INTERNATIONAL JOURNAL OF
ENERGY AND ENVIRONMENT

Volume 9, Issue 5, 2018 pp.499-514
Journal homepage: www.IJEE.IEEFoundation.org

A suggested analytical investigation of heat generation inducing into vibration beam subjected to harmonic loading

Diyaa H.J. Al-Zubaidi[1], Muhannad Al-Waily[2], Emad Q. Hussein[1], Maher A.R. Sadiq Al-Baghdadi[2]

[1] University of Kerbala, Faculty of Engineering, Mechanical Engineering Department, Iraq.
[2] University of Kufa, Faculty of Engineering, Mechanical Engineering Department, Iraq.

Received 10 April 2018; Received in revised form 25 June 2018; Accepted 10 July 2018; Available online 1 Sep. 2018

Abstract
The vibration of structure are cusecs generation heat on the machine parts, the heat generation dependent onto the natural frequency supplied on the structure and the materials types (mechanical and thermal properties). Therefore, in this paper investigation the heat generation in the beam due to vibration of beam under frequency excitation (harmonic load excitation), for the beam supported with various boundary conditions, simply supported; clamped supported; and cantilever supported beam. Also, the results included evaluated the heat generation as a function of time with different frequency applied of harmonic load, less and more than the natural frequency of beam. The heat generation due to vibration of beam with time is evaluated by analytical investigation and comparison with numerical study, by using finite element method with CFD package program. The results evaluated are shown the generation heat is decreasing with increasing the frequency of the applied harmonic load. In addition to, the value of natural frequency of beam, dependent is effect on the heat generation of vibration beam, therefore, the supported types of beam are effect of the heat generation due to harmonic load, since the natural frequency of beam is dependent on supported types of beam. The comparison of results between analytical investigation and numerical work had shown the good agreement between with maximum error about (1.36%).

Keywords: Heat induce vibration; Heat generation; Thermal induce vibration; Beam vibration; Harmonic vibration; CFD heat vibration; Finite element heat generation.

1. Introduction
The heat generation in the structure subjected to the high frequency due to mechanical viscoelastic losses in the materials. The thermo-elastic damping "represent between mechanical and thermal energy of the energy conversion" occur due to the slow temperature rise in a vibration structure. Due to internal damping of the materials, the loss of energy occur when s structure vibration under frequency effect. And, when the structure subjecting to high frequency and vibration under the high frequency, generation the heat and high displacement due to internal damping characteristic of materials. When a structure is exposed to vibrations of high frequency, an important amount of heat will be generated within the structure because of mechanical losses in the material due to the viscoelastic effects. A second mechanism contributing to the

slow temperature increase in a vibrating structure is called thermo-elastic damping, and represents the energy conversion between mechanical and thermal energy.

Vibration damping in solids is the phenomena whereby vibrational energy is converted to heat due to internal mechanisms within the solid. These mechanisms include internal friction between individual atoms, friction between different phases or precipitates, the movement of dislocations within a material.

Every material has different damping characteristics due to its unique structure. Thus, in general the crack effect on the mechanical behaviors are investigation by multi researchers with various crack parameters effect, as effect of crack on natural frequency of beam structure, [1-5], crack effect on natural frequency on plate structure, [6-10], crack effect on natural frequency of pipe induce vibration behavior, [11, 12], crack effect on buckling behavior of beam, [13], and other application. Where, the researchers are presented that the crack defect causes decrease the stiffness for structure, in addition to increase responses for structure with various load applied.

The thermal induce vibration of structure are studied from many researchers with different technique as shown in same present review below,

In 1980, S. Pantellou and A. Dimarogonas, [14], presented the analysis of heat generation for rotation shaft from materials damping, assuming both lumped mass and continuous system are considered. And, the forced torsional vibration is assumed. The elastic deformation rang and an elastoplastic materials, used hysteretic model for materials damping to yield the heat generation, assumed in the plastic range. From the heat conduction solved for cooling surface cylindrical shaft and the maximum temperatures and the maximum surface temperature obtained, shown that the develop of substantial temperature in shafts undergoing torsional vibration.

At 1997, Michael I. F. et. al, [15], studied the active damping of thermal vibration smart structure, the simply supported baem made of aluminum materials is presented to studied the effect of thermal induce vibration beam. The control system design studied with effect of large temperature changes. The results of paper shown that the smart structure can be used to damping the vibration with thermal effect.

In 2013, Jadwiga K. K., [16], presented the analysis of vibration cantilever beam with thermal effect. The periodically time varying heat source acts onto surface of beam are assumed in the paper. The variables of thermal moment effect are add to the Bernoulli-Euler equation to vibration beam. And, Green's function method used to solving the heat and vibration problem of beam.

After this, at 2014, I. Jafarsadeghi P. et. al, [17], studied the dynamic behaviors for cantilever beam made of graded materials and subjected to the harmonic thermal load. The metal and ceramic materials are used to product the graded materials of the beam used in this paper. The equation of coupled energy and equation of motion are derived in addition to Euler-Bernoulli beam theory is used. Also, the numerical method presented in the paper.

Also, in 2014, Wenbo Zhang et. al, [18], presented the investigation of thermal effect on high frequency of beam vibration, and developed the energy flow analysis to predict frequency response beam. The methods used in the paper are EFA model and numerical simulations to study the effect of temperature effect onto thermal stress and materials properties changed.

In addition to other researchers studied the problem with different way to evaluating the heat generation and temperature distribution, but, not presented the analytical solution of problem as in this work. Where, the paper presented analytical solution evaluating the heat generation as a function of time and dependent onto natural frequency subjected onto beam and with different boundary condition of beam and various beam vibration modes. In addition to, comparison the analytical results are evaluated with numerical results production by finite element method, with using CFD program.

2. Analytical investigation

The analytical investigation is techniques used to give exact solution of problem with various parameters effect, [19-23], in addition to, the analytical solution is given agreement results comparison with other technique used, [24-31]. Thus, the analytical part included two section, first evaluated of natural frequency of beam, and then, in the section part derived the equation determine the generation heat due to supplied frequency onto beam and evaluated the equation of temperature as a function of time and temperature as a function of length for beam (distribution of temperature through length of beam).

2.1 Vibration of beam

To evaluated the natural frequency of beam, solution of general equation of motion, [32-35], of free vibration uniform cross section and constant modulus beam, as, [36],

$$\mathrm{EI}\frac{\partial^4 \mathrm{w(x,t)}}{\partial \mathrm{x}^4} + \rho\mathrm{A}\frac{\partial^2 \mathrm{w(x,t)}}{\partial \mathrm{t}^2} = 0 \tag{1}$$

Where, $E.I.\rho.A$ are modulus of elasticity, moment of inertia, density, and cross section area of beam, respectively, x is beam direction through length of beam, t is time, and w is the deflection of beam through lateral direction of beam.
Then, by using separation of variable as,

$$W(x,t) = W_n(x) \times W_n(t) \tag{2}$$

Then, by substation Eq. 2 in to Eq. 1, and solve equation, get,

$$W(x) = \begin{bmatrix} C_1(\cos\beta x + \cosh\beta x) + C_2(\cos\beta x - \cosh\beta x) + \\ C_3(\sin\beta x + \sinh\beta x) + C_4(\sin\beta x - \sinh\beta x) \end{bmatrix} \tag{3}$$

Where, $C_1, C_2, C_3, and\ C_4$ are constant. And, β can be determine from boundary conditions of beam. And, the natural frequency of beam can be determine from, [36],

$$\omega_n = (\beta l)^2 \sqrt{\frac{EI}{\rho A l^4}} \tag{4}$$

Where, l is the length of beam and n is the number of mode of beam.
Therefore, to evaluated the value of natural frequency must be selected the boundary condition of beam, then, can be selected three types of boundary condition beam as,

1. Cantilever beam, the boundary condition of cantilever beam included first ends fixed and other edges free as shown in Figure 1, as,

$$W = 0, \frac{\partial W(x)}{\partial x} = 0 \text{ at fixed end, and, } \frac{\partial^2 W(x)}{\partial x^2} = 0, \frac{\partial^3 W(x)}{\partial x^3} = 0 \text{ at free end.} \tag{5}$$

Then by substitution Eq. 5 into Eq. 3 and solution, get (first four natural frequencies),

$$\beta_1 l = 1.875104, \quad \beta_2 l = 4.694091, \quad \beta_3 l = 7.854757, \quad \beta_4 l = 10.995541 \tag{6}$$

And, the normal mode functions of beam, as,

$$W_n(x) = \left[(\sin\beta_n x - \sinh\beta_n x) - \left(\frac{\sin\beta_n l + \sinh\beta_n l}{\cos\beta_n l + \cosh\beta_n l}\right)(\cos\beta_n x - \cosh\beta_n x)\right] \tag{7}$$

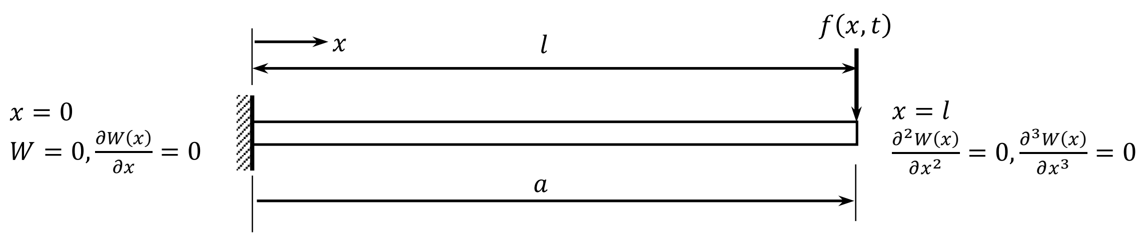


Figure 1. Boundary condition of cantilever beam supported.

