Simulation of the thermal borehole resistance in groundwater filled borehole heat exchanger using CFD technique

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Abstract

The thermal borehole resistance in a groundwater-filled borehole heat exchanger (BHE) is affected by both conductive and convective heat transfer through the borehole water. To calculate this heat transport, different models are required compared to calculation of only conductive heat transfer in a back-filled BHE. In this paper some modelling approximations for groundwater-filled, single U-pipe BHEs were investigated using a 3D CFD model. The purpose is to find approximations that enable to construct a fast, simple model including the convective heat transfer that may be used in thermal response test analyses and BHE design programs. Both total heat transfer calculations (including convective and conductive heat transport) and only conductive heat transfer calculations were performed for comparison purposes. The approximations that are investigated are the choice of boundary condition at the U-pipe wall and using a single pipe in the middle of the borehole instead of the U-pipe. For the total heat transfer case, it is shown that the choice of boundary condition hardly affects the calculated borehole thermal resistance. For the only conductive heat transfer case, the choice of boundary condition at the pipe wall gives large differences in the result. It is also shown that using an annulus model (single pipe in the middle of the borehole) results in similar heat transfer as the U-pipe model provided that the equivalent radius is chosen appropriately. This approximation can radically decrease the number of calculation cells needed.

1. Introduction

In the 2005 worldwide review of geothermal heat pumps, Sweden was in the “top five” countries with regard to largest installed capacity and annual energy use. About 275,000 residential units (~12 kW) were in operation in Sweden, which is almost half as many as in United States of America at that time [1]. In Sweden and in some other places, groundwater is used to fill the space between the U-pipe and borehole wall instead of some backfilling material. During operation, natural convection will be induced in the borehole water due to occurring temperature and density gradients. This will increase the heat
transfer resulting in quite low borehole thermal resistances \( R_b = 0.06-0.08 \text{ m·K·W}^{-1} \) using heat injection) compared to many other filling materials.

The thermal resistance in the borehole is of great importance for the design of the system. A high resistance will result in a larger temperature difference between the borehole wall and the circulating fluid. If e.g. heat is extracted from the borehole, a high borehole thermal resistance will result in a low return temperature to the heat pump, which decreases the efficiency of the pump compared to a lower resistance. In groundwater-filled boreholes the borehole thermal resistance will change depending on water temperatures and injection or extraction rate. It is therefore important to include this when designing the system, since different seasons and/or injection rates will result in different borehole thermal resistances, which changes the efficiency of the system.

In today’s design and analysis tools for BHEs, the convective heat flow is approximated to a constant equivalent thermal conductivity. The conductive heat transfer was investigated using analytical, semi-numerical and numerical models and for long-term and short-term conditions, [e.g. 2-8]. Since the aspect ratio is small, the heat transfer is often treated as transient in the bedrock and steady-state inside the borehole using the borehole thermal resistance to describe the heat transfer through the circulating heat carrier fluid, U-pipe wall and borehole filling material. The changes in the convective flow due to different injection/extraction rates is thereby disregarded, which may result in poorly designed BHE systems.

A common approximation for BHE models is using annular geometry instead of the more complex U-pipe geometry in order to perform 1D or 2D calculations that diminish the calculation time. There are several described methods for calculating the equivalent radius for conductive heat models where the most commonly used method is to give the equivalent radius pipe the same cross-section area as the two U-pipe legs [2]. It was also shown by Gu and O’Neal [3] in 1998 that the equivalent diameter was dependent on the U-pipe diameter and the leg spacing. In 1999 Paul and Remund [4-6] gave an expression for the borehole thermal resistance that depended on the grout thermal conductivity and a borehole shape factor determined by the borehole geometry. It would be of advantage if this approximation also could be used when including the convective heat transfer and this paper therefore investigates which, if any, equivalent radius is appropriate.

