INTERNATIONAL JOURNAL OF
ENERGY AND ENVIRONMENT
ISSUE ON APPLIED MECHANICS RESEARCH
Volume 8, Issue 6, 2017 pp.501-508
Journal homepage: www.IJEE.IEEFoundation.org

Identification of complicated structure by using MATLAB; a case study on crankshaft

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Received 18 Dec. 2016; Received in revised form 9 Jan. 2017; Accepted 10 Jan. 2017; Available online 1 Nov. 2017

Abstract
In MATLAB System Identification SID refers to the method for estimating the system transfer function from experimental tests by using computer software so in this work the SID method is employed for analyzing practical structure for crankshaft. The validity of this method is firstly checked by applying it on beam model under boundary condition (simply support) where the required parameters for this simple system are evaluated in two ways. First theoretically by using Modal analysis approach and second experimentally by using SID method. From comparing the results, it is found that; the accuracy of using SID method is within acceptable limits, where the error is not exceeded 6.7% for case of simply support. Then the method is extended for using for the crank shaft it is found that the transfer function parameters at the mid-section are increased as compared with the crank ends.

Keywords: Transfer function; System identification; Modal analysis; Crankshaft; FFT.

1. Introduction
Investigating the dynamical behavior of complicated structures from analyzing experimental data took wide attention of researchers and engineers. The main goal of this analysis is to evaluate the system parameters associated with the resulting curve relating the input and output data. In this method the system is subjected to an input excitation to produce an output response. Both input and output signal waves are sampled at equal time interval to generate set of data. The data is then transformed to frequency domain by Fast Fourier Transformation FFT and proceed in computer to find the optimum transfer function. By evaluating the transfer function the dynamical parameter of System such as, natural frequency, damping, gain can be evaluated. Wide information about the vibration, stability and control characteristics can be clearly investigated for the target system.
System Identification SID is a computer software used for performing such an analysis, as example in Matlab. This field of research has wide practical applications in engineering such as fault detection in automatic machines Basseville et. al [1], damage detection Mevel, et.al.[2], health monitoring Zimmerman et.al. [3], car body, sabri [4] airplane Goethals [5], automotive and aerospace applications, Bart Peeters [6] mobility analyzer (DMA) in the nanometer size range Hummes, [7] ducted laminar premixed flame Karimi, et.al. [8] and vehicle component testing Peeters et.al [9]
In simple or regular systems and structures the transfer function can be evaluated in straight forward analysis, however for complicated systems it is not so because of the complication of stiffness and mass as well as the existed of nonlinearity. In such systems the theoretical analysis becomes tedious. In this

regard; finite element and state space methods are normally used Ewins [10]. The accuracy obtained in these methods depend on the number of elements used which are normally large Heylen et. al. [11]
The main challenge of determination of the system parameters is how to fit the complex transfer function curve to the standard forms of transfer function with certain confidence Cauberghe [12].
Researchers attempted many numerical and statistical methods to improve the optimization (minimizing error and time labor) process; for example Guillaume et.al. [13] used frequency-domain maximum likelihood, Vander et.al.[14] employed fast-stabilizing frequency domain method, Guillaume et.al.[15] introduced a poly-reference implementation of the least-squares and Canales and Mevel [16] applied the quasi newton based system.
Due to the recent developments in the field of prosthetic which involve the robotic and intelligent devices such as; the C-leg, feeling leg and robotic prosthesis hand the demand for evaluating the system parameter becomes essential for design and implement.
In this paper, the estimation of the transfer function is attempted for the crankshaft. In order to simulate with the practical requirements. For the validation purpose a classical beam with simply support boundary condition is used in which the system parameters are evaluated in both theoretical and experimental methods.

2. Theoretical considerations

In order to check the validity of the present method a typical beam model will be investigated to evaluate the system parameters theoretically by using Modal Analysis method. The beam model is shown in Figure 1.

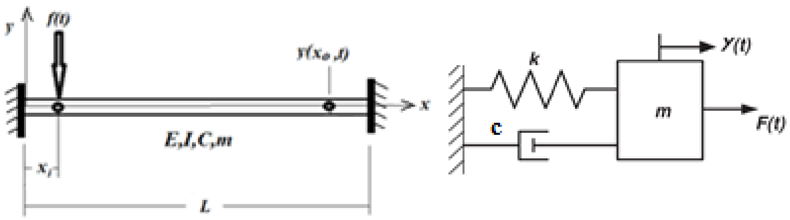


Figure 1. The beam System and equivalent m-c-k system [17].