2. Simply supported beam, the boundary condition of simply supported beam are pinned ends for two edges of beam, as shown in Figure 2, as,

$$W = 0, \frac{\partial^2 W(x)}{\partial x^2} = 0 \text{ at two edges of beam.} \tag{8}$$

Also, by substation Eq. 8 in to Eq. 3, get (first four natural frequencies),

$\beta_1 l = 3.141593$, $\beta_2 l = 6.283185$, $\beta_3 l = 9.424778$, $\beta_4 l = 12.566371$ (9)

And, the normal mode functions of beam, as,

$$W_n(x) = (\sin\beta_n x) \quad (10)$$

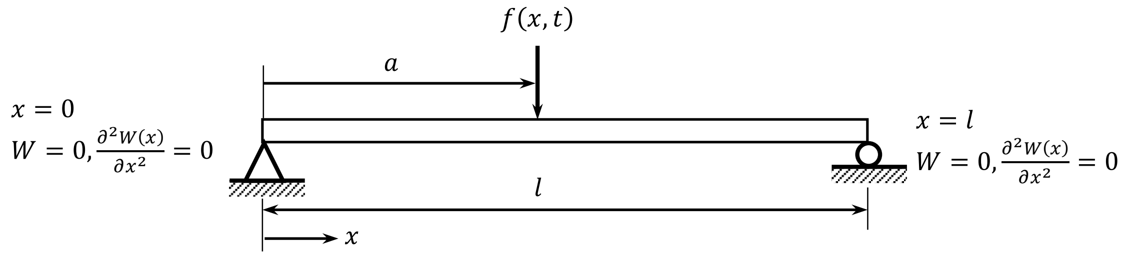


Figure 2. Boundary condition of simply supported beam.

3. Clamped beam, the boundary condition of clamped supported beam are fixed ends for two edges of beam, as shown in Figure 3, as,

$W = 0, \frac{\partial W(x)}{\partial x} = 0$ at two edges of beam. (11)

Also, by substation Eq. 11 in to Eq. 3, get (first four natural frequencies),

$\beta_1 l = 4.730041$, $\beta_2 l = 7.853205$, $\beta_3 l = 10.995608$, $\beta_4 l = 14.137165$ (12)

And, the normal mode functions of beam, as,

$$W_n(x) = \left[(\sinh\beta_n x - \sin\beta_n x) + \left(\frac{\sinh\beta_n l - \sin\beta_n l}{\cos\beta_n l - \cosh\beta_n l}\right)(\cosh\beta_n x - \cos\beta_n x)\right] \quad (13)$$

Then, applied the frequency on the beam less than the natural frequency of beam and evaluated the heat generation on the beam. And then, applied frequency on the beam equal to natural frequency of beam (resonance of beam occurred) and comparison the different between the two case.

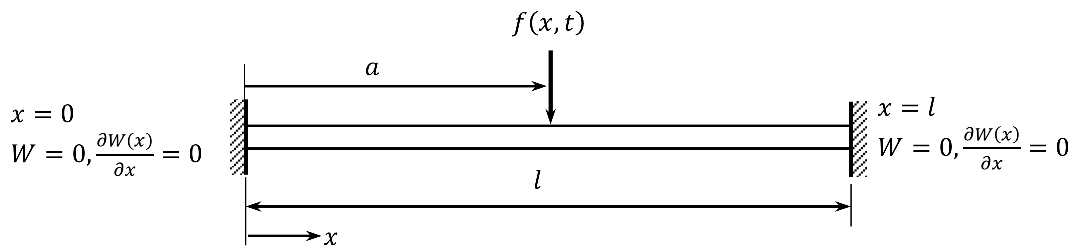


Figure 3. Boundary condition of clamped beam supported.

Therefore, the general equation of motion of force vibration beam can be used,

$$\omega_n^2 W_n(x) W_n(t) + W_n(x)\frac{d^2 W_n(t)}{dt^2} = \frac{1}{\rho A} f(x,t) \quad (14)$$

Where, $f(x,t)$ general force applied on the beam at distance a (N/m), as shown in Figures. 1, 2, and 3. By multiplying Eq. 14 throughout by $W_n(x)$, and integrating from 0 to l, and using the orthogonally, get,

$$\frac{d^2 W_n(t)}{dt^2} + \omega_n^2 W_n(t) = \frac{1}{\rho A b} Q_n(t) \quad (15)$$

Where, $Q_n(t)$ is generalized forced (N),

$$Q_n(t) = \int_0^l f(x,t)W_n(x)\, dx. \tag{16a}$$

And, b is constant (m), and can be expressed as,

$$b = \int_0^l W_n^2(x)\, dx \ \text{(m)} \tag{16b}$$

Then, by using Duhamel integral, the solve of Eq. 16, for zero initial conditions (displacement and velocity), and substation the solution into Eq. 2, get,

$$w(x,t) = W_n(x)\left(\frac{1}{\rho A b \omega_n}\int_0^t Q_n(\tau)\sin\omega_n(t-\tau)\ d\tau\right) \tag{17}$$

Therefore, applied harmonic load $f(x,t) = f_o \sin\omega t$ with frequency of ω, at $x = a$, and substitution it's in Eqs. 16 and 17, to evaluating the general equation of beam deflection as a function of x and time.

2.2 Heat generation

The forced vibration of the beams results in heat generated due to material damping. The heat generated is due to higher temperatures that reduce the natural frequency, which may affect the work of the beam. The temperatures can arrive high premiums in some states, so the reliability of some of the components of the device may be affected [37]. To calculate the amount of heat generated, the energy of elastic deformation value must first be calculated. The energy of elastic deformation for a unit length, longitudinal direction, under stress can be written as [38].

The energy of elastic deformation for a unit length, longitudinal direction, under stress is, [38],

$$\mathrm{U} = \frac{1}{2}\mathrm{b_w}\int_{-h/2}^{h/2}\sigma_{xx}.\varepsilon_{xx}\,\mathrm{dz} \tag{18}$$

Where, b_w is the width of beam, $\sigma_{xx}, \varepsilon_{xx}$ are stress and strain of beam in x-direction, h is the beam thickness, and z is the direction of lateral direction beam.

Therefore, the strain of beam in x-direction can be evaluated from,

$$\varepsilon_{xx} = \frac{du}{dx} \tag{19}$$

Where, u is the displacement of beam in x-direction, can be evaluated as,

$$u = -z\frac{dw}{dx} \tag{20}$$

Then, by substation Eq. 20 in to Eq. 19, get,

$$\varepsilon_{xx} = -z\frac{d^2w}{dx^2} \tag{21}$$

Therefore, with using of Hook's low, get,

$$\sigma_{xx} = E\varepsilon_{xx} = -Ez\frac{d^2w}{dx^2} \tag{22}$$

Then, by substation Eqs. 22 and 21 into Eq. 18, get,

$$\mathrm{U} = \frac{1}{2}\mathrm{b_w E}\int_{-h/2}^{h/2}\varepsilon_{xx}^2\,\mathrm{dz} = \frac{1}{2}\mathrm{b_w E}\int_{-h/2}^{h/2} z^2\left(\frac{d^2W(x,t)}{dx^2}\right)^2\ \mathrm{dz} \tag{23}$$

Then, for symmetrical uniform beam can be integration Eq. 23 and get,

$$U = \frac{1}{24} b_w E h^3 \left(\frac{d^2 W(x,t)}{dx^2}\right)^2 \quad \text{(J/m)} \tag{24}$$

Where, $W(x,t)$ is the lateral deflection beam evaluated form Eq. 17.
Hence, the heat generation rate due to materials damping of beam, [14],

$$q = \frac{1}{2}\pi * f_n * \eta * U = \frac{\pi}{48} b_w E h^3 \left(\frac{d^2 W(x,t)}{dx^2}\right)^2 f_n * \eta \quad \text{(W/m)} \tag{25}$$

Where, f_n is the frequency [Hz], and η is the loss factor ($\eta = 0.0028$) [39].
Then by substation Eq. 17 into Eq. 25, get the general heat generation of beam with x-direction and time for different beam boundary condition, as,

