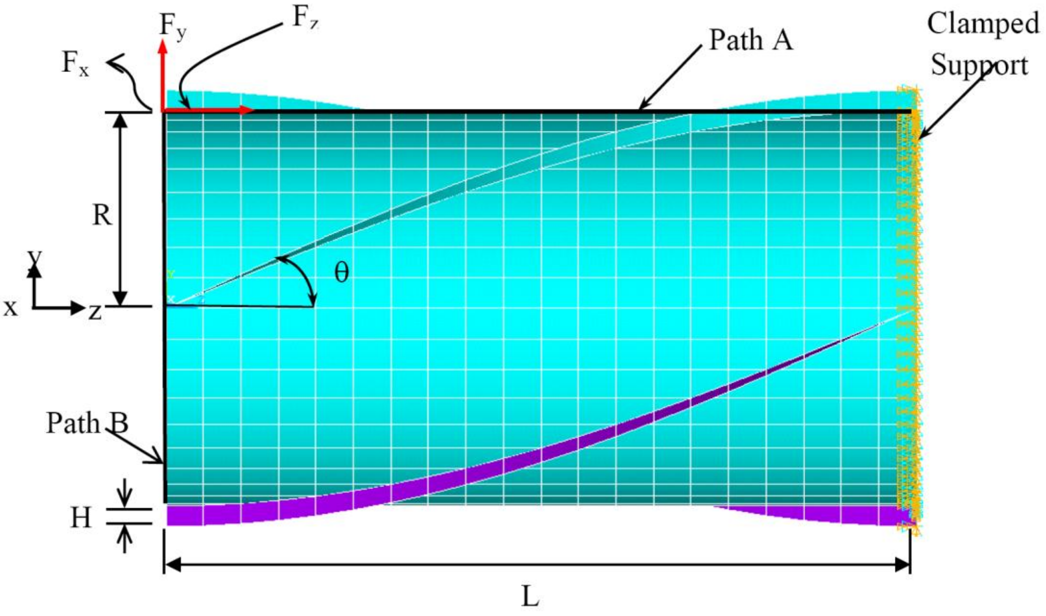
The material properties depending on reference [1] and geometrical dimensions of stiffened cylindrical shell are shown in Table 1. A cantilever supporting is used and applying a concentrated load in three directions (F_x, F_y, F_z) at the upper free edge of cylinder. Two paths (path A is a longitudinal one and path B is a circumferential path) are used to obtain the results along these paths, the finite element of stiffened shell geometry is shown in Figure 1.

The helix angle of stiffeners ranges from 0° to 90° increasing by 22.5° (i.e. 0°, 22.5°, 45°, 67.5° and 90°). These stiffeners may be internal or external stiffeners (see Figure 2).

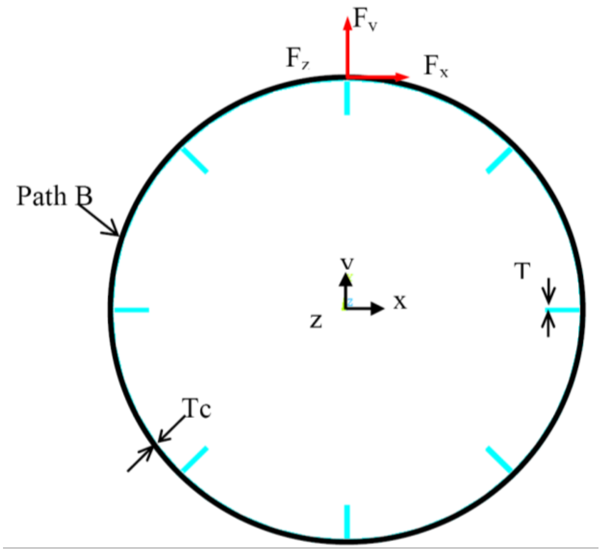
Also, the other variable in the present research is the thickness of stiffeners ranged between 0.4 mm and 3.2 mm.

Table 1. Geometrical and material properties for the current models

Parameters	Symbols	Value	Units
Radius of cylinder	R	19.0985	cm
Length of cylinder	L	60	cm
Thickness of cylinder	Tc	0.5	mm
Load	F_x, F_y, F_z	$F_x = F_y = F_z = 100$	N
Modulus of elasticity	E	201	Gpa
Mass density	ρ	7823	Kg/m³
Poisson's ratio	υ	0.3	/