Another common approximation is to disregard the fluid flow inside the collector and instead choose a suitable boundary condition at the outer pipe wall. The most common method is to use a constant heat flux [e.g. 7-8], but another alternative is to use a constant temperature. The effect of these boundary conditions is investigated for both conductive and total heat transfer (including convective heat transfer), since groundwater-filled boreholes may freeze during heat extraction and a calculation model therefore should be accurate for both liquid and solid conditions.

In this paper a 3 m long section of a BHE is simulated using a 3D computer fluid dynamic (CFD) model. The length was chosen to be the same as in [11, 12]. The model is used to investigate how two common approximations work when using total heat transfer calculations (including convective heat flow) instead of only conductive heat transfer. One approximation is the influence of the boundary conditions on the pipe wall where constant heat flux and constant temperature are compared. The other is the equivalent radius approximation, which is compared to a three-dimensional U-pipe model for a water-filled borehole heat exchanger. These two approximations, if appropriate for total heat transfer calculations, may truly decrease the required computational capacity and time for groundwater-filled BHE models.

2. Models and simulations

Two three-dimensional computer fluid dynamics (CFD) models are in this paper used to investigate how the two approximations mentioned above affect the heat transfer in a groundwater-filled BHE. The models are built and simulated in the commercial software Fluent using steady-state conditions and Boussinesq approximation for density. The basis of the code is a conservative finite-volume method. The program is able to model fluid flow and heat transfer in different geometries with complete mesh flexibility. The scaled residuals are useful indicators of solution convergence; a decrease to \( 10^{-3} \) is normally sufficient for a converged solution according to the supplier of the software [9].

The first model is the U-pipe model \( (M_u) \), which is a 3 m long section of a groundwater-filled single U-pipe BHE (Figure 1a). The borehole is surrounded with solid bedrock out to a radius of 1 m with material parameters similar to granite. The U-pipe has an outer diameter of 0.04 m and the shank spacing (pipe centre to pipe centre) is 0.05 m. A total of 634,200 hexahedron and wedge-shaped volume element cells are used in the model. The large amount of cells required limiting the length of the borehole to 3 m. For
the comparison presented here between different simulation approximations, the length of the BHE does not affect the result.

The equivalent radius model (Mer) is used to investigate if this common approximation is appropriate for total heat transfer (THT, including convective heat flow) calculations. The U-pipe legs are replaced with one larger pipe placed in the middle of the borehole (Figure 1b). This 3D model has a total amount of 540,000 hexahedron and wedge-shaped volume element cells. The annular-shaped geometry enables it to be reduced to a 2D axisymmetric model, which considerably reduces the total number of calculation cells. However, in this paper, both models (Mu and Mer) use 3D calculations. In that way they use the same Fluent calculation models and may thereby be compared to each other.

There are different ways of choosing the equivalent radius as discussed in the introduction. Those mentioned there are valid for conductive heat transfer using constant heat flux at the pipe wall. The choice of the equivalent radius (req) will be different for other boundary conditions and heat transfer situations. In this paper conductive heat transfer (CHT) calculations will be used for both constant heat flux and constant temperature at the pipe wall. For those calculations Fourier’s law, Eq. (1), was used to calculate req with the result from the simulations using the U-pipe model (Mu). Notice that when using constant heat flux at the pipe wall, the calculated req results in the same cross-section area as the U-pipe as described in the literature [2], but not for constant temperature at the pipe wall. For the simulations using total heat transfer (THT), req is chosen so that the heat transfer area is the same as for the U-pipe. This will be shown to be suitable for both boundary conditions at the pipe wall. Table 1 shows the used equivalent radius for the different model conditions.

![Figure 1. Outline of the model geometries (a) U-pipe model (Mu), and (b) equivalent radius model (Mer).](image)

\[
q' = \frac{2\pi \lambda}{\ln(r_{bhw}/r_{eq})(T_{pw} - T_{bhw})}
\]

where \(q'\) is the heat flow (W·m⁻¹), \(\lambda\) is the thermal conductivity (W·m⁻¹·K⁻¹), \(r_{bhw}\) is the radius to the borehole wall (m), \(r_{eq}\) is the equivalent radii (m), \(T_{pw}\) is the temperature at the pipe wall (K) and \(T_{bhw}\) is the temperature at the borehole wall (K).