1. Cantilever beam,

$$q = \frac{\pi}{2} \times f_n \times \eta \times U$$
$$= \frac{\pi}{48} f_n \eta b_w E h^3 \beta_n^4 \left[\begin{pmatrix} \left(\frac{\sin\beta_n l + \sinh\beta_n l}{\cos\beta_n l + \cosh\beta_n l}\right)(\cos\beta_n x + \cosh\beta_n x) \\ -(\sin\beta_n x + \sinh\beta_n x) \end{pmatrix} \left(\frac{1}{\rho A b \omega_n}\int_0^t Q_n(\tau)\sin\omega_n(t-\tau)\ d\tau\right)\right]^2$$
$$q = \frac{\pi}{4}\frac{f_n \eta}{\rho A b^2}\left[\begin{pmatrix} \left(\frac{\sin\beta_n l + \sinh\beta_n l}{\cos\beta_n l + \cosh\beta_n l}\right)(\cos\beta_n x + \cosh\beta_n x) \\ -(\sin\beta_n x + \sinh\beta_n x) \end{pmatrix} \left(\int_0^t Q_n(\tau)\sin\omega_n(t-\tau)\ d\tau\right)\right]^2 \tag{26}$$

2. Simply supported beam,

$$q = \frac{\pi}{2} \times f_n \times \eta \times U = \frac{\pi}{48} \times f_n \times \eta \times b_w E h^3 \beta_n^4 \left[(\sin\beta_n x)\left(\frac{1}{\rho A b \omega_n}\int_0^t Q_n(\tau)\sin\omega_n(t-\tau)\ d\tau\right)\right]^2$$
$$q = \frac{\pi}{4} \times \frac{f_n \times \eta}{\rho A b^2}\left[(\sin\beta_n x)\left(\int_0^t Q_n(\tau)\sin\omega_n(t-\tau)\ d\tau\right)\right]^2 \tag{27}$$

3. Clamped beam,

$$q = \frac{\pi}{2} \times f_n \times \eta \times U$$
$$= \frac{\pi}{48} f_n \eta b_w E h^3 \beta_n^4 \left[\begin{pmatrix} (\sinh\beta_n x + \sin\beta_n x) + \\ \left(\frac{\sinh\beta_n l - \sin\beta_n l}{\cos\beta_n l - \cosh\beta_n l}\right)(\cosh\beta_n x + \cos\beta_n x) \end{pmatrix} \left(\frac{1}{\rho A b \omega_n}\int_0^t Q_n(\tau)\sin\omega_n(t-\tau)\ d\tau\right)\right]^2$$
$$q = \frac{\pi}{4}\frac{f_n \eta}{\rho A b^2}\left[\begin{pmatrix} (\sinh\beta_n x + \sin\beta_n x) + \\ \left(\frac{\sinh\beta_n l - \sin\beta_n l}{\cos\beta_n l - \cosh\beta_n l}\right)(\cosh\beta_n x + \cos\beta_n x) \end{pmatrix} \times \left(\int_0^t Q_n(\tau)\sin\omega_n(t-\tau)\ d\tau\right)\right]^2 \tag{28}$$

Therefore, by integrating Eq. 26, 27 or, 28, get the general equation of heat generation due to vibration beam as a function of time and longitudinal beam. Where, and by building computer program by using Matlab program to evaluate the heat generation of beam with different beam boundary condition and as a function of time and x-direction of beam. But to calculate the amount of heat in the program Matlab for the purpose of giving results in the form of data and also to draw the figures to compare the amount of heat per unit volume, so the following equation is applied: $\dot{q} = \frac{q}{A}$.

3. Numerical investigation

The numerical investigation is technique used to evaluation approximate solution for problem, [40-43], thus, the its technique in more application used to analysis the difficult structure its cannot solution by analytical techniques, [44, 45]. In addition, the numerical technique used to comparison the numerical results by other results are evaluated with other techniques, to given the agreement of results, [46-48].
Therefore, the COMSOL program will be used, which will depend on the finite element method (FEM) for design and analysis of the models, and give the outcomes required by simulations using computer programming to reach the outputs and outcomes of the completion of the research and development of the

mechanism of work, and compared to other outcomes extracted with the analytical solution, and given the percent error of its results.

3.1 COMSOL program

This program is used to find precise numerical solutions and different designs and structures and to find the results of complex mathematical equations that are difficult to solve analytically, [49-53].

In this work, the COMSOL/CFD program was used to find the heat generated values by the vibration of the beam exposed to the harmonic force at different frequencies for different cases of supported. Follow the main steps when using the software in the computer as follows,

1. Design of the models according to properties, dimensions and geometries of the material to be used.
2. Defining the meshing geometry.
3. Applying the boundary conditions, frequency and force.
4. Solving and visualizing your results.

3.2 Modelling and computational domain

The beam design length (84 cm) and cross-section (2.5 cm * 2.5 cm) as in the Figure 4. The material used is carbon steel (carbon=1.5%) with a density of (7750 kg/m^3) and modulus of elasticity (200 Gpa). The supported is determined by the type of beam where the use of three types including (cantilever beam, simply supported beam, clamped beam) and each type of supported has special boundary conditions, the required frequency and force are applied to the beam, after which the program is executed and the desired results and figures are included.

The design of the model form on the mesh as in Figure 5 which follows a finite element method (FEM) to give the results the elements, the results will be more accurate as the mesh is accurate if the distance between the nearest mesh lines, the mesh was represented symmetrically for all types of supported, so that the accuracy of the results is uniform for all cases to be used in comparison

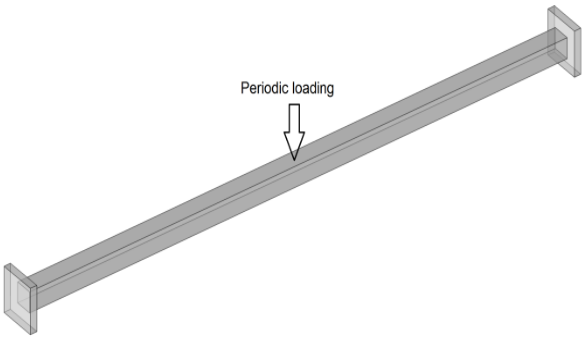


Figure 4. Three-dimensional computational domain.

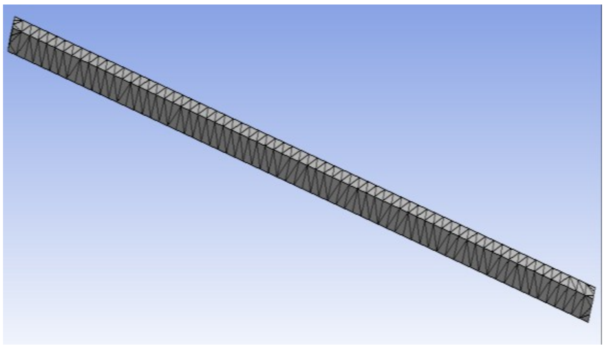


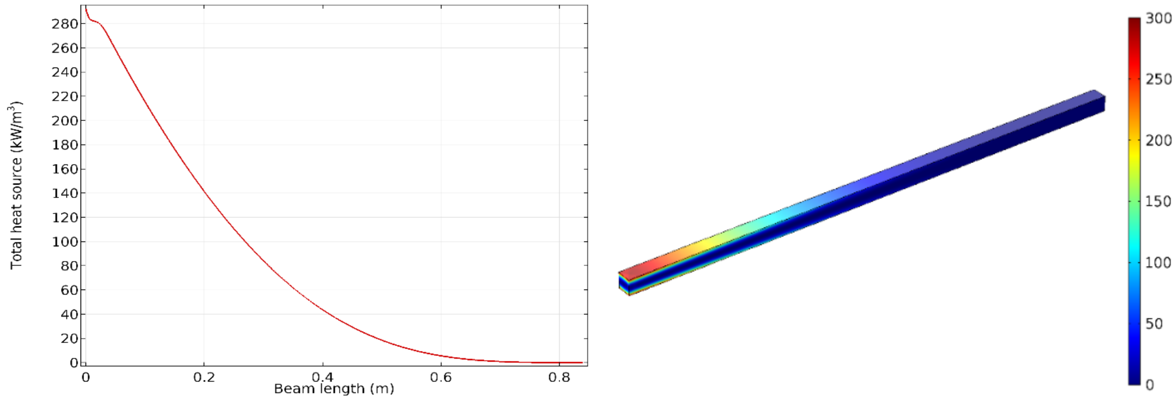
Figure 5. Mesh of the computational domain.