The equation of force vibration of damped beam according to Euler-Bernoulli theory can be written as [17];

$$EI\frac{\partial^4 y}{dx^4} + C\frac{\partial y}{dt} + \rho A\frac{\partial^2 y}{dt^2} = f(t)\delta(x - x_j) \tag{1}$$

where the concentrated force is located at xj. For Modal Analysis one can seek the following solution;

$$y(x,t) = \sum_{s=0}^{N} \phi_s(x) q_s(t) \tag{2}$$

Where $\phi_s(x)$ and $q_s(t)$ stand for mode shape and generalize coordinates. Substituting the solution (2) into the equation of motion (1) results;

$$EI\sum \phi_s^{IV} q_s + C\sum \phi_s \dot{q}_s + \rho A\sum \phi_s \ddot{q}_s = .f(t)\,\delta(x - x_j) \tag{3}$$

Where $\phi_s(x)$ and q(t) for simplicity are replaced by qs and ϕs respectively.

Now, multiplying Eq. (3) by the boundary residual series $\phi_r(x) = \sum_{r=1}^{N} \phi_r(x)$ and integrating over the whole beam length (0 to L), the following matrix equation can be obtained [18];

$$[K]\{q\}+[C]\{\dot{q}\}+[M]\{\ddot{q}\}=f(t)\varphi(xj) \tag{4}$$

The matrices [K] and [M] are diagonal due to the orthogonally property of the normal modes that is;

$$\int_0^L \phi_s(x)\phi_r(x)dx = \begin{cases} 1 for..s=r \\ 0 for...s\neq r \end{cases}$$

And,

$$\int_0^L \phi_r^{IV}(x)\phi_s(x)dx = \begin{cases} \lambda_s^4 for..s=r \\ 0 for..s\neq r \end{cases} \tag{5}$$

The elements of the stiffness and mass matrices are as the follows;

$$K_{s,s}=EI\int_0^1 \phi_s^{IV}(x)\phi_s(x)dx \tag{6}$$

$$m_{s,s}=\rho A\int_0^1 \phi_s(x)^2\,dx \tag{7}$$

To evaluate the transfer function the harmonic analysis must be considered, the forms of harmonic motion and harmonic force one can written as the follows;

$$q_s(t)=Q_s e^{i\omega_s t} \text{ and } f(t)=Fe^{i\omega t} \tag{8}$$

Substituting equations (8) into equation (4) and making the use of transfer function definition the following equation is resulted;

$$H_s(\omega)=\frac{Q_s(\omega)}{F_s(\omega)}=\frac{K_s\varphi_s(xj)}{[1+2\zeta\frac{\omega}{\omega_s}+\frac{\omega^2}{\omega_s^2}]} \tag{9}$$

For equivalent single degree of m-c-k system can be obtained by putting s=1, hence the corresponding transfer function becomes;

$$H_1(\omega)=\frac{K_s\varphi_1(xj)}{[1+2\zeta\frac{\omega}{\omega_1}+\frac{\omega^2}{\omega_1^2}]} \tag{10}$$

Although the above analysis is general for any beam boundary condition in this paper a focus is made on simply support. In this case the mode shape function $\phi_s(x)$ is [17];

$$\phi_s(x)=\sin\lambda sL \tag{11}$$

Where the eigenvalues λ s are the roots of the characteristics equation sin λsL=0. The natural frequency can be obtained as;

$$\omega_s=(\lambda_s L)^2\sqrt{\frac{EI}{ML^4}} \tag{12}$$

Equations 10 together with equations 6, 7, 11 and 12 can give the theoretical transfer function and system parameters of the simply support beam.
Due to the difficulty of evaluating the damping theoretically. The damping was measured experimentally by using the concept of the logarithmic decrement of the transient response curve which is available from hummer test.