Both models (Mu and Mer) are simulated with either a constant temperature (cTpw) or a constant heat flux (cq pw) applied over the pipe wall. For the other boundary conditions a constant temperature is applied at the outer vertical bedrock boundary (cTbrb) and the top and bottom boundaries are adiabatic. Material parameters for the water in the groundwater-filled borehole depend on the temperature in each simulation and are taken from a standard parameter table [10]. All parameters are held constant during each
simulation except the density, which uses the Boussinesq approximation during THT modelling. All simulations were calculated until the scaled residuals were less than $5 \cdot 10^{-5}$.

Table 1. Equivalent radius used in the model for the different boundary conditions, heat flows

<table>
<thead>
<tr>
<th></th>
<th>$r_{eq}$ [m]</th>
<th>$cT_{pw}$</th>
<th>$r_{eq}$ [m]</th>
<th>$cq''_{pw}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total heat transfer (THT)</td>
<td>0.04</td>
<td>0.04</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Conductive heat transfer (CHT)</td>
<td>0.0355</td>
<td>0.0283</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 2 shows a flow chart of the simulations performed for this paper. The simulations were performed for the two models, U-pipe model ($M_u$) and Equivalent radius model ($M_{er}$) in order to see if $M_{er}$ could be used as an appropriate approximation for the more complex U-pipe geometry when simulating groundwater-filled BHEs. The result is presented in Section 4. The models are investigated both for only conductive heat transfer flow (CHT) and total heat transfer flow (THT, including both convective and conductive heat transfer). The result for THT is presented in Section 3.1 and for CHT in Section 3.2 and is further discussed in Section 4 during comparison of the two models. Since the heat transfer in the fluid and through the pipe wall is disregarded in the model, a boundary condition has to be given at the outer U-pipe wall. The most common choice of boundary condition is either constant temperature ($cT_{pw}$) or constant heat flux ($cq''_{pw}$) at the outer U-pipe wall. In a full-scale BHE the temperatures and heat flux will change along the length of the borehole so that neither approximation will cover the real case. The effect of choosing either boundary condition is therefore also investigated; simulations $M_{1-6}$ and $M_{13-18}$ use constant temperature and $M_{7-12}$ and $M_{19-24}$ use constant heat flux.

Table 2 shows the boundary conditions for simulation $M_{1-12}$. $M_{1-6}$ uses a constant temperature at the pipe wall. The achieved mean heat flux value at the pipe wall is then used in simulations $M_{7-12}$, which use a constant heat flux over the pipe wall. Since steady-state conditions are used and $M_1$ and $M_7$ have the same mean heat flux over the pipe wall and the same temperature applied at the outer bedrock boundary ($T_{brb}$), the total heat flow in the bedrock must be the same. The two simulations will therefore receive the same mean heat flow per metre borehole, which is a parameter commonly used in discussions of BHE systems. If the boundary condition affects the result, this will be seen as different thermal resistances in the borehole water, $R_w$ (Eq. 2). The mean borehole wall temperature ($T_{bhw}$) will remain the same since both boundary conditions have the same $cT_{brb}$ and $q'$, while the mean temperature at the pipe wall ($T_{pw}$) will change resulting in a different temperature difference between the borehole wall and the pipe wall.

$$R_w = \frac{T_{pw} - T_{bhw}}{q'}$$

(2)

where $R_w$ is the thermal resistance in the water (m·K·W⁻¹), $T_{pw}$ is the temperature at the pipe wall (K), $T_{bhw}$ is the temperature at the borehole wall (K) and $q'$ is the heat flow (W·m⁻¹).
Table 2. Boundary conditions for simulations M1-M12 for U-pipe model (Mu) and received heat flow per metre of borehole using total heat transfer flow (THT)