3.3 Meshing solved and processing

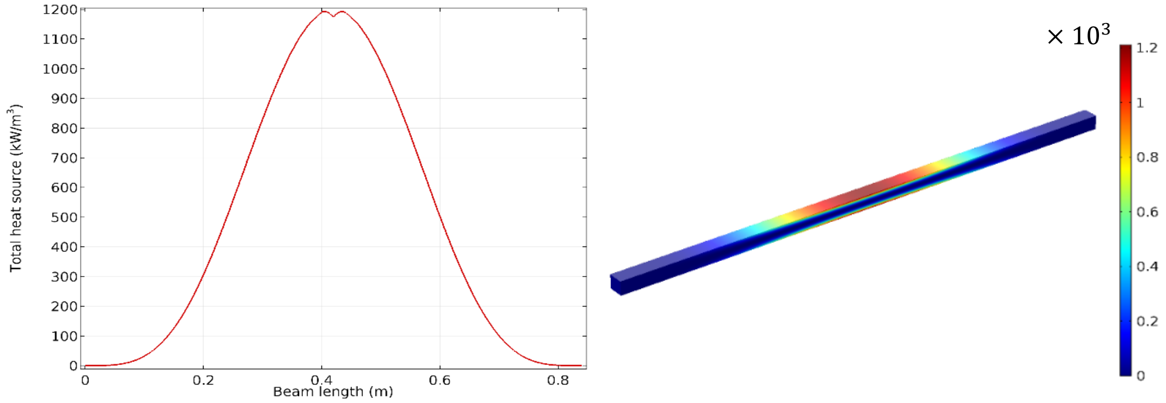
Default mesh setups were utilized. The model solution at transient boundaries. It is plotted measured values of displacement, heat generation and the temperature distribution utilizing domain plotting parameters. The Eigen natural frequency analysis outcomes initially then, the resonance are appeared in the distorted shape with sub- domain outcomes. The temperature distribution and heat generation plot outcomes are appeared of normal sub- domain outcomes.

3.4 Work states

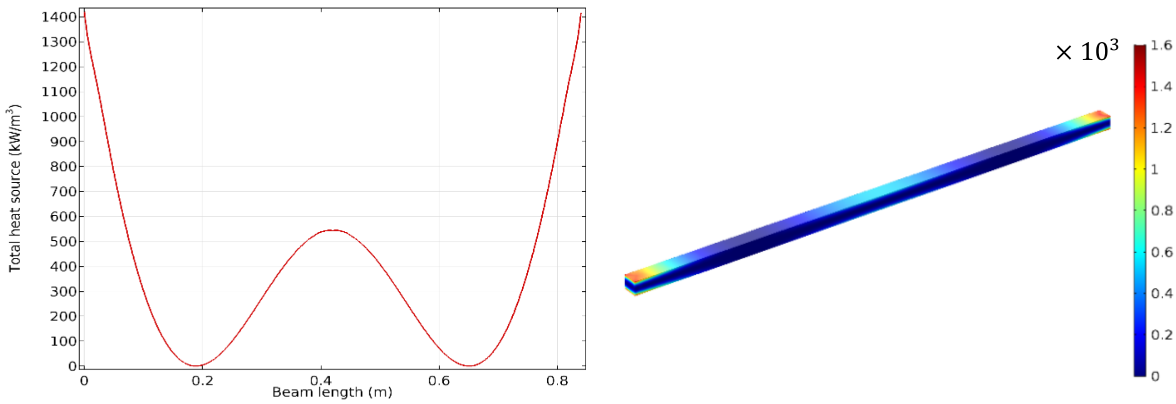
The results obtained through the use of the COMSOL program by simulating similar cases in the analytical solution. The results shown in the forms were obtained to determine the relationship between the variables and the behavior of the curves of the model in the cases of assuming that the harmonic force causes vibration very near to the resonance state of each case to show the variables clearly. The heat generated along the beam during a fixed period (60 seconds) as shown in Figure 6 (a, b and c) for different supported beam as shown:



(a) Heat generated contour for cantilever beam.



(b) Heat generated contour for simply supported beam.



(c) Heat generated contour for clamped beam.

Figure 6. The relationship between the heat generated & the length of the beam with constant time.

4. Results and discussion

The verification and comparison of the results and figures extracted from the theoretical part of the analytical solution and the numerical solution, for the heat generated by the vibration of the beam exposed to the harmonic force with time.

Since the analytical solution can calculate the magnitude of heat generated in the beam through the energy of elastic deformation, so the use of the COMSOL program to calculate the magnitude of heat generated and temperature changes in the beam due to vibration. The numerical solution will be implemented using the COMSOL program by designing models similar to the beam specifications in terms of dimensions, geometry and properties of the material used the same forces exerted during vibration of the beam calculated by analytical solution with compared between them.

The magnitude of heat generated is compared between the results calculated in the analytical solution and the results in the numerical solution for the same models for multiple cases of supported.

4.1 Analytical results

The amount of heat generated in the assumed beam (length (L=0.84 m), width, height (w=h=2.5 cm)) is calculated analytically by using the Matlab program during the vibration of the beam under the influence of harmonic force at different frequencies compared to the natural frequency. Sometimes the heat generated and the deflection is calculated for each location along the length of the beam on demonstrated constant time at other times it is calculated with time in a location where the heat generated and deflection the highest value in the beam for several cases of supported, as follows

4.1.1 The heat generated in the cantilever beam

Figure 7 shows the amount of heat generated in the beam changes according to the time and length of the beam when the beam is vibrated at a frequency ($\omega = 0.9 \times \omega_n$).

Now, in Figure 8, the relationship between the heat generated with the length of the beam is explained at constant time for different frequencies (ω = (0.5, 0.6, 0.7, 0.8, 0.9 and 1.2) ω_n) as shown (a), (ω = 0.99 ω_n) as shown (b) and ($\omega = \omega_n$) as shown (c).

The Figure (9), the relationship between the heat generated with the time is explained at the fixed end (x = 0) for the appearance of the highest heat generated as shown in the previous Figure 9 for different frequencies (ω = (0.8, 0.9, 1.1 and 1.2) ω_n) as shown (a) and (ω = (0.99, 1 ω_n) as shown (b).

4.1.2 The heat generated in the simply supported beam

Figure 10 shows the amount of heat generated in the beam changes according to the time and length of the beam when the beam is vibrated at a frequency ($\omega = 0.9 \times \omega_n$). Now, in Figure 11, the relationship between the heat generated with the length of the beam is explained at constant time for different frequencies ($\omega = \omega_n$) as shown (a) and (ω = (0.85, 0.9, 0.95, 0.99, 1.05, 1.1, 1.15 and 1.2) ω_n) as shown (b). The Figure 12, the relationship between the heat generated with the time is explained at the middle of the beam (x = 0.42 m) for the appearance of the highest heat generated as shown in the previous Figure for different frequencies (ω = (0.85, to 1.2 ω_n) as shown in Fig. 12.(a) and (ω = (0.99 and 1) ω_n) as shown in Figure 12 (b).

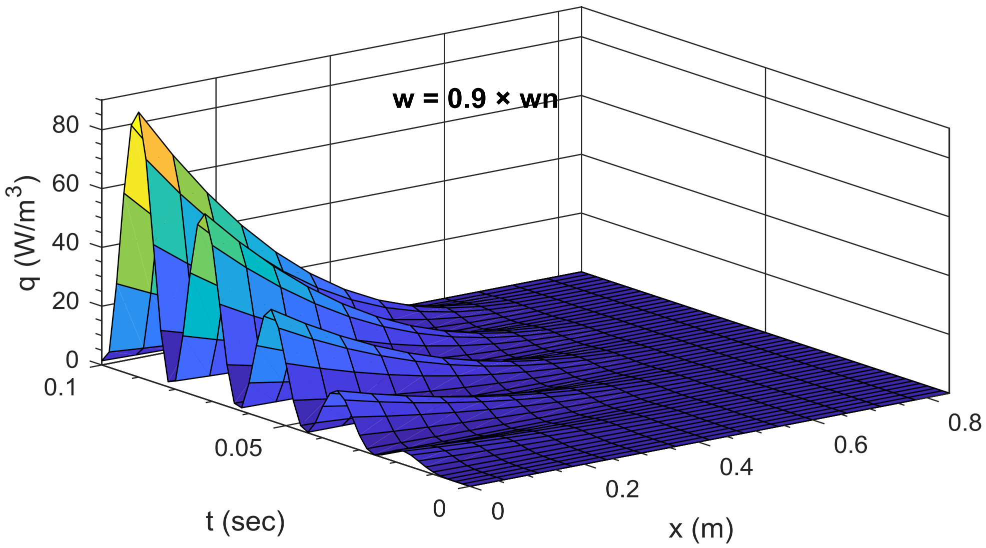
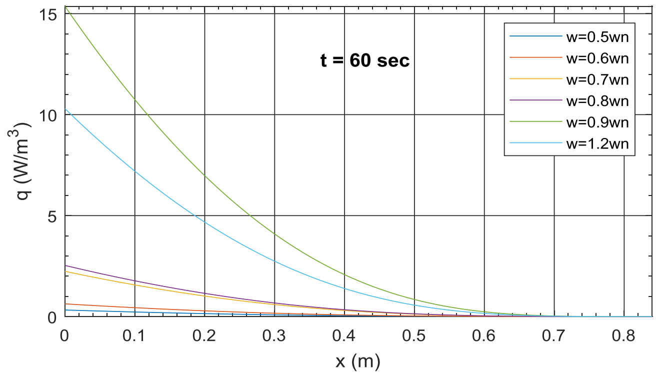
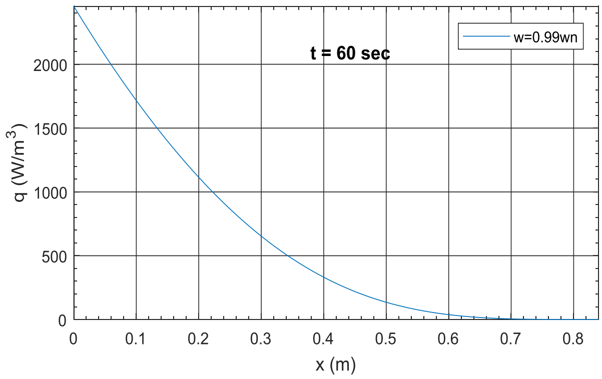