(a) External four stiffener



(b) Internal eight stiffener

Figure 1. Finite element shell geometry

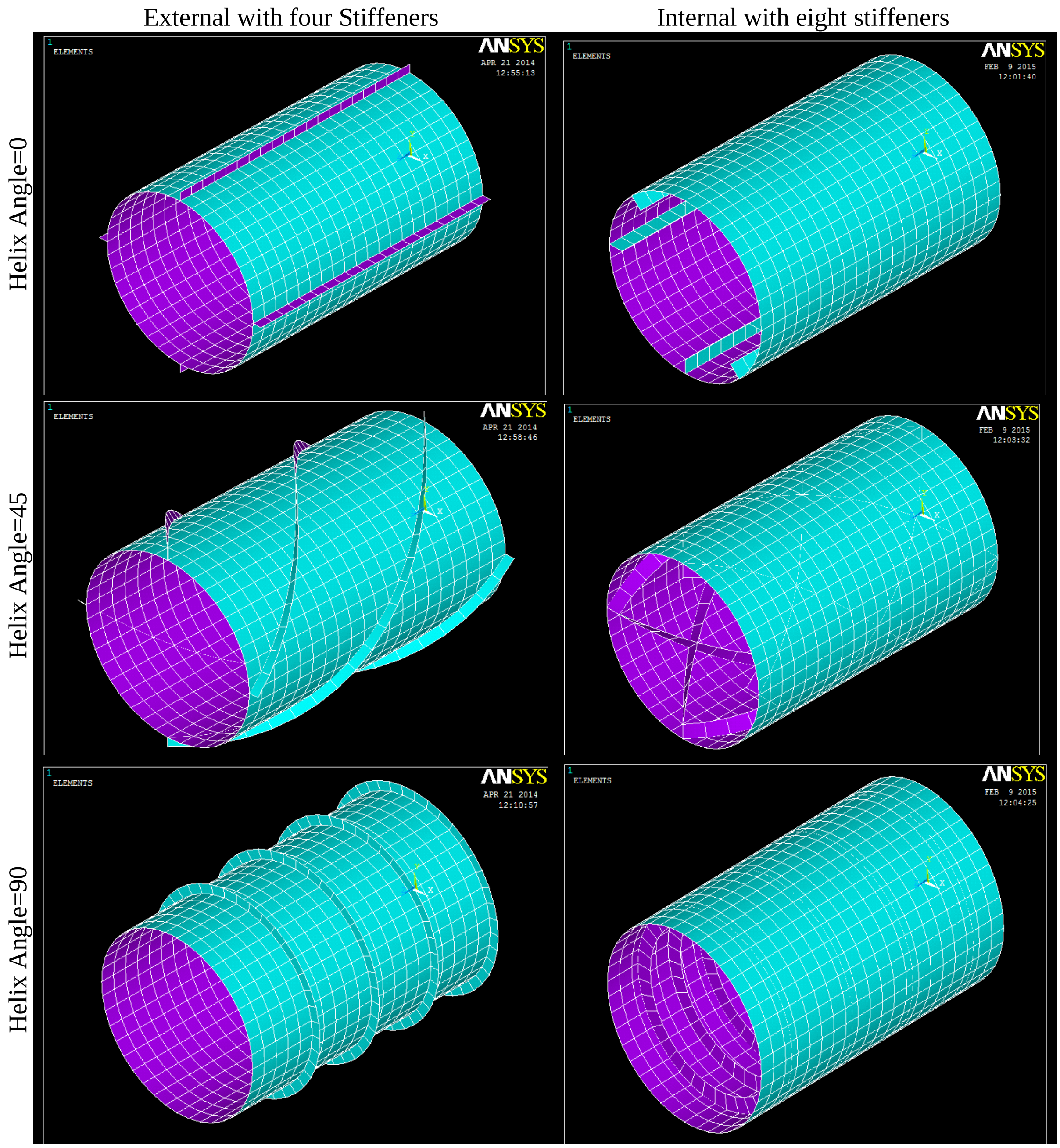


Figure 2. Some finite element models

4. Experimental study

The experimental work includes the experimental tensile and vibration tests of stiffened cylindrical shell specimen. The tensile test has been done to evaluate the yield, ultimate stresses and the elongation for steel plate. Also, the vibrational test has been done to calculate the fundamental natural frequency of the stiffened cylindrical shell and damping ratio of the structure.

4.1 Tensile test

The tensile experimental test of steel plate includes the determination of the yield, ultimate stresses and the elongation for this plate. The dimensions of the samples used in the tensile test were taken from ASTM (A370-2012) as shown in Figure 3.

The tensile test machine used to calculate yield, ultimate stresses and the elongation for plate is shown in Figure 4.

The environmental conditions of the laboratory that the tensile test done in it are (Temperature =25°C and Moisture = 40%). The speed of the tensile test machine is (5 mm/min). The results that obtained from the tensile test for the three specimens (take medium one) are shown in Figure 5.

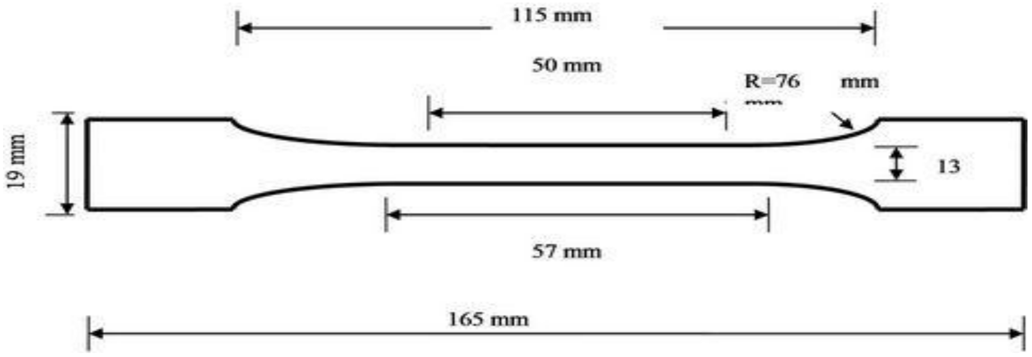


Figure 3. Shape and dimensions of tensile test sample according to ASTM number (A370-2012)

