Nyquist's theorem in active vibration control system of conservative and non conservative pipes conveying fluid

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Received 22 Dec. 2019; Received in revised form 15 Mar. 2020; Accepted 25 Mar. 2020; Available online 1 June 2020

Abstract
Fluid conveying pipes are used in all hydraulic systems and enter in all industrial fields, such as water, petroleum products and gases of all kinds. In spite of this widespread and critical importance of fluid conveying pipes, they suffer from major problems. One of these problems is the problem of vibrations that cause the collapse of the systems completely and cause significant economic losses if not avoided. For this reason, the researchers have dealt with this issue in all years, but the problem is not over. This research will highlight the problem of controlling the vibrations resulting from fluid flow inside the pipes, in addition to the study of reducing the vibrations of these pipes. Therefore, after investigation the control and stability for pipe with various methods, with previous papers, presenting in this paper investigation for active control for pipe by using Nyquist’s method. There, the research has studied the response and the natural frequency; the dynamic behavior of conservative and non-conservative pipes conveying fluid using Nyquist's Theorem in the presence with no hydraulic damping (active control); and monitoring the response and stability of each case of stabilization. This study is carried out by deriving differential equations for pipes and for different types of fixation. Where, the analytical solution included drive the general equation of motion for pipe conveying fluid and then solving its equation by using Galerkin’s technique, and then, calculating the stability by using Nyquist’s techniques.

Keywords: Nyquist’s Theorem; Active vibration control; Pipe conveying fluid; S-plane, SISO; Single input and output; MIMO, Multiple input and output systems; Frequency range.

1. Introduction
Systems pipes of conveying fluid is widely used in aircraft power plants, ships, nuclear industry, oil, and energy industry, metallurgy industry, power industry, biological engineering, marine engineering, and in everyday life. The main purpose of the pipes is to transmit energy or energy flow, mass liquid flow. Every year huge economic losses occur in advanced countries is caused due to the vibration of pipes. The annual damage due to vibrations in pipelines in developed countries is estimated at 10 billion dollars, according to estimates by Canadian experts. Therefore, research on the study of vibrations in fluid pipe conveying fluid systems has engineering and economic value. In all industries pipes are used to transport fluids of all kinds despite this proliferation and extreme importance of the pipes these pipes are used in the different fields of the system and the presence of vibrations, which leads to damage to the systems or damage the pipes in addition to the resulting noise therefore, it is necessary to find control systems to reduce the vibrations generated in the pipes.
Some methods can be used to control vibrations. These control methods reduce the dynamic forces that cause vibration. However, these methods can not completely terminate these forces. The practical situations have shown that the most important methods of control are, [1],
1. Reducing the transmission of the forces of excitation from part of the machine to the machine by vibration isolators.
2. Through input of mechanism of the damping process or energy-dissipation, the attenuation system can show extravagant response, even in the status of resonance.
3. Control of natural system frequencies and elusion resonance under the influence of external excitations.
4. Decreasing the system response through adding an assistant mass neutralizer or a vibration absorber. Therefore, various researchers investigation the effect of flow fluid induced pipe on vibration and control behavior with various technique, as,
At, 1981, L. Thompson et. al., [2], studied an elastic pipe in a balanced arrangement of arbitrary giant, without taking into account the rotating mechanism of Coriolis force on the pipe length because of its movement within the static examination. They terminated that a discharged pipe with end forces and moments is statically resembling a beam or strut with an equivalent end forces and moments and the reversed momentum vector \( \rho Au^2 \) that is the similar to the static centrifugal and resistance drag forces. Also at same year, B. Laither et. al., [3], derived the equation of the motion, using Hamilton’s principle, the representation of Hamilton’s principle gained the 1st for an open system, wherever the instant total mass of that will not essentially stay constant within the system. They derived the equation of motion severally for clamped tube and cantilever system, and once more for cantilever tube with the system thought-about to be quasi-losed, wherever all the effects of flow induced were included during the default work as if they were all the forces of external. They compared the ensuing equation with those using the Newton force balance process. Then, at 1988, G. Kuiken, [4], mentioned one plugged pipeline exposure to an oscillating flow, the structure interaction of fluid, taking under consideration not solely flow conditions at the inlet and outlet of the tube and also the pressure, but also B.C. for the longitudinal and radial tube movement at the inlet and outlet. It absolutely was shown that interaction of fluid-structure appear by permitting the tube to maneuver release in the direction of axial at the outlet ought to be taken under consideration so as to get an actual analysis for the response of frequency at a pipeline. In its follow-up, it absolutely was not economically good to install all pipes stiffly in a network of a pipeline. Designers of systems for pipeline ought to be conscious in that the primary resonances in non-rigid installed systems happen at a lot of lower frequencies than the kind of the system which is fully installed. In addition, at same year, A. Misra et. al., [5-6], looked into the curvilinear pipes vibrations by the technique of finite element, and three models were developed: (1) inextensible modified theory, (2) extensible theory and (3) inextensible theory. In pipes of each ends supported, the anticipated results by protractile theory area unit very getting ready to the results of non-extensible changed one. Then, at 1993, M. P. Paidousiss et. al., [7], have known and studied the vital variables for fluid flow within the pipe (pipe mass, flow rate, fluid mass, the internal pressure of the fluid). The most inference of this analysis was that the natural frequency increases if the speed of the fluid is reduced. Additionally, their square measure sure speeds where pipes lose their stability and reckoning on the kind of pipe installation. Then, at 2001, X. Wang et al., [8], studied the static and dynamic instabilities of submerged and inclined concentrically pipes transporting fluid. The equation governing the interior hollow beam was by product below the tiny deformational assumption. They earned the dynamic discretized equations employing schemes of abstraction finite-difference. In steady flow, each buckling and flutter instabilities were examined. Also, at 2002, O. Doare et. al., [9], created the relationship between the global and local motions of bending for fluid pipes conveying on a flexible floor or elastic foundation. The process of global imposed a pipe of finite with a given collection of boundary conditions, while in the process of local it referred to a pipe which has infinite length with a note without taking in consideration its finite ends. Various kinds of diffusion disturbances are specific from the relation of dispersion, namely evanescent, unstable and neutral waves, such as the pipe length which is increased, the global base for instability which is established to coincide with the neutrality of local, by dint of neutral waves generated only a local harmonic impact. Also, the collection of boundary conditions give lift only to instabilities of static. The process for global instability of the very long pipe is neutral static waves present. Contrariwise, for the collection of boundary conditions which permit dynamic instabilities, the process for global instability of the very long pipe is compatible for the presence of indifferent waves of nonzero finite frequency. These results are discussed in a relationship with the theory of Kulikovskii work and another comparable process.