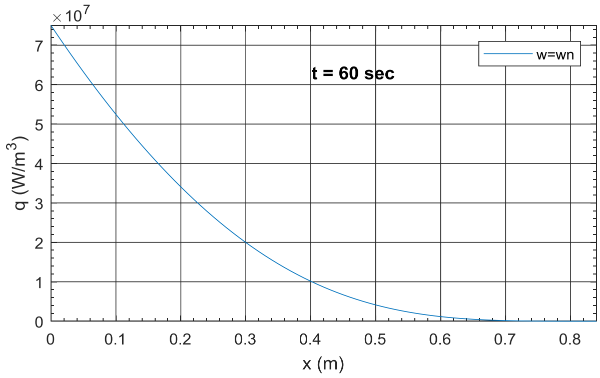
Figure 7. Heat generated with time and length of the beam, (Three dimension).



(a) Various natural frequency.

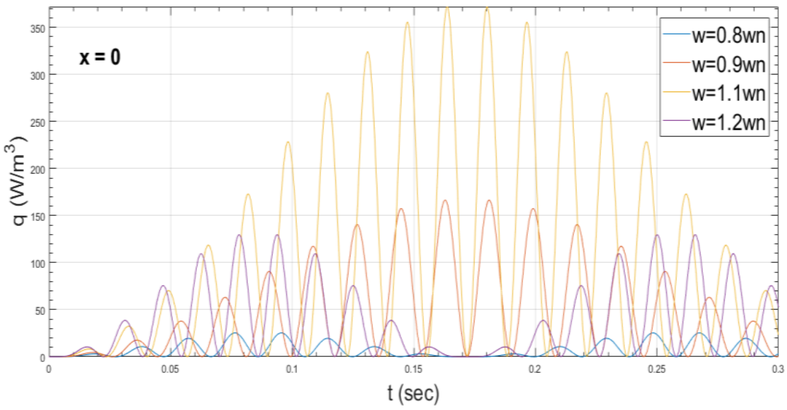


(b) $\omega = 0.99\omega_n$.

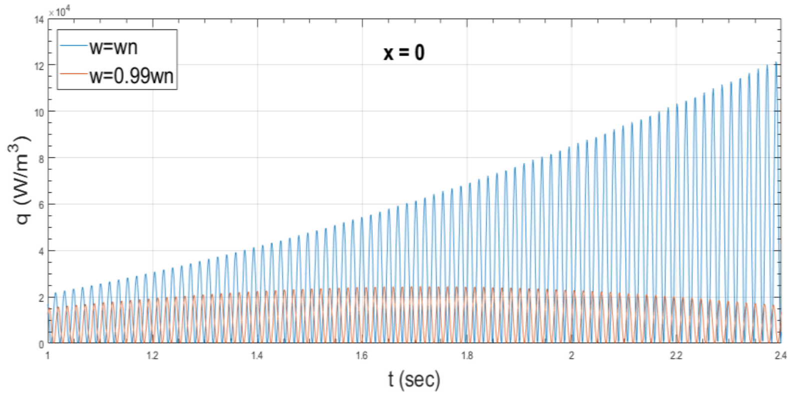


(c) $\omega_n = \omega$.

Figure 8. The heat generated with length of the beam.



(a) Various natural frequency.



(b) for $\omega = 0.99\omega_n, and\ \omega_n = \omega$.

Figure 9. The heat generated with time with various natural force applied.

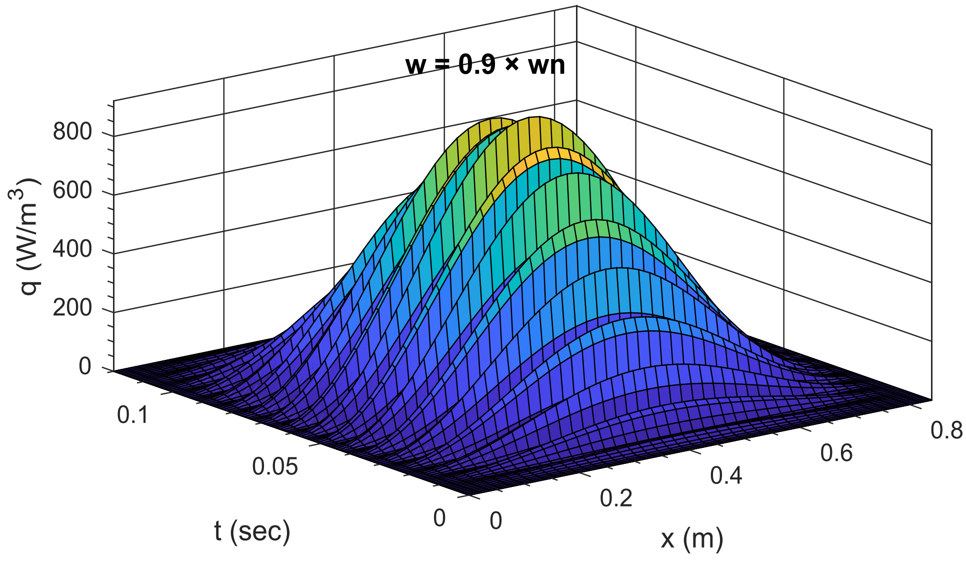
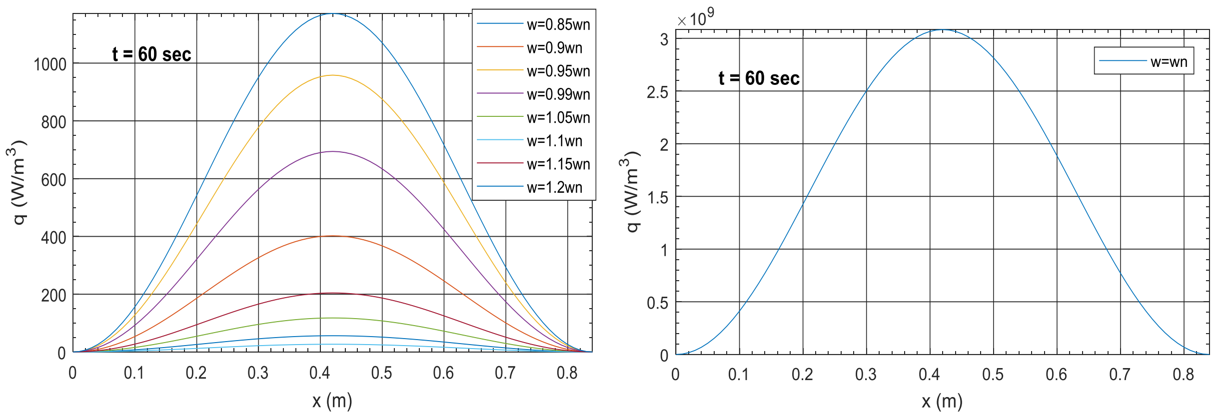
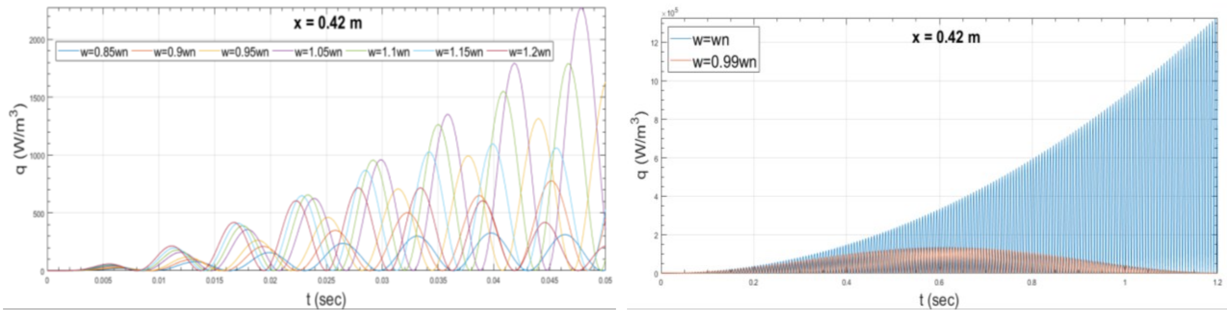


Figure 10. Heat generated with time and length of the beam, (Three dimension).



(a) various natural frequency. (b) $\omega = \omega_n$.

Figure 11. The heat generated with length of the beam.



(a) various applied load frequency. (b) for $\omega = 0.99\omega_n, and\ \omega_n = \omega$.

Figure 12. Heat generated with the time at the middle of the beam (x = 0.42 m).