<table>
<thead>
<tr>
<th>Boundary conditions: cTpw</th>
<th>Tpw [K]</th>
<th>Tbrb [K]</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>285.08</td>
<td>273.94</td>
</tr>
<tr>
<td>M2</td>
<td>293.88</td>
<td>283.79</td>
</tr>
<tr>
<td>M3</td>
<td>302.62</td>
<td>293.63</td>
</tr>
<tr>
<td>M4</td>
<td>307.10</td>
<td>298.65</td>
</tr>
<tr>
<td>M5</td>
<td>293.88</td>
<td>289.25</td>
</tr>
<tr>
<td>M6</td>
<td>293.88</td>
<td>286.60</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Boundary conditions: cq&quot;pw</th>
<th>q&quot;pw [W·m⁻²]</th>
<th>Tbrb [K]</th>
</tr>
</thead>
<tbody>
<tr>
<td>M7</td>
<td>237.15</td>
<td>273.94</td>
</tr>
<tr>
<td>M8</td>
<td>219.45</td>
<td>283.79</td>
</tr>
<tr>
<td>M9</td>
<td>197.49</td>
<td>293.63</td>
</tr>
<tr>
<td>M10</td>
<td>186.30</td>
<td>298.65</td>
</tr>
<tr>
<td>M11</td>
<td>98.85</td>
<td>289.25</td>
</tr>
<tr>
<td>M12</td>
<td>156.97</td>
<td>286.60</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Boundary conditions: M1 / M7</th>
<th>M2 / M8</th>
<th>M3 / M9</th>
<th>M4 / M10</th>
<th>M5 / M11</th>
<th>M6 / M12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Received q' [W·m⁻¹]</td>
<td>59.49</td>
<td>55.05</td>
<td>49.54</td>
<td>46.74</td>
<td>24.68</td>
</tr>
</tbody>
</table>

3. Results for the U-pipe model (Mu)
Both total heat transfer (THT) and only conductive heat transfer (CHT) simulations were performed and are presented in Sections 3.1 and 3.2, respectively. The focus will be on investigating how the heat transfer is affected by the boundary conditions applied at the outer U-pipe wall.

3.1 Total heat transfer (THT)
Water close to the U-pipe wall will have a more rapid increase in temperature during heat injection than water close to the borehole wall. The induced temperature gradient results in density differences with warmer, lighter water rising and colder, heavier water sinking. In the model one large convective cell is achieved. However, the boundary condition applied at the pipe wall will affect the temperature distribution in the BHE water. For the boundary condition constant temperature (cTpw), the temperature in the water close to the U-pipe wall will reach almost the same temperature as the wall. The temperature distribution at different heights in the borehole will therefore be similar. Using constant heat flux (cq"pw) at the outer U-pipe wall results in an increase in temperature along the borehole with higher temperatures at the top of the borehole, because the rising water receives a constant heat input along the way up to the top. The two boundary conditions will therefore affect the achieved convective heat flow differently.

Figure 3a shows the temperature in and around the borehole at a borehole length of 1.5 m for boundary condition constant temperature at the pipe wall (cTpw, Mu4). An un-radial pattern is seen inside the borehole due to both U-pipe legs acting as heat sources. The heat transfer becomes radial after a distance out in the bedrock (r radial). This un-radial heat flow in the water changes the heat transfer compared to using an equivalent radius model (Me). In Figure 3b the radial temperature difference between the x and z directions is shown in the bedrock for both boundary conditions: constant temperature, cTpw (Mu4) and constant heat flux, cq"pw (Mu10). It may be seen that constant temperature (cTpw) results in a slightly higher temperature difference between the x and z directions. Already at a distance of less than 0.2 m from the centre of the borehole, the temperature difference is however less than 0.01°C and the radial pattern is established for both boundary conditions. This is valid for all heights.