3. Experimental estimation

When a set of experimental input data X(t) and output data Y(t) are available, the transfer function H(ω) can be obtained from the following relation [12],

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{\text{FT of output}}{\text{FT of input}} \tag{13}$$

Equation 13 leads to complex transfer function with magnitude and phase shift which can written as;

$$H(\omega) = A(w)e^{i\theta(\omega)} \tag{14}$$

Now if equation 14 is plotted against a range of frequencies and fitted to optimal bode plot it is possible to estimate the TF in terms of the system parameters (gain, natural frequencies, damping …etc).
The method of auto-regression can be used which leads to a classical linear least squares. In order to reduce the fitting error the following cost function J(b) is used [10];

$$J(b) = \int_0^\infty \left|G(i\omega, b) - A(w)e^{i\theta(\omega)}\right|^2 d\omega \tag{15}$$

Where $G(i\omega, b)$ denotes the desired transfer function.
For desecrate data this integration is replaced by summation;

$$J(b) = \frac{1}{2}\sum_{n=1}^{N}\left|G(i\omega, b) - A(w)e^{i\theta(\omega)}\right|^2 \Delta\omega \tag{16}$$

The System transfer function is assumed to have a linear structure expressed in factored form. This is the general form familiar to Bode plot.
Which can be represented in the following factored form [10];

$$G(s, b) = \prod_{j=1}^{M} g_m(s, b)_j \tag{17}$$

Where can take any of the following forms;

$$s, (1 + \tau s), (1 + 2\zeta s + \omega^2 s^2), \frac{1}{(1+2\zeta s+\omega^2 s^2)} \dots\dots\dots.\text{etc} \tag{18}$$

In Matlab software the so called “System Identification“ is available to work with measuring data or observations.
The set of input and output data in time domain are imported for 1024 points. Firstly the user can allowed to work out with the data to reject the noise by introducing several types of filters. The cleaned data then applied to FFT to convert to frequency domain. The method of least square is used to fit the data for desired transfer function. Figure 2 is a flow chart of the procedures used in System Identification by Matlab software.

4. Experimental work

The block diagram of the instruments used for measuring the transient excitation and the response are shown in Figure 3.
To provide the simply support fixing in which the transverse displacement and the bending moment are zero two half circular rings are used. Figure 4 shows the supports configurations.
The crankshaft boundary conditions are simply supported at both ends. These achieved by using two half circular shape rings. Three tested points are considered as shown in Figure 5.
In performing tests at every position the accelerometer is attached and the hammer is used for excited the model at the other points show that in Figure 6. The data then extracted from oscilloscope by USB to use it in computer in software programs to estimate the transfer function and another characteristics vibration.

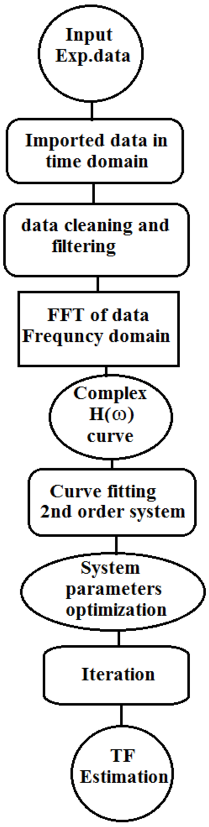


Figure 2. Flow chart of the procedures used in SID.

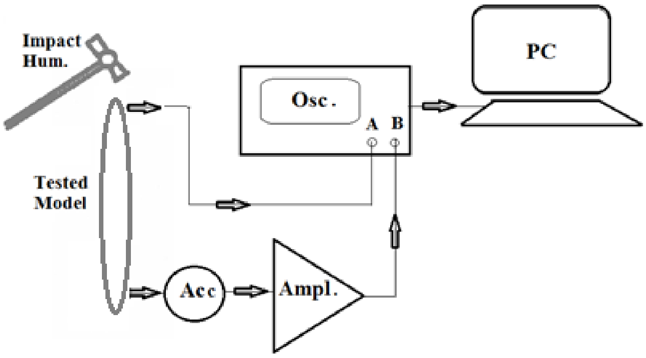


Figure 3. Block diagram of the instruments.

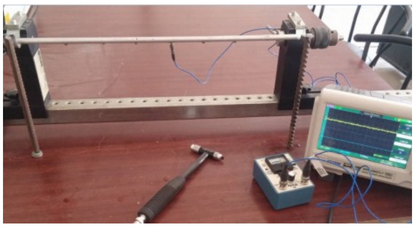


Figure 4. Simply support fixing.