4.1.3 The heat generated in the clamped beam

Figure 13 shows the amount of heat generated in the beam changes according to the time and length of the beam when the beam is vibrated at a frequency ($\omega = 0.9 \times \omega_n$). Now, in Figure 14, the relationship between the heat generated with the length of the beam is explained at constant time for different frequencies ($\omega = \omega_n$) as shown (a), ($\omega = 0.99\omega_n$) as shown (b) and (ω = (0.85, 0.9, 0.95, 1.05, 1.1 and 1.15) ω_n) as shown (c). The Figure 15, the relationship between the heat generated with the time is explained at the fixed end (x = 0) for the appearance of the highest heat generated as shown in the previous Figure 14 for different frequencies (ω = (0.85, 0.9, 0.95, 1.05, 1.1, 1.15 and 1.2) ω_n) as shown (a) and (ω = (0.99 and 1) ω_n) as shown (b, c).

4.2 The comparison the analytical solution and the numerical solution

The comparison the relationship between the heat generated with the length of the beam is explained for the analytical solution and the numerical solution at constant time and constant frequency for different supported beam, as, Figure 16a for cantilever beam, Figure 16b for simply supported beam, and, Figure 16c for clamped beam, with $\omega = 0.9\omega_n, t = 60\ sec$.

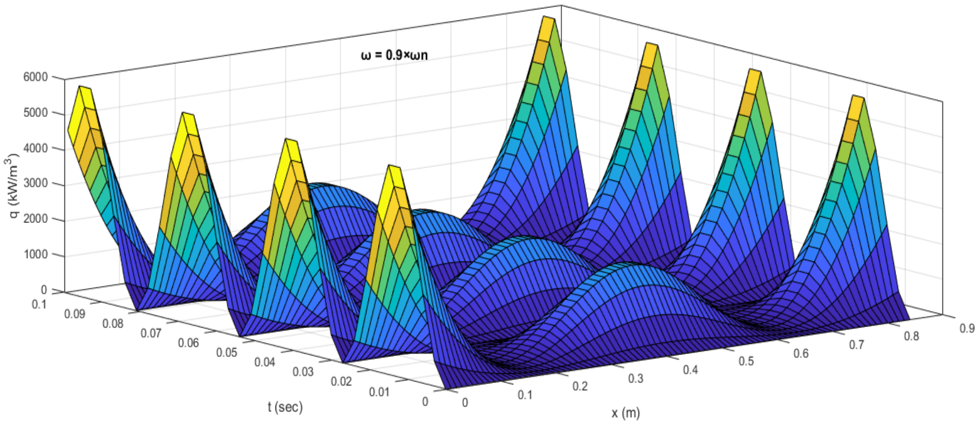
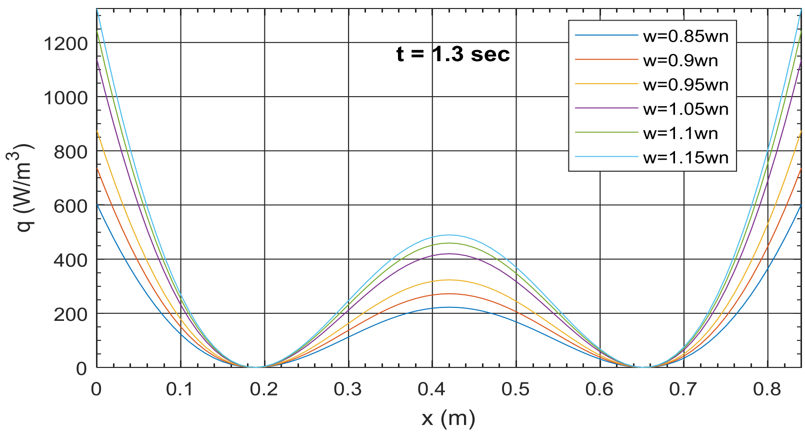
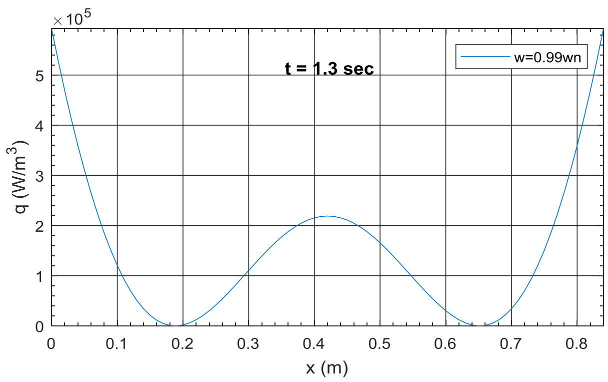


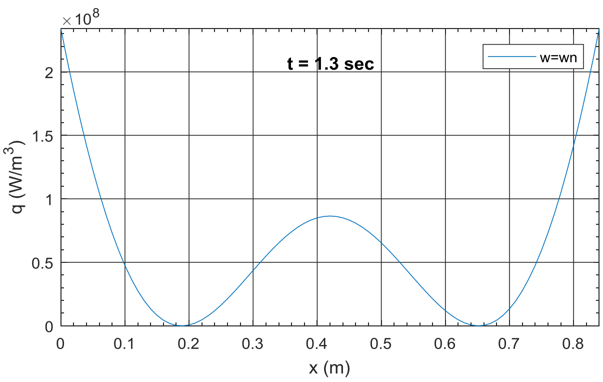
Figure 13. Heat generated with time and length of the beam.



(a) various applied load frequency.

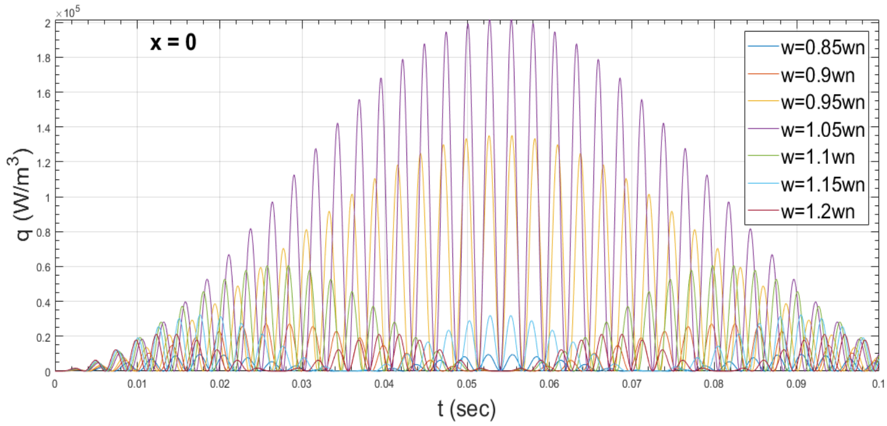


(b) $\omega = 0.99\omega_n$.

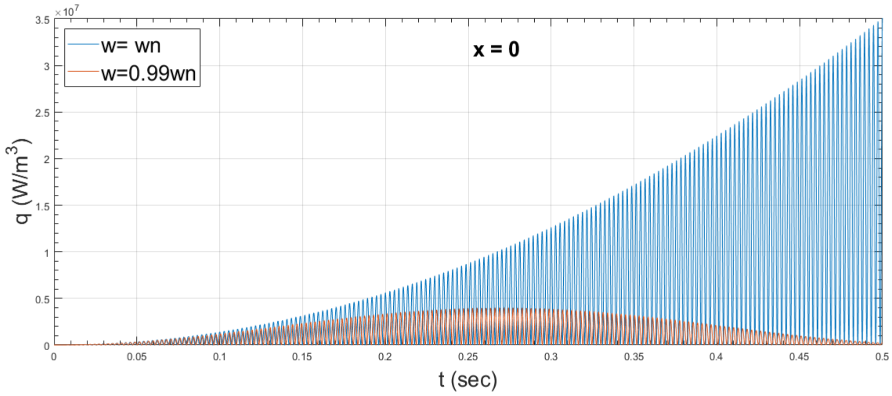


(c) $\omega_n = \omega$.

Figure 14. The heat generated with length of the beam.

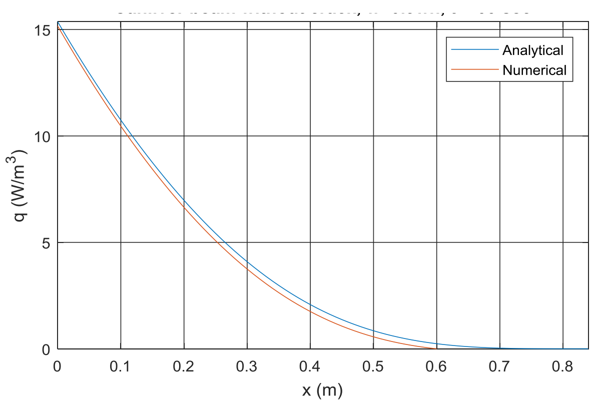


(a) various applied load frequency.