Figure 4 shows the mean temperature difference between the U-pipe wall and borehole wall for simulations Mu1-Mu6 (cTpw) and simulations Mu7-Mu12 (cq"pw). Notice that simulations Mu1 and Mu7 and so on simulate the same basic condition: the same mean heat flow per borehole length, temperature at the outer bedrock boundary (Tbrb) and temperature level in borehole water. It may be seen that the two boundary conditions result in almost the same temperature difference between pipe and borehole wall. The maximum deviation in the result is 0.14°C between the two boundary conditions. It may therefore be concluded that for total heat transfer calculations (THT), the choice of boundary condition at the pipe wall hardly affects the result using mean values over the whole borehole length.
Figure 3. (a) Temperatures [K] in and around the borehole at the vertical level 1.5 m for Mu4, and (b) temperatures difference between the x and z directions in the bedrock for cTpw & cq"pw.

Figure 4. The mean temperature difference between pipe wall and borehole wall for cTpw (Mu1-6) and cq"pw (Mu7-12)
3.2 Conductive heat transfer (CHT)
The conductive heat transfer (CHT) simulations 
$M_{u13-24}$ were performed assuming stagnant liquid water in the
borehole, i.e. the water is treated as a solid and no convective
flow can occur. Simulations $M_{u13-18}$ use constant temperature at the
pipe wall ($cT_{pw}$) and have the same boundary conditions as
$M_{u1-6}$ using total heat transfer (THT) in Table 2. The new
achieved mean heat flux at the pipe wall is then used in
simulations $M_{u19-24}$, which use constant heat flux at the pipe wall ($cq''_{pw}$). The heat transport through the
stagnant water is less effective, whereby the temperature difference between pipe wall and borehole wall
must be larger for CHT compared to THT.

Using the same heat transfer parameters in the bedrock, outer bedrock temperature ($T_{bed}$), pipe wall
temperature ($cT_{pw}$) and a less effective heat transport through the borehole water will result in a reduced
heat transfer rate ($q'$) for CHT. The achieved mean heat transfer rate is approximately 70% of the values
given in Table 2 for total heat transfer (THT). The conductive heat transfer case (CHT) thus receives a
radical change in borehole thermal resistance (eq. 2). It is therefore not possible to use calculations with
only conductive heat transfer when liquid water is filling the borehole.

Figures 5a and b show the temperature gradient received for the two boundary conditions, constant
temperature ($cT_{pw}$, $M_{u16}$) and constant heat flux ($cq''_{pw}$, $M_{u22}$), at a borehole length of 1.5 m. The
difference between the two boundary conditions may clearly be seen when conductive heat transfer
(CHT) calculations are used. Without the convective flow mixing the water, larger temperature
differences are achieved. Using constant temperature at the pipe wall results in peanut-shaped isotherms
around the pipe legs, and this un-radial heat pattern is transferred far out in the bedrock. Using a constant
heat flux instead results in a higher temperature in the middle of the borehole as a result of twice as much
heat input in this area giving a more radial heat pattern.

In Figure 5a ($cT_{pw}$) the change in temperature around the borehole wall is 1.4°C while the pipe wall has
constant temperature. For $cq''_{pw}$ (Figure 5b) the larger temperature difference is around the pipe wall with
a 9.9°C change and only 0.3°C difference around the borehole wall. As a result the radial heat transfer
pattern is achieved approximately 3 times further out for the boundary condition constant temperature
($cT_{pw}$, CHT) compared to when the convective flow is included ($cT_{pw}$, THT) and 1.5 times for constant
heat flux ($cq''_{pw}$, CHT).

Figure 6 shows the mean temperature difference between the pipe and borehole wall for $cT_{pw}$ ($M_{u13-18}$)
and $cq''_{pw}$ ($M_{u19-24}$) for only conductive heat transfer (CHT). Using constant heat flux at the pipe wall
($cq''_{pw}$) results in 60% larger temperature difference than $cT_{pw}$, even though the mean heat flow per metre
of borehole is the same. This is because a constant heat flux at the pipe wall results in higher
temperatures in the middle of the borehole, while constant temperature results in a more even spread of
the heat in the borehole. Using constant heat flux at the pipe wall ($cq''_{pw}$) thus results in higher thermal
borehole resistance (eq. 2) than using constant temperature at the pipe wall ($cT_{pw}$). The choice of
boundary condition would thereby affect the result greatly if water and ice conditions were to be
simulated. Since a full-length borehole has both changing temperature and heat flux along the length
neither is fully correct. The most common approximation in BHE models is the constant heat flux.