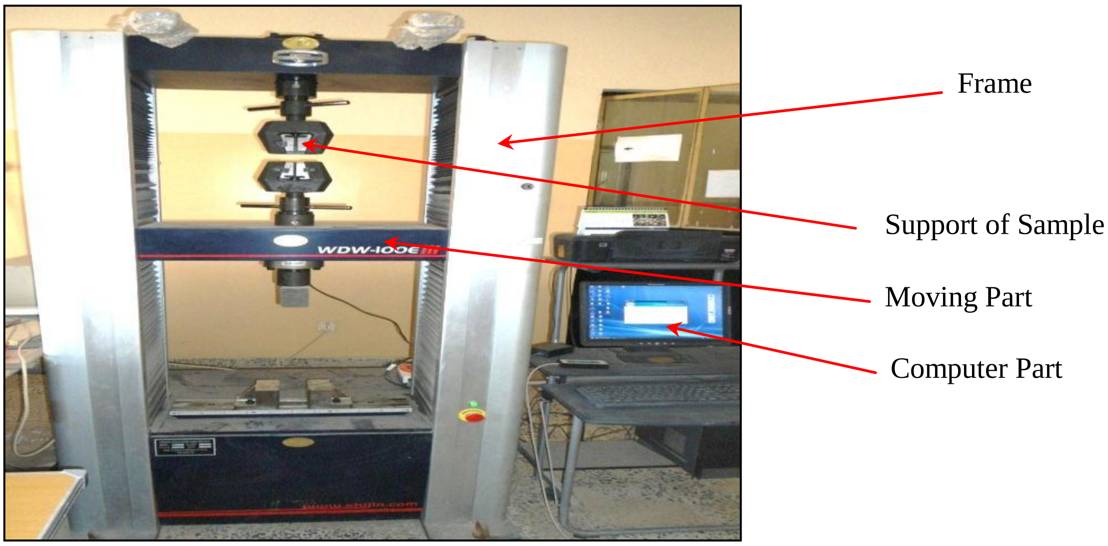
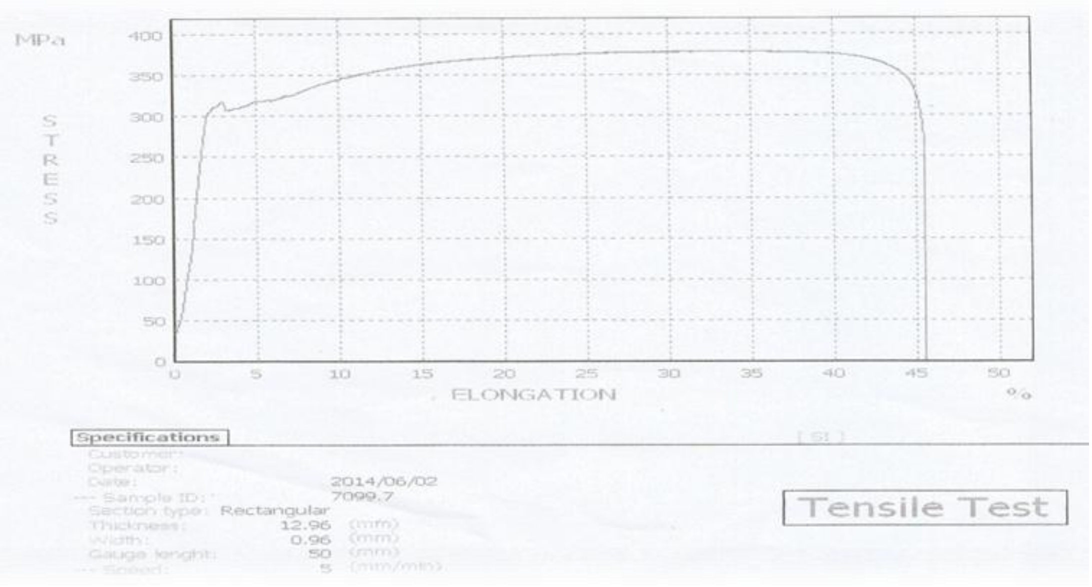


Figure 4. Tensile test machine used in this work



Specifications

Customer:
Operator:
Date: 2014/06/02
Sample ID: 7099.7
Section type: Rectangular
Thickness: 12.96 (mm)
Width: 0.96 (mm)
Gauge lenght: 50 (mm)
Speed: 5 (mm/min)

Figure 5. Tensile test result

4.2 Vibration test of stiffened cylindrical shell samples

The vibration test involves studying the fundamental natural frequency and damping ratio for stiffened cylindrical shell.

Figure 6 shows that the stiffened cylindrical shell manufactured from flat plate with dimensions shown in the same figure mentioned and then rolled to make a cylinder. Then, the stiffeners are made from the same plate and welding with the cylinder.

The stiffened cylinder specimen is made from 12-stiffeners (4-longitudinal, 4-circumferential and 4-inclained with angle=67.5^{o}) with clamped free boundary condition.

The parts and machines that are used in the vibration test are shown in Figure 7.