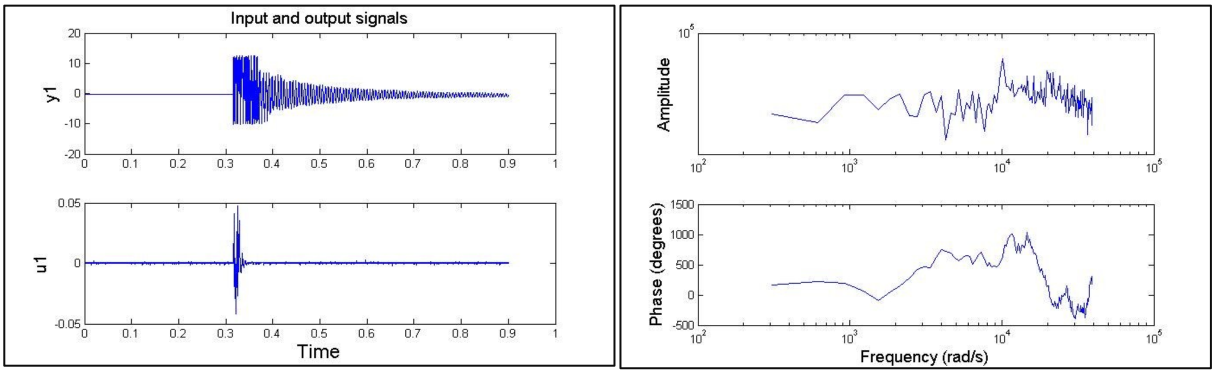
Figure 5. Crank shaft test points.

Figure 6. Crank shaft test.

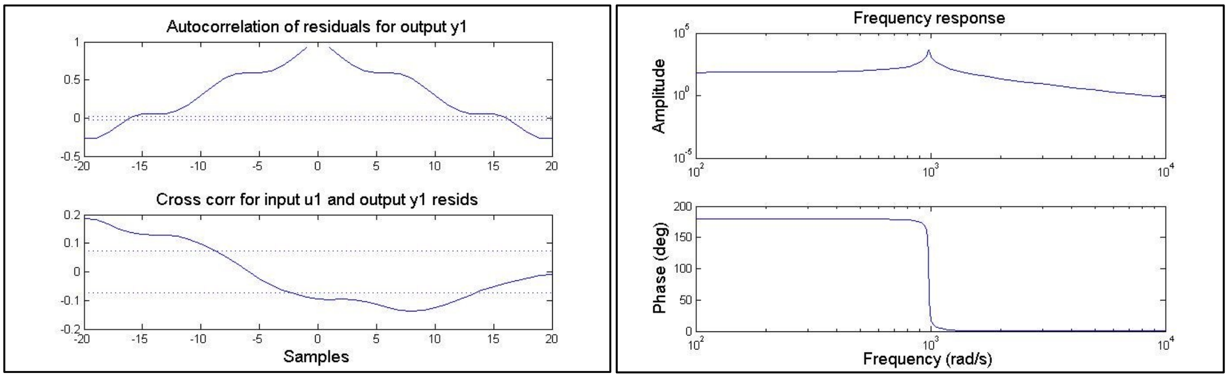
5. Results and discussions

The experimental results for estimating the System parameters of the simply support beam by using system identification available in Matlab are shown in Figures 7 (a to d). In Figure (7-a) the input and output data are plotted in time domain. The FFT of data is performed and plotted in Figure (7-b). Figure 7-c shows the autocorrelation and cross correlation of data exported for best fitting to the slandered 2nd transfer function. Finally Figure (7-d) presents the bode plot of the transfer function. The estimated transfer function is given as,

$$H(\omega) = \frac{88.304}{[1+2x0.0064\frac{\omega}{1003}+\frac{\omega^2}{1003^2}]} \tag{19}$$



a. b.



c. d.

Figure 7. SID plots of simply support beam.

The main System parameters of the beam, namely; natural frequency, damping ratio and static gain are calculated by using equations 6 and 7 as well as the experiment for measuring the damping. The experimental SID and theoretical results and the percentage error are collected in Table 1.
In this case the crankshaft is used since it has a complicated geometry and it has wide application in mechanical engineering, three points are selected for exciting and measuring the response, one point is near the one end, the second is at mid center and the third is near the last end of crank. The experimental results for estimating the SID System parameters of the crankshaft at themed point are shown in Figure 8 (a to d).

Table 1. Theoretical and experimental parameters of simply support.