4. Comparison between Equivalent radius model ($M_{eq}$) and U-pipe model ($M_{u}$)
The complex geometry in the U-pipe model requires a large number of cells and is thereby
computationally heavy. A common approximation is the Equivalent radius model ($M_{eq}$) using the annular
geometry with one pipe in the middle instead of two U-pipe legs (Figure 2). The different equivalent
radii ($r_{eq}$) used in the simulations are presented in Table 1, in Section 2. All simulations for the
Equivalent radius model ($M_{eq}$) have the same boundary conditions as the U-pipe model ($M_{u}$), discussed in
Sections 2 and 3. It will be investigated whether the un-radial heat transfer pattern in the U-pipe model
changes the total heat transfer pattern for $M_{u}$ compared to $M_{eq}$. If the two models have similar borehole
thermal resistance results, $M_{eq}$ is counted as an appropriate approximation.

Figure 7a shows the mean temperature difference between the pipe wall and borehole wall using total
heat transfer THT ($r_{eq}=0.04$ m). As may be seen for both boundary conditions, $cT_{pw}$ ($\square$) and $cq''_{pw}$ ($\bigcirc$), the
results differ slightly when comparing the two models $M_{u}$ and $M_{eq}$. Such small changes hardly affect the
borehole thermal resistance, however. The maximum variation in temperature difference for these
simulations is 0.08°C between $M_{u}$ and $M_{eq}$. The deviation between the two boundary conditions (0.14°C,
section 3.1) is thus larger than for the two models. The received mean heat flow per metre of borehole is
also almost the same for the two models, with a deviation of only 0.5%. It may thereby be concluded that
the chosen $r_{eq}$ is appropriate for both boundary conditions.
Figure 5. (a) Temperatures [K] in and around the borehole for constant temperature (cT_{pw}, M_{u16}) at a borehole length of 1.5m, and (b) temperatures [K] in and around the borehole for constant heat flux (cQ_{pw}, M_{u22}) at a borehole length of 1.5m.

In Figure 7b the temperature difference between the pipe wall and borehole wall is shown for the only conductive heat transfer (CHT) case. Here, r_{eq} = 0.0355 m when using constant temperature at the pipe wall (cT_{pw}) and 0.0283 m using constant heat flux (cQ_{pw}). The resulting temperature differences from the two models (M_{u} and M_{er}) do not deviate at all. The difference between the result from cQ_{pw} (○) and cT_{pw} (□) is 60%, the same magnitude as discussed in Section 3. The U-pipe (M_{u}) and the Equivalent radius model (M_{er}) give the same result as regards area-weighted mean values in spite of the un-radial heat pattern across the borehole wall, as expected.
Figure 6. The mean temperature difference between pipe wall and borehole wall using stagnant water in the borehole

Figure 7. (a) Comparison between \( M_u \) and \( M_{er} \) for total heat transfer (THT), and (b) comparison between \( M_u \) and \( M_{er} \) for conductive heat transfer (CHT)