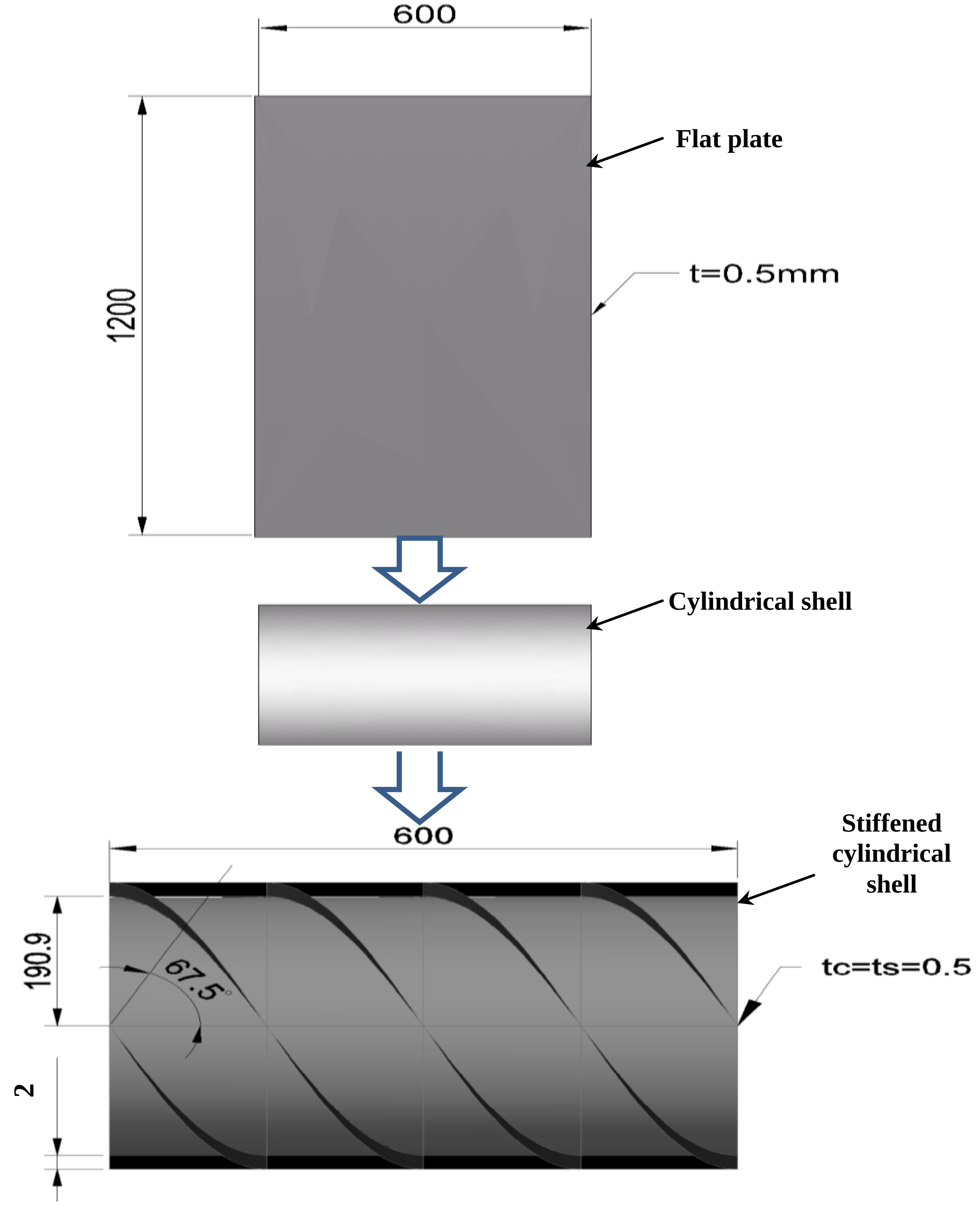


Figure 6. Dimensions and stages of manufacturing of stiffened cylindrical shell that used in vibration test (all dimensions in mm)

The vibration structure and components are composed to the following parts:

1. Structure to support the sample, made of steel plate with (10 mm) thickness.
2. Impact hammer instrument has the model (086C01-PCB Piezotronic vibration division)
3. The accelerometer was used to read the signal from structure which fixed on the model by magnetic. The model of this accelerometer is (353C68).
4. The model of amplifier is (480E09) used to measure the response signal from accelerometer and gives output signal to the digital storage oscilloscope.
5. Digital storage oscilloscope model ADS 1202CL+ and the serial No.01020200300012.

In the present work, the roving accelerometer test is used. The stiffened cylindrical shell is impacted at single point in the upper free edge and the single accelerometer is used in different five positions as shown in Figure 8.