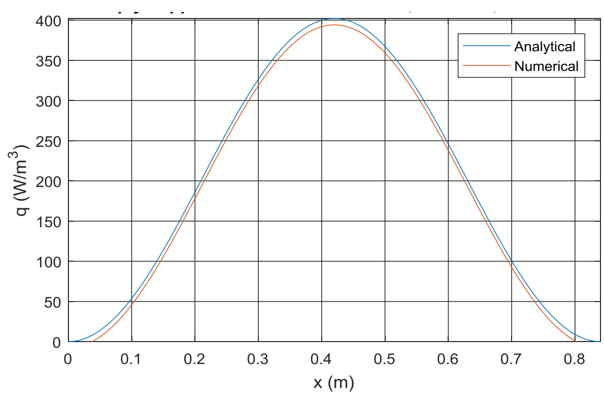


(b) for $\omega = 0.99\omega_n, and\ \omega_n = \omega$.

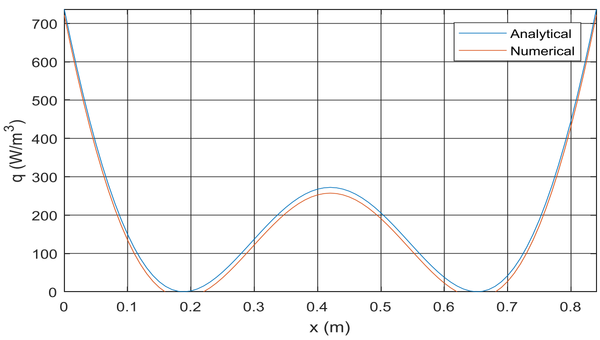
Figure 15. The heat generated with time.



(a) cantilever beam.



(b) simply supported beam.



(c) clamped beam.

Figure 18. The heat generated (analytical & numerical), , $\omega = 0.9\omega_n, t = 60\ sec$.

5. Conclusions

From previous results, the following conclusions can be shown briefly as follows:

1. The theoretical method was followed by accurate and logical results to calculate the amount of heat generated in the beam for the different types of supported according to the change of time or location in the beam and frequencies different according to the natural frequency. Also calculate the change in the natural frequency.
2. In the cantilever beam, the higher of heat generated at the fixed end. In the simply supported beam, the higher of heat generated in the middle of the beam. In the clamped beam, the higher of heat generated in the fixed ends and starts decreasing until it reaches approximately zero at (x =(0 .25, 0.75) L) and then rise up to the middle of the beam, but less than the ends.
3. To compare the amount of heat generated in the beam for the three types of supported the maximum heat generated in the fixed ends of the clamped beam is less than the heat generated in the middle of the beam for the simply supported beam and the lower is at the fixed end of the cantilever beam.
4. In general and for any type of supported it was observed that the heat generated increases as the frequency supplied is approaching to the natural frequency, and the generated heat is reduced as the frequency supplied goes away the natural frequency significantly.
5. The analytical solution is a good tool technique used to evaluation the heat generation for vibration beam subjected to harmonic load. In addition, the analytical solution was given a good agreement of heat generation results with compassion by numerical technique used.

Acknowledgement

The authors would like to express thanks for the Mechanical Engineering Department-Faculty of Engineering at University of Kufa, and College of Engineering at Kerbala University. In addition, the authors express thanks to Maher A.R. Sadiq Al-Baghdadi-President of the International Energy and Environment Foundation, Editor-in-Chief of the International Journal of Energy and Environment.

Reference

[1] Luay S. Al-Ansari, Muhannad Al-Waily, Ali M. H. Yusif Al-Hajjar ‘Experimental and Numerical Study of Crack Effect on Frequency of Simply Supported Beam’ Al-Khwarizmi Engineering Journal, Vol. 8, No. 2, 2012.

[2] Muhannad Al-Waily ‘Theoretical and Experimental Vibration Study of Continuous Beam with Crack Depth and Location Effect’ Indian Journal of Applied Research (IJAR), Vol. 3, No. 9, 2013.

[3] Muhannad Al-Waily ‘Theoretical and Numerical Vibration Study of Continuous Beam with Crack Size and Location Effect’ International Journal of Innovative Research in Science Engineering and Technology (IJIRSET), Vol. 2, No. 9, 2013.

[4] Muhannad Al-Waily ‘A Suggested Analytical Solution of Oblique Crack Effect on the Beam Vibration’ International Journal of Energy and Environment, Vol. 6, No. 3, 2015.

[5] Muhannad Al-Waily ‘Analytical and Experimental Investigations Vibration Study of Isotropic and Orthotropic Composite Plate Structure with Various Crack Effect’ International Energy and Environment Foundation, 2017.

[6] Muhsin J. Jweeg, Ali S. Hammood, Muhannad Al-Waily ‘A Suggested Analytical Solution of Isotropic Composite Plate with Crack Effect’ International Journal of Mechanical & Mechatronics Engineering IJMME / IJENS-Vol. 12, No. 5, 2012.

[7] Muhannad Al-Waily ‘Investigation of Health Monitoring of Composite Plate Structures-Using Vibration Analysis’ Ph. D. Thesis, Alnahrain University, College of Engineering, Mechanical Engineering Department, 2012.

[8] Muhsin J. Jweeg, Ali S. Hamood, Muhannad Al-Waily ‘A Suggested Analytical Solution of Orthotropic Composite Plate Structure with Crack Effect’ International Journal of Mechanical Engineering (IJME), IASET, Vol. 1, No. 2, 2012.

[9] Muhsin J. Jweeg, Ali S. Hammood, Muhannad Al-Waily ‘Analytical Solution to Oblique Crack Effect in Different Types of Composite Plates’ ARPN Journal of Science and Technology, Vol. 2, No. 8, 2012.

[10] Muhsin J. Jweeg, Ali S. Hammood, Muhannad Al-Waily ‘Experimental and Numerical Study of Oblique Crack Effect on Natural Frequency of Different Composite Plate Structure Types’ AT, Asian Transactions on Engineering, Vol. 2, No. 5, 2012.

[11] Muhannad Al-Waily, Maher A. R. Sadiq Al-Baghdadi, Rasha Hayder Al-Khayat ‘Flow Velocity and Crack Angle Effect on Vibration and Flow Characterization for Pipe Induce Vibration’ International Journal of Mechanical & Mechatronics Engineering IJMME-IJENS, Vol.17, No.5, 2017.

[12] Muhsin J. Jweeg, E. Q. Hussein, K. I. Mohammed 'Effects of Cracks on the Frequency Response of a Simply Supported Pipe Conveying Fluid' International Journal of Mechanical & Mechatronics Engineering IJMME-IJENS, Vol. 17, 05, 2017.

[13] Muhannad Al-Waily, Alaa Abdulzahra Deli 'A suggested analytical solution of buckling investigation for beam with different crack depth and location effect' International Journal of Energy and Environment, Vol. 7, No. 3, 2016.

[14] S. Pantellou, A. Dimarogonas 'Heat Propagation Due to Torsional Vibration of Shafts' Journal of Sound and Vibration, Vol. 73, No. 2, pp. 239-245, 1980.

[15] Michael I. Friswell, Daniel J. Inman, Ralph W. Rietz 'Active Damping of thermally Induced Vibrations' Journal of intelligent Material System and Structures, Vol. 8, 1997.

[16] Jadwiga Kidawa-Kukla 'Thermally Induced vibration of a Cantilever beam with Periodically Varying Intensity of a Heat Source' Journal of Applied Mathematics and Computational Mechanics, Vol. 12, No. 4, pp. 59-65, 2013.

[17] I. Jafarsadeghi-pournaki, G. Rezazadeh, M. R. Zamanzadeh, R. Shabani 'Thermally Induced Vibration of an Electro-Statically Deflected Functionally graded Micro-Beam Considering thermo-Elastic Coupling Effect' Sharif University of Technology, Scientia Iranica B, Vol. 21, No. 03, pp. 647-662, 2014.

[18] Wenbo Zhang, Hualing Chen, Danhui Zhu, Xiangjie Kong 'The Thermal Effects on High-Frequency Vibration of Beams using Energy Flow Anaysis' Journal of Sound and Vibration, Vol. 333, pp. 2588-2600, 2014.

[19] Muhsin J. Jweeg, Muhannad Al-Waily 'Determination of Inter-Laminar Shearing Stresses Using a Suggested Analytical Solution in the Composite Laminated Plates' International Journal of Mechanical Engineering (IJME), IASET, Vol. 2, No. 5, 2013.

[20] Muhsin J. Jweeg, Muhannad Al-Waily 'Dynamic Analysis of Stiffened and Unstiffened Composite Plates' The Iraqi Journal For Mechanical And Material Engineering, Vol. 13, No. 4, 2013.

[21] Muhannad Al-Waily 'Dynamic Analysis Investigation of Stiffened and Un-Stiffened Composite Laminated Plate Subjected to Transient Loading' International Energy and Environment Foundation, 2015.

[22] Muhannad Al-Waily, Zaman Abud Almalik Abud Ali 'A Suggested Analytical Solution of Powder Reinforcement Effect on Buckling Load for Isotropic Mat and Short Hyper Composite Materials Plate' International Journal of Mechanical & Mechatronics Engineering IJMME-IJENS, Vol.15, No. 4, 2015.