The thermal resistance in the borehole water (\( R_w \)) may now be calculated with the result from the numerical simulations according to Eq. (2). The result for total heat transfer (THT) is shown in Table 3 for both models (\( M_u \) & \( M_{er} \)). The thermal resistance in the borehole water is presented in the first row for constant temperature at the pipe wall (\( cT_{pw} \)) as \( M_{u1-6} \) and \( M_{eer1-6} \). If the result is the same only one value is given while different results are presented as \( M_u / M_{er} \). At the second row, constant heat flux at the pipe wall is presented for \( M_{u7-12} \) and \( M_{eer7-12} \). Notice that simulations \( M_7 \) and \( M_{er7} \) have the same basic simulation conditions; the same mean heat flow per borehole metre and mean water temperature. In each column the difference between the two boundary conditions (\( cT_{pw} \) and \( cq''_{pw} \)) may be seen.
The maximal difference in $R_w$ between the two models ($M_u$ and $M_e$) is as small as 0.002 m·K·W⁻¹ or 7%. The deviation between the two boundary conditions is slightly higher and results in a maximum difference of 12% (0.003 m·K·W⁻¹) for the investigated heat rates and temperature interval. The boundary condition constant heat flux at the pipe wall ($cq''_{pw}$) gives in general lower resistance than constant temperature. A borehole heat exchanger system is however not affected by such small differences. It may therefore be concluded that the Equivalent radius model ($M_e$) is an appropriate approximation for the U-pipe model ($M_u$) for THT calculations, and the result is independent of the choice of boundary condition at the pipe wall.

### Table 3. Calculated thermal resistances in the borehole water ($R_w$) for total heat transport (THT)

<table>
<thead>
<tr>
<th></th>
<th>$M_1$</th>
<th>$M_2$</th>
<th>$M_3$</th>
<th>$M_4$</th>
<th>$M_5$</th>
<th>$M_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$cT_{pw}$</td>
<td>$M_u/M_e$</td>
<td>0.030/0.028</td>
<td>0.026</td>
<td>0.024/0.025</td>
<td>0.024</td>
<td>0.030/0.029</td>
</tr>
<tr>
<td>$cq''_{pw}$</td>
<td>$M_u/M_e$</td>
<td>0.028</td>
<td>0.024/0.025</td>
<td>0.023</td>
<td>0.022/0.023</td>
<td>0.028</td>
</tr>
</tbody>
</table>

In Table 4 the thermal resistances are shown for conductive heat transfer (CHT). The results from the two models differ as little as 0.001 m·K·W⁻¹ or 1% using $cT_{pw}$, while the result for $cq''_{pw}$ does not differ at all. The Equivalent radius model is thereby also an appropriate approximation for CHT calculations, which has been shown earlier in several published papers for boundary condition constant heat flux.

### Table 4. Calculated thermal resistances in the borehole water ($R_w$) for conductive heat transport

<table>
<thead>
<tr>
<th></th>
<th>$M_{13}$</th>
<th>$M_{14}$</th>
<th>$M_{15}$</th>
<th>$M_{16}$</th>
<th>$M_{17}$</th>
<th>$M_{18}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$cT_{pw}$</td>
<td>$M_u/M_e$</td>
<td>0.102</td>
<td>0.099/0.100</td>
<td>0.098</td>
<td>0.097</td>
<td>0.099/0.100</td>
</tr>
<tr>
<td>$cq''_{pw}$</td>
<td>$M_u/M_e$</td>
<td>0.163</td>
<td>0.159</td>
<td>0.156</td>
<td>0.155</td>
<td>0.159</td>
</tr>
</tbody>
</table>

In Figure 8 the average thermal resistance in the borehole water for the six different simulation conditions is shown for each model ($M_u$, $M_e$), heat transport (THT, CHT) and boundary condition ($cT_{pw}$, $cq''_{pw}$). It is clearly seen here that $M_u$ and $M_e$ result in almost the same values for all modelling approximations and boundary conditions. It is also seen that THT results in almost the same value independent of boundary condition and model. The choice of boundary condition will radically change the result for CHT, with a lower value using constant temperature at the pipe wall ($cT_{pw}$). Notice also that using only conductive heat transfer calculation (CHT) for liquid water and thereby disregarding the effect of the convective flow result in clearly too high thermal resistance in the borehole water. This together with the large difference between the two boundary conditions may result in incorrect BHE system design and a less efficient system.