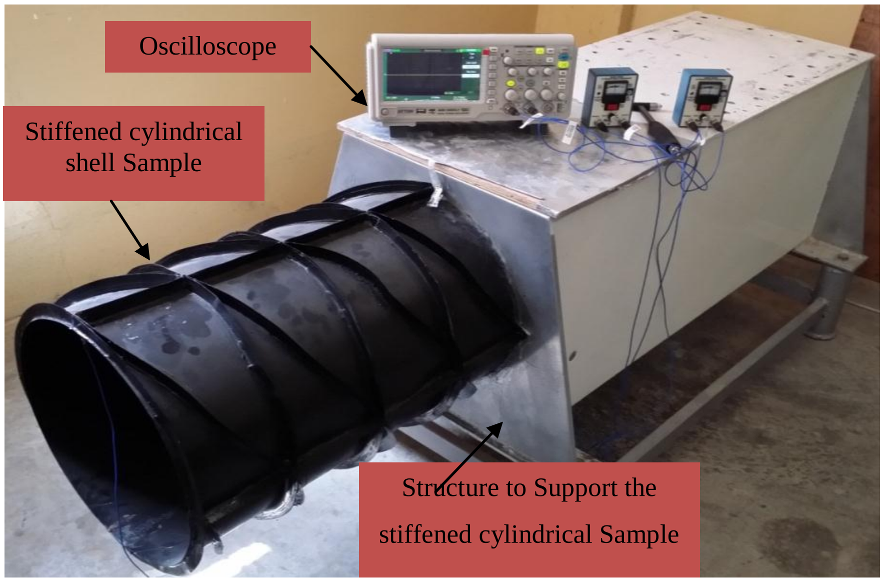


Figure 7. Rig and vibration test machine of stiffened cylinder structure

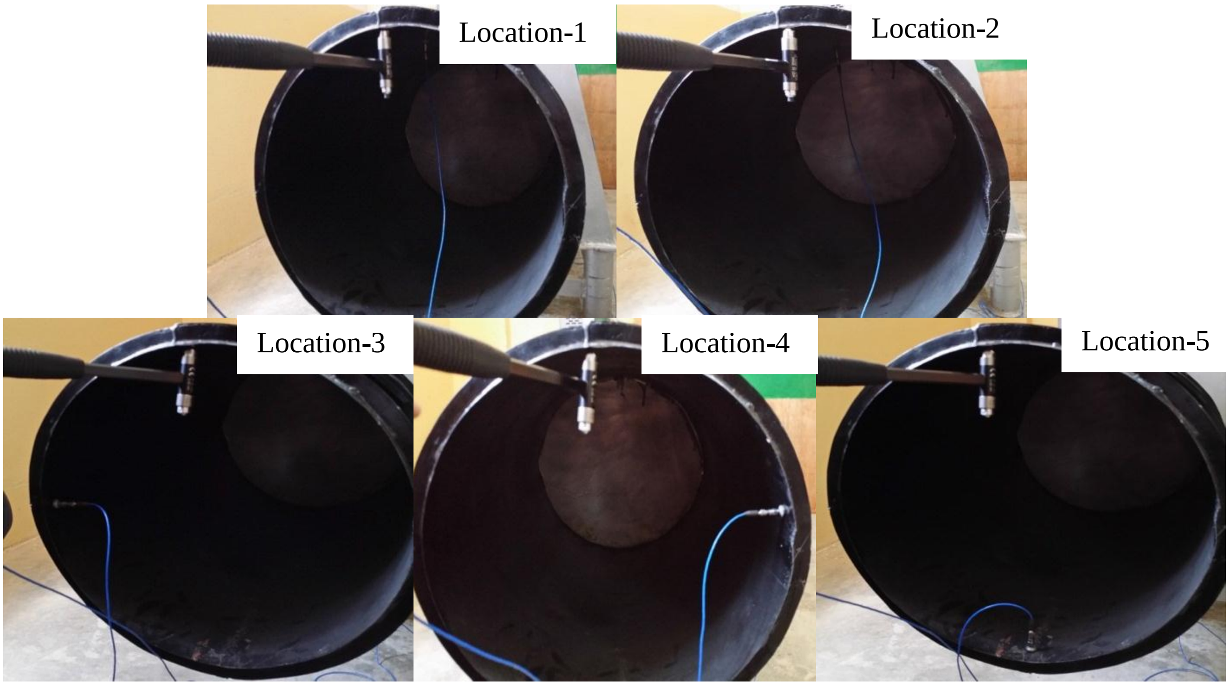


Figure 8. Location of accelerometer and impact hammer on the stiffened cylindrical shell sample

5. Results and discussions

5.1 Verification case study

To verify this case study, steel cylindrical stiffened shell as shown in Figure 9 is taken from M. Bagheri and A.A. Jafari [1] with dimensions and material properties illustrated in Table 2. In the present work, the model is solved numerically by FE approach using ANSYS program.

The comparison between numerical present work and experimental part of M. Bagheri and A.A. Jafari [1] is shown in Figure 10 for first five modes of natural frequencies. It is found that the maximum difference between them estimated by 3%.

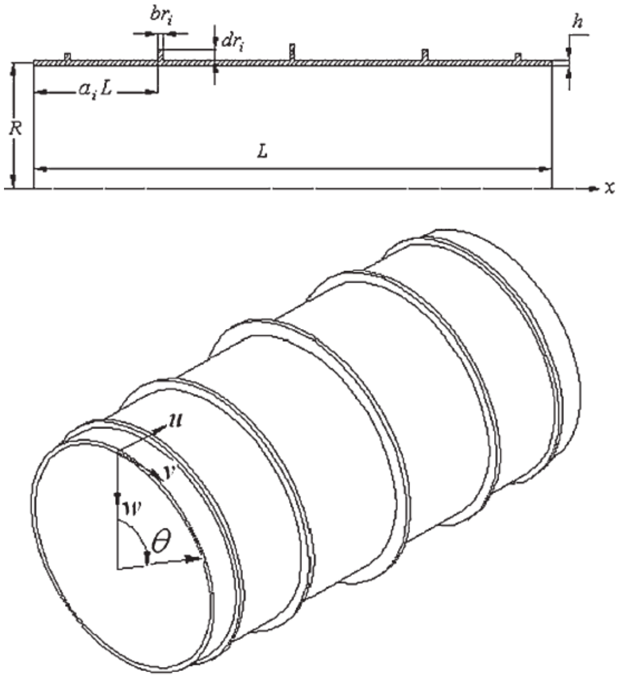


Figure 9. Geometry of reference [1]

Table 2. Geometrical and material properties of reference [1]

Number of Rings (N)	4
Shell Radius R (m)	0.0825
Shell Thickness h (m)	0.0025
Shell Length L (m)	0.2475
Ring Depth dr (m)	0.0037
Ring Width br (m)	0.002
Modulus of Elasticity E (Gpa)	201
Mass Density ρ $\left(\frac{kg}{m^3}\right)$	7823
Poisson's Ratio υ	0.3
Stiffening Type	External

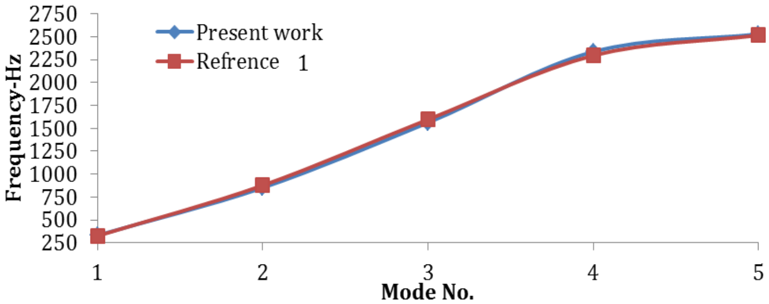


Figure 10. A comparison of the natural frequencies with the mode numbers of both works

5.2 Experimental results

The experimental results include the calculation of the first mode of the natural frequency and damping ratio of the stiffened cylinder. Through the analysis of the accelerometer signal with sig-view software, the fundamental natural frequency of the stiffened cylinder is computed. This software is used to transform the signal obtained from time domain into frequency domain by using (FFT) function as shown in Figure 11.