[23] Abdulkareem Abdulrazzaq Alhumdany, Muhannad Al-Waily, Mohammed Hussein Kadhim 'Theoretical analysis of fundamental natural frequency with different boundary conditions of isotropic hyper composite plate' International Journal of Energy and Environment, Vol. 7, No. 3, 2016.

[24] Muhsin J. Jweeg, Sameer Hashim Ameen 'Experimental and theoretical investigations of dorsiflexion angle and life of an ankle-Foot-Orthosis made from (Perlon-carbon fibre-acrylic) and polypropylene materials' 10th IMEKO TC15 Youth Symposium on Experimental Solid Mechanics, 2011.

[25] Muhsin J. Jweeg, Ali S. Hammood, Muhannad Al-Waily 'Experimental and Theoretical Studies of Mechanical Properties for Reinforcement Fiber Types of Composite Materials' International Journal of Mechanical & Mechatronics Engineering IJMME / IJENS-Vol. 12, No. 4, 2012.

[26] Muhannad Al-Waily 'Theoretical and Numerical Analysis Vibration Study of Isotropic Hyper Composite Plate Structural' International Journal of Mechanical and Production Engineering Research and Development (IJMPERD), (TJPRC), Vol. 3, No. 5, 2013.

[27] Mohsin Abdullah Al-Shammari, Muhannad Al-Waily 'Theoretical and Numerical Vibration Investigation Study of Orthotropic Hyper Composite Plate Structure' International Journal of Mechanical & Mechatronics Engineering IJMME / IJENS-Vol. 14, No. 6, 2014.

[28] Muhsin J. Jweeg, Muhannad Al-Waily, Alaa Abdulzahra Deli 'Theoretical and Numerical Investigation of Buckling of Orthotropic Hyper Composite Plates' International Journal of Mechanical & Mechatronics Engineering IJMME-IJENS, Vol.15, No. 4, 2015.

[29] Muhannad Al-Waily 'Analytical and Numerical Buckling and Vibration Investigation of Isotropic and Orthotropic Hyper Composite Materials Structures' International Energy and Environment Foundation, 2015.

[30] Muhannad Al-Waily 'Analytical and Numerical Thermal Buckling Analysis Investigation of Unidirectional and Woven Reinforcement Composite Plate Structural' International Journal of Energy and Environment, Vol. 6, No. 2, 2015.

[31] Abdulkareem Abdulrazzaq Alhumdany, Muhannad Al-Waily, Mohammed Hussein Kadhim Al-Jabery 'Theoretical and Experimental Investigation of Using Date Palm Nuts Powder into Mechanical Properties and Fundamental Natural Frequencies of Hyper Composite Plate' International Journal of Mechanical & Mechatronics Engineering IJMME-IJENS, Vol.16, No. 1, 2016.

[32] Dependent Loading' M.Sc. Thesis, University of Kufa, Faculty of Engineering, Mechanical Engineering Department, 2005.

[33] Muhsin J. Jweeg, Muhannad Al-Waily 'Analytical Investigation of Time Dependent Tensional Loading of Stiffened and Un-Stiffened Composite Laminated Plates' International Journal of Mechanical and Production Engineering Research and Development (IJMPERD), (TJPRC), Vol. 3, No. 4, 2013.

[34] Muhannad Al-Waily 'Theoretical and Numerical Study of Natural Frequency Investigation for Orthotropic Unidirectional and Woven Hyper Composite Materials Beam' First International Conference on Engineering Sciences' Applications, ICESA, College of Engineering, University of Kerbala, 24-25 /December, 2014.

[35] Muhsin J. Jweeg 'A Suggested Analytical Solution for Vibration of Honeycombs Sandwich Combined Plate Structure' International Journal of Mechanical & Mechatronics Engineering IJMME-IJENS, Vol. 16, No. 02, 2016.

[36] Singiresu S. Rao 'Mechanical Vibrations-Fifth Edition' Prentice Hall, 2011.

[37] D. N. Walker, C. Bowler, R. Jackson 'Results of Subsynchoronous Resonance Tests at Mohave' Transactions on Power Apparatus and Systems, Institute of Electrical and Electronic Engineers, PAS-94, pp. 1878-1889, 1975.

[38] Irving H. Shames, Clive L. Dym 'Energy and Finite Element Methods in Structural Mechanics' Book, Hemisphere Publishing Corporation, 1991.

[39] Sung Soo Jung, Yong Bong Lee, Byeong Su Jeon, Su Hyun Shin 'Measurement of the Loss Factor and the Young's Modulus in Structural Steel by Using a Laser Beam Reflection Method' Journal of the Korean Physical Society, Vol. 65, No. 07, pp. 1024-1027, 2014.

[40] Zainab Yousif Hussien, Kadhim Kamil Resan 'Effects of Ultraviolet Radiation with and without Heat, on the Fatigue Behavior of Below-Knee Prosthetic Sockets' International Journal of Mechanical and Production Engineering Research and Development (IJMPERD), Vol. 07 , No. 06, 2017.

[41] Muhannad Al-Waily, Kadhim K. Resan, Ali Hammoudi Al-Wazir, Zaman Abud Almalik Abud Ali 'Influences of Glass and Carbon Powder Reinforcement on the Vibration Response and Characterization of an Isotropic Hyper Composite Materials Plate Structure' International Journal of Mechanical & Mechatronics Engineering IJMME-IJENS, Vol.17, No.6, 2017.

[42] Ayad M. Takhakh 'Manufacturing and Analysis of Partial Foot Prosthetic for the Pirogoff Amputation' International J. of Mechanical & Mechatronics Engineering IJMME-IJENS, Vol.18, No.3, pp.62-68, 2018.

[43] Ameer A. Kadhim, Muhannad Al-Waily, Zaman Abud Almalik Abud Ali, Muhsin J. Jweeg, Kadhim K. Resan 'Improvement Fatigue Life and Strength of Isotropic Hyper Composite Materials by Reinforcement with Different Powder Materials, International Journal of Mechanical & Mechatronics Engineering IJMME-IJENS, Vol. 18, No. 02, 2018.

[44] Muhsin J. Jweeg 'Application of finite element analysis to rotating fan impellers' Doctoral Thesis, Aston University, 1983.

[45] Mohsin Abdullah Al-Shammari, Lutfi Y. Zedan, Akram M. Al-Shammari 'FE simulation of multi-stage cold forging process for metal shell of spark plug manufacturing' 1st International Scientific Conference of Engineering Sciences-3rd Scientific Conference of Engineering Science, ISCES 2018–Proceedings, 2018.

[46] Muhannad Al-Waily 'Experimental and Numerical Vibration Study of Woven Reinforcement Composite Laminated Plate with Delamination Effect' International Journal of Mechanical Engineering (IJME), IASET, Vol. 2, No. 5, 2013.

[47] Muhsin J. Jweeg, Muhannad Al-Waily 'Experimental and numerical analysis of cross-ply and angle-ply composite laminated plates having various shapes of cut outs' International Journal of Energy and Environment, Vol. 7, No. 6, 2016.

[48] Jawad K. Oleiwi, Ahmed Namah Hadi 'Experimental and numerical investigation of lower limb prosthetic foot made from composite polymer blends' International Journal of Mechanical and Production Engineering Research and Development, Vol. 08, No. 02, pp. 1319-1330, 2018.

[49] Maher A.R. Sadiq Al-Baghdadi 'A CFD Study of Hygro-Thermal Stresses Distribution in PEM Fuel Cell During Regular Cell Operation' Renewable Energy Journal, Vol. 34, No. 03, pp.674-682, 2009.

[50] Maher A.R. Sadiq Al-Baghdadi 'Prediction of Deformation and Hygro-Thermal Stresses Distribution in Ambient Air-Breathing PEM Fuel Cells using Three-Dimensional CFD Model' Recent Patents on Mechanical Engineering, Vol. 02, No. 01, pp. 26-39, 2009.

[51] Maher A.R. Sadiq Al-Baghdadi 'A CFD Analysis of Transport Phenomena and Electrochemical Reactions in a Tubular-Shaped Ambient Air-Breathing PEM Micro Fuel Cell' HKIE Transactions Hong Kong Institution of Engineers, Vol. 17, No. 02, 2010.

[52] Maher A.R. Sadiq Al-Baghdadi, K. K. Resan, Muhannad Al-Waily 'CFD investigation of the erosion severity in 3D flow elbow during crude Oil contaminated sand transportation' The 1st International Scientific Conference of Nanotechnology and Advanced Materials in Petroleum & Gas Industry, 3-4 May 2017, Engineering and Technology Journal, Vol. 35, Part A, No. 9, 2017.

[53] Rasha Hayder Al-Khayat, Maher A.R. Sadiq Al-Baghdadi, Ragad Aziz Neama, Muhannad Al-Waily 'Optimization CFD Study of Erosion in 3D Elbow During Transportation of Crude Oil Contaminated with Sand Particles, International Journal of Engineering & Technology, Vol. 07, No. 03, pp. 1420-1428, 2018.