![Figure 8](image.png)
5. Summary and conclusions
Using only conductive heat transfer calculations results in 4-6 times higher thermal resistance than using total heat transfer calculations (including the convective heat flow) for a groundwater-filled BHE according to the simulations presented in this paper. The reduction in thermal resistance due to convective flow was also shown in an experimental investigation made in 1999 by Kjellsson and Hellström [11, 12]. In ref. [13] the BHE design program EED [14] was used to determine the effects of different borehole thermal resistances. For a fictive 15-borehole system, the change in borehole thermal resistance from 0.07 to 0.1 altered the total required borehole length by more than 200 m, which is an extra borehole. Simulations of groundwater-filled BHEs therefore require that the induced convective flow is included in the model. Most existing models only included conductive heat transfer and it is therefore of interest to study how the common model approximations used in this paper affect total heat transfer calculations.

One common approximation is to disregard the fluid flow inside the collector and instead use a boundary condition at the outer U-pipe wall, most commonly constant heat flux. Two boundary conditions given at the U-pipe wall were investigated; constant temperature (cTpw) and constant heat flux (cq"pw). The comparison was performed for average temperature values at the pipe and borehole wall as well as water thermal resistance. It was shown that for total heat transfer calculations (THT), the choice of boundary condition at the pipe wall hardly changed the result at all. Even though the temperature distribution differed in and along the borehole for the two boundary conditions, the mean values over the borehole lengths yielded almost the same result. The choice of boundary condition using only conductive heat transfer (CHT) resulted in a 60% difference in calculated borehole thermal resistance, with higher values using constant heat flux at the pipe wall. The U-pipe geometry receives an un-radial heat pattern in and around the borehole, since the two U-pipe legs function as two separate heat sources, which is more obvious for CHT calculations. A radial pattern will be established at a certain distance from the centre of the borehole (r_radial). For total heat transfer calculations (THT), the convective flow will decrease the temperature differences in and around the borehole water. The radial pattern is therefore achieved at a distance smaller than 0.2 m for both boundary conditions for THT. For only conductive heat transfer (CHT), the un-radial pattern is visible further out in the bedrock and more evident for constant temperature at the pipe wall. The radial pattern for cTpw is established at approximately 3 times the distance found for THT calculations and cq"pw results in 1.5 times the THT. This might be used by reducing the radius of the surrounding bedrock in future models.

Another common approximation for the U-pipe model (M_u) is using one pipe centred in the middle of the borehole instead of the U-pipe, the Equivalent radius model (M_eq). This approximation is shown to be valid for both total heat transfer (THT) and conductive heat transfer (CHT) using both boundary conditions (cTpw & cq"pw), if the appropriate equivalent radius is used. The differences between the two models were as small as 7% for THT and only 1% for CHT. The Equivalent radius model (M_eq) may easily be converted to a 2D axisymmetric model, which reduces the required number of cells considerably and thereby the computational constraints.

As has been shown in several papers [2-6], the choice of the equivalent radius (r_eq) in the Equivalent radius model is dependent on several parameters. In these simulations the two models and boundary conditions required different equivalent radii in order to get the same result as the U-pipe model. For total heat transfer (THT), the equivalent radius should be chosen, so that the heat transfer area is the same as for the two U-pipe legs (r_eq=0.04m). This is valid for both boundary conditions. For only conductive heat transfer (CHT) using constant heat flux at the pipe wall (cq"pw) the suitable r_eq is, as stated in the literature, to achieve the same cross-section area as for the two pipe legs (r_eq=0.0283 m). For constant temperature at the pipe wall (cTpw) and CHT, an equivalent radius of 0.0355 was shown to be appropriate. When constructing a new Equivalent radius model using different conditions, the value of r_eq should be chosen with great care.

So, when constructing a BHE model, convective heat flow must be included when modelling groundwater-filled boreholes. It is also appropriate to use the equivalent radius model if only a liquid or solid state is modelled. If both states (liquid and solid) are to be simulated e.g. during freezing conditions, each state requires a different equivalent radius. For total heat transfer calculations the choice of boundary condition (constant temperature or constant heat flux) does not affect the result, while by using only conductive heat transfer large differences are achieved.
References