The comparison between numerical fundamental natural frequency of stiffened cylinder of geometry and mechanical properties shown in Table 3 with experimental result showed that a close agreement between them estimated by (18.43%). The natural frequency that evaluated numerically is (302.04 Hz) and value that it's computed experimentally is (246.36 Hz). This natural frequency which obtained experimentally is the average value of multi reading of accelerometer that firm up in different positions on the structure as shown in Table 4. The positions of the accelerometer on the cylinder with fixed hammer location are shown in Figure 8.

The damping ratio of the stiffened cylinder structure is also computed by drawing the exponential decay out curve on the signal obtained as shown in Figure 11. Also, a curve fitting to this curve has been conducted to find the equation that gives the value of damping ratio.

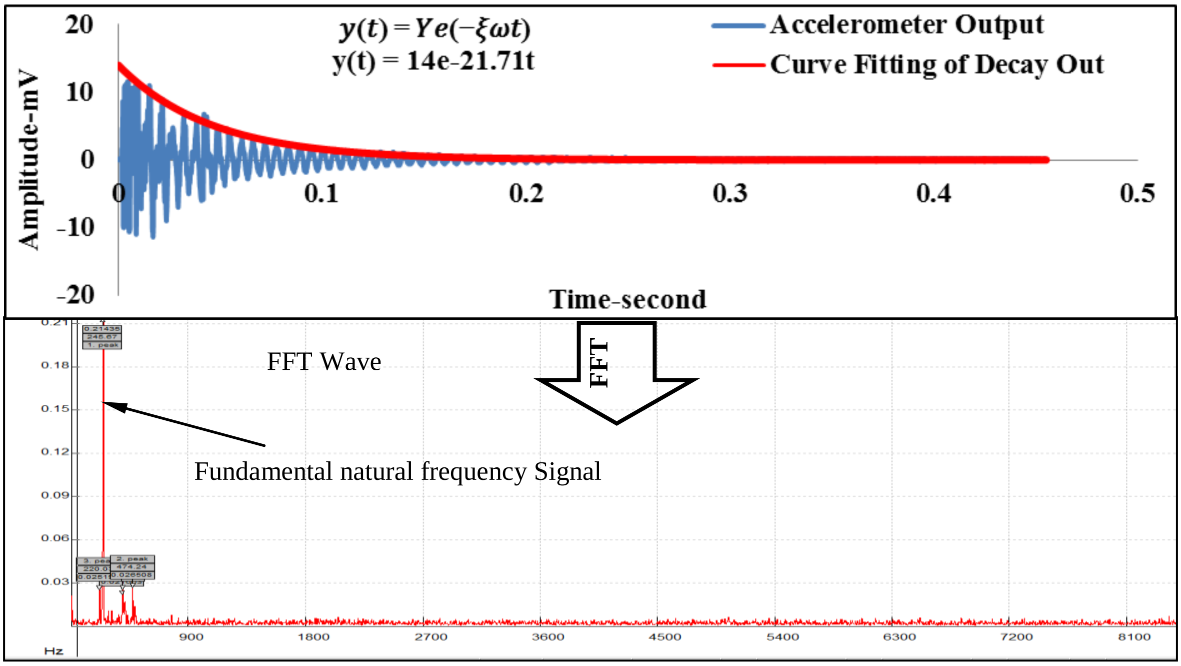


Figure 11. Experimental signal analyzed by sig-view software

Table 3. Geometrical and material properties of experimental sample

Parameters	Symbol	Value	Unit
Radius of cylinder	R	19.0985	cm
Length of cylinder	L	60	cm
Angle	θ	0, 67.5 and 90	Degree
Thickness of cylinder	Tc	0.5	mm
Thickness of stiffeners	Ts	0.5	mm
Height of stiffeners	H	2	cm
No. stiffeners	N	12	/
Type of stiffeners	T	(External)	/
Modulus of elasticity	E	201	Gpa
Mass density	ρ	7823	Kg/m³
Poisson’s ratio	υ	0.3	/

The value of damping ratio (ξ) is calculated as follows:

$y(t) = Ye^{-\xi\omega_n t}$ (General Form of Equation of Decay out Curve)

$y(t) = 14e^{-21.7t}$ (Equation of the Current Decay out Curve)

By comparing them, the following result has been obtained:

$\xi\omega_n = 21.7$ (From the Exponential Power)

Since the value of experimental (ω_n) is ($246.36*2\pi=1547.92\frac{rad}{sec}$)

So $\xi = \frac{21.7}{1547.92} = 0.014$

Table 4. Experimental natural frequency results in different locations

Location	Natural Frequency (Hz)
1	252.3 251.1
2	246.4 245.8
3	232.6 231.1
4	250.3 250.1
5	251.8 252.1
Average	246.36

5.3 Numerical results (Natural frequency results)

The numerical results include the natural frequencies and von Mises stresses results. The von Mises stresses obtained for different FEMs under various effects of loads. These loads are static, harmonic, and transient loads that applied to stiffened cylindrical shell structures.

The structures that created have many stiffeners arrangement (numbers, location, height and helix angle of stiffeners).

The flowchart below shown in Figure12 illustrates the environmental loads that models exposed to it.