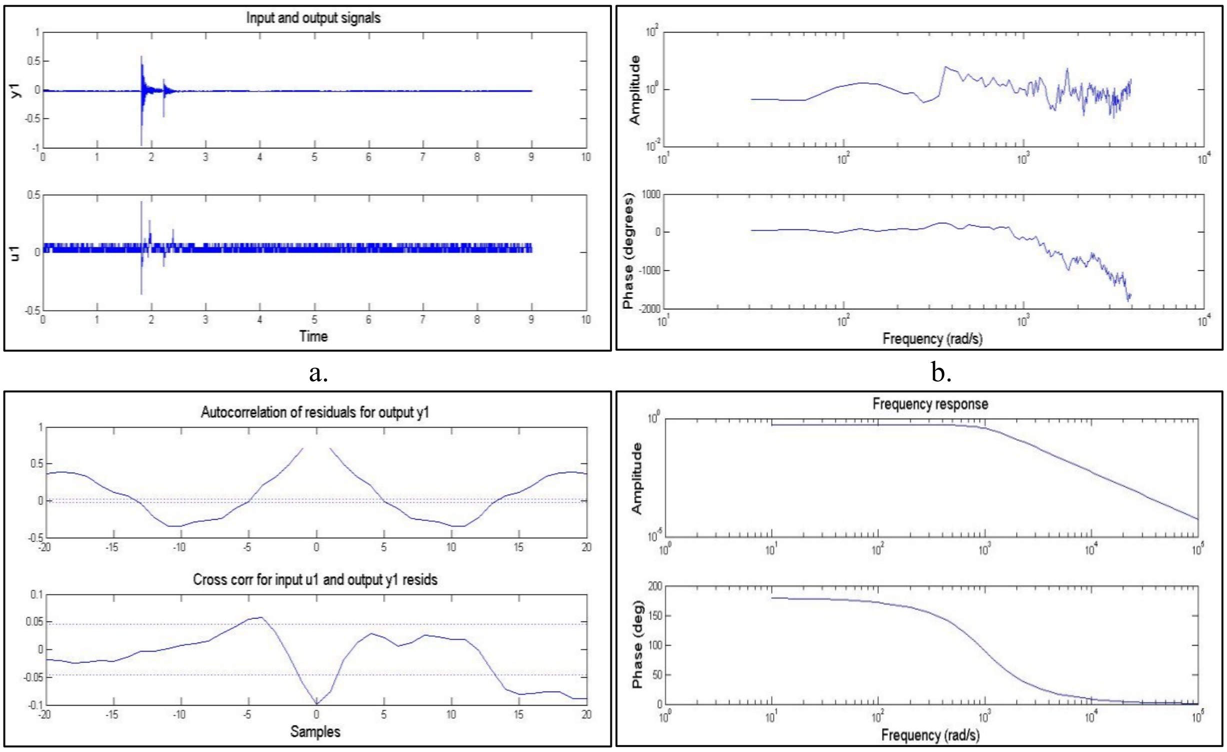
System parameters	Theoretical	System Ident.	Error (%)
Natural frequency (r/s)	937	1003	6.7
Damping ratio	0.0068	0.0064	5.8%
Gain (KN/m)	91.3	88.304	3.2%

The estimated transfer Function of the crank shaft;

$$H(\omega) = \frac{336.11}{[1+2x0.0056\frac{\omega}{597}+\frac{\omega^2}{597}]} \quad \text{(first end point)} \tag{20}$$

$$H(\omega) = \frac{547.89}{[1+2x0.0072\frac{\omega}{1232}+\frac{\omega^2}{1232}]} \quad \text{(mid point)} \tag{21}$$

$$H(\omega) = \frac{488.8}{[1+2x0.0059\frac{\omega}{378}+\frac{\omega^2}{378}]} \quad \text{(second end)} \tag{22}$$



a. b. c. d.

Figure 8. SID plots of crankshaft at mid point.

Comparing the transfer function parameter, it is clear that; the largest value of the gain is at the center of crank while lowest value at the first point. This is also true for the natural frequencies in which the largest value is at the center and the lowest at the end of crank shaft. However the damping ratios have very small different between each case.

6. Conclusions

From the discussions of the results the following conclusions can be derived;

1. The validity of the present method is checked; it is found that the percentage error in the transfer function parameters between the theory and experiments is not exceeded (6.7 %).
2. The transfer function of the crankshaft is affected by the position point in the crankshaft the natural frequency increasing the midpoint and decrease in the end point and the damping is not affected.
3. The experimental estimation of transfer function via SID method can be simply performed using hummer test and simple instrumentation without the needing of elaborate analytical analysis.

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