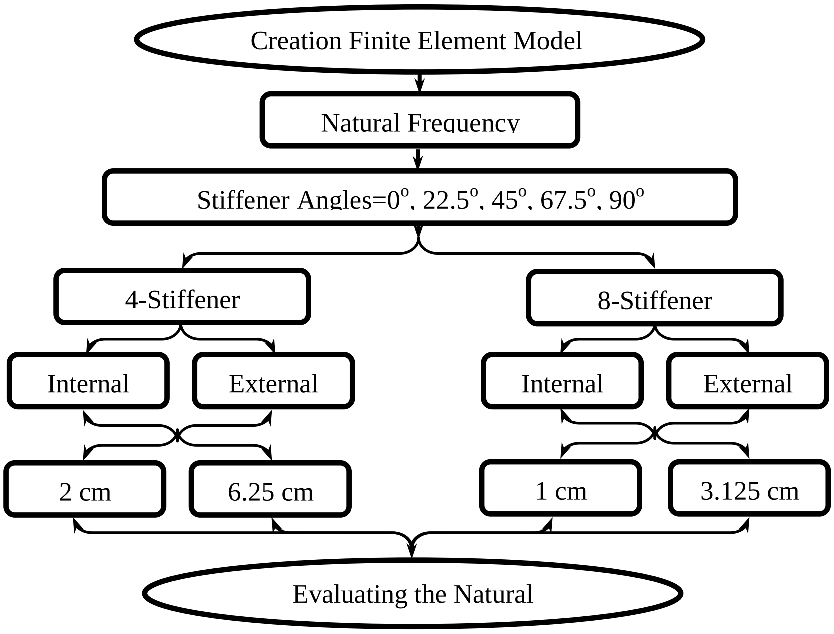


Figure 12. Flowchart for finite element steps to evaluate the natural frequencies and von-Mises stresses for different models

The structures are analyzed by using modal analysis to estimate the first five natural frequencies to models of different stiffeners configurations (location, orientation, number and height of stiffeners located on the cylinder).

Figures 13, 14 show the effect and behavior of stiffeners on the natural frequencies for cylindrical shell models.

For internal stiffeners, Figure 13 shows the effect of helix angles on the natural frequencies for two height (2cm and 1cm), where the natural frequencies increase with increasing the stiffener orientation to reach the maximum value at (67.5°). But, the natural frequency decrease when the stiffener orientation exceeded the angle of (67.5°). For external stiffeners, the natural frequencies of FEMs are also increased with increasing stiffeners orientation where the maximum value occurred at (90°).

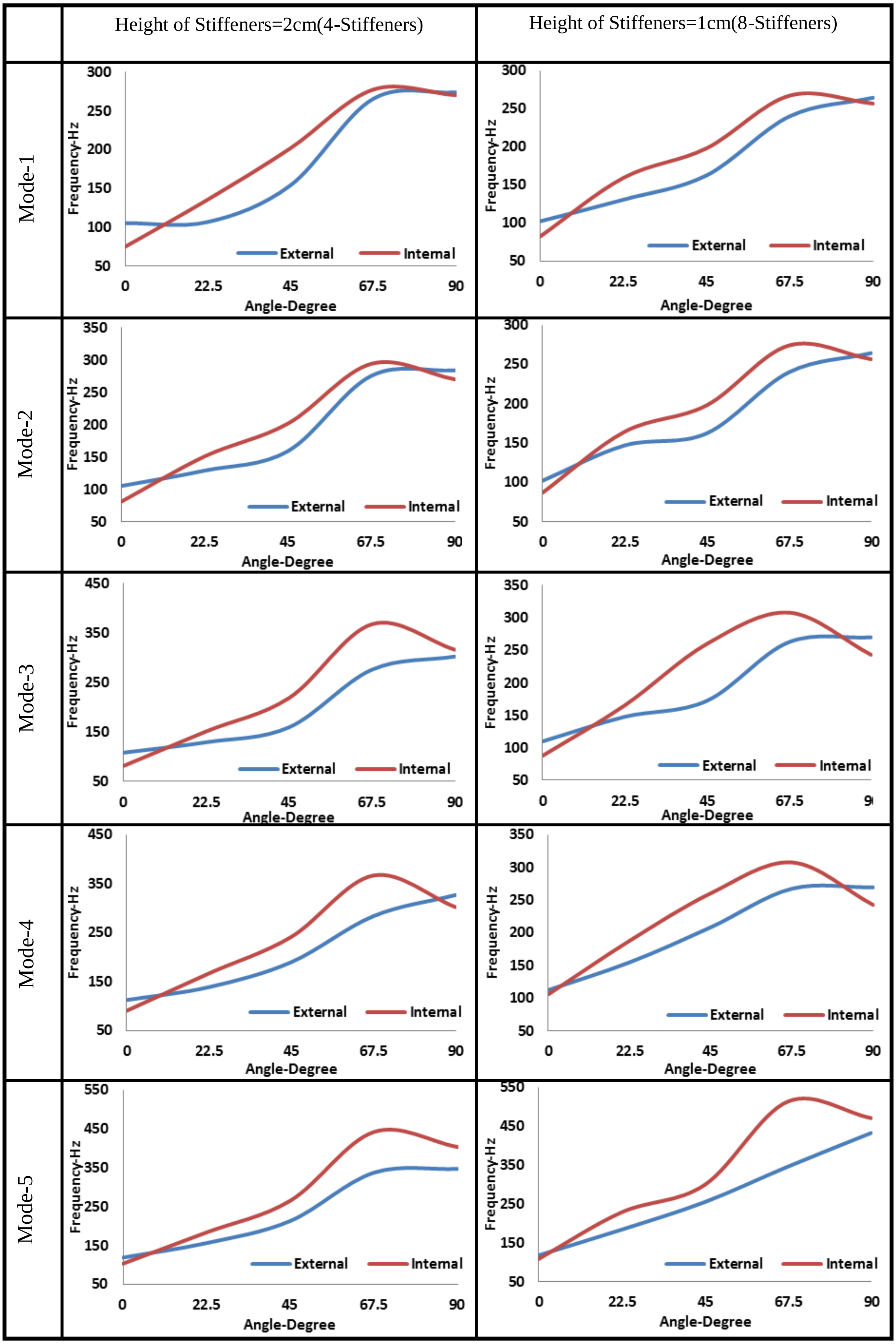


Figure 13. Variations of the natural frequencies as a function of helix angle for the different models

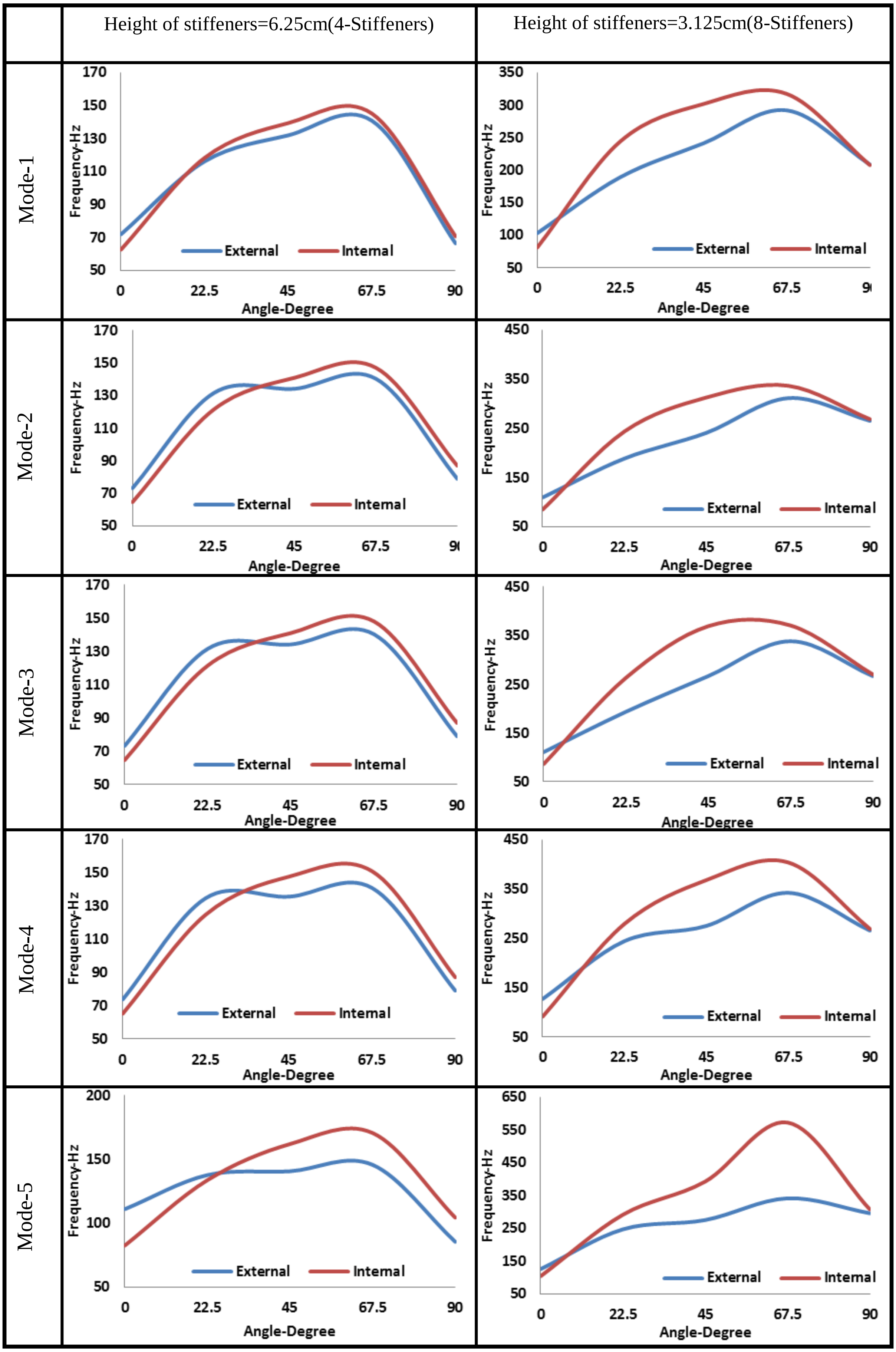


Figure 14. Variations of the natural frequencies as a function of helix angle for the different models

For internal and external stiffener, Figure 14 represents an increasing in the natural frequencies of shell structures for two heights (6.25cm and 3.125cm) until reached (67.5^o) of stiffener angle. In addition, the natural frequency decrease when stiffener angle exceed the angle of (67.5^o).
It is found that the number of stiffeners affect the natural frequencies value as shown in Figures 13, 14 where the natural frequency value increases when increasing the number of stiffener because the stiffness of the structures are increased when the number of stiffeners increased.
The minimum natural frequencies are occurred at internal stiffener of height (6.25 cm) and 4-stiffener at angle (0^o) which have values (first mode=62.8 Hz, second mode=64.6 Hz, third mode=64.6 Hz, fourth mode=65.1 Hz and fifth mode=82.34 Hz). But the maximum values occurred at angle (67.5^o) of height (3.125 cm) with 8 stiffeners which have the magnitude (first mode=317.6 Hz, second mode=317.7 Hz, third mode=370.3Hz, fourth mode=404.6 Hz and fifth mode=573.5 Hz).

6. Conclusions

According to the obtained results, the following conclusions have been obtained:

1. From the modal analysis, it is found that the natural frequency of the structure for all modes is high when the stiffeners locate inside the cylinder.
2. In the modal analysis, when the height of the stiffeners is small, frequency is increased when increasing of the helix angle.
3. In the modal analysis the first five natural frequencies are calculated and showed that the optimal value of natural frequency in all modes appear at stiffeners (helix angle=67.5^o, height=3.125cm, internal with eight stiffeners) where the value of natural frequency equals to (317.63Hz at first and second mode, 370.23Hz at third mode, 404.65Hz at fourth mode and 573.54Hz at fifth mode).

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