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in the theory of hydrodynamic stability. After this, at 2006, J. Lumijärvi, [10], treated with the optimal design for conveying pipes for cantilevered type fluids. The goal is to maximize the critical fluid velocity of the fluid by additional masses or springs or support dampers over the pipe length. And the improvement problem was similar to the one by FEM, using Euler-Bernoulli for beam elements. These elements (damping constant, mass and spring constant) were selected as design parameters. A significant increase in the velocity of critical flow for the fluid realized in cases studied the remarkable sensitivity for the system to the properties extra elements and locations were observed in optimal designs. Also, the footnote relatively to stability seems somewhat small in some optimum designs that are being considered.

Then, at 2017, O. Kavianipour, [11], addressed the uniform cantilever for Euler-Bernoulli at beam exposure to transversal and follower force at the free end for uniform cantilever as such a model for a pipe fluid conveying under damper of the electromagnetic force. The damper of electromagnetic consists of DC motor, a permanent-magnet, a nut , and a ball screw. The main aim of the present work is to reduce the pipe vibration eventual from the velocity of the fluid and allow it to transform to the energy of electric. To implement this goal, the vibration and stability of the beam sample were studied using Newmark and Ritz methods. It was noticed that increasing the velocity of fluid results in a decrease in the movement of the free end for the pipe . The simulation showed the results which the designed damper controlled of electromagnetic semi-active by the control strategy off-on damping where the vibration amplitude was lowered for the pipe around 5.9 %, energy of regenerated about 1.9 (MJ/sec). It showed also that the designed electromagnetic damper semi-active has a best energy regeneration and performance more than the electromagnetic damper passive. Also, at 2017, M. J. Jweeg et. al., [12], investigation the dynamic behavior for pipe conveying fluid with various crack depth and position effect, by using analytical technique. In addition, also at same year, M. Al-Waily et. al., [13], investigated the effect of crack angle on the vibration characterizations for pipe by using experimental and numerical techniques. Finally, at 2019, D. Hussein et. al., [14], studied the active control for vibration behavior of pipe conveying fluid by using state space technique. Where, the investigation included drive the general equation of motion for vibration pipe conveying fluid and then solving its equation by using state space method to calculate the active control for vibration. There, the results included study for different flow parameters on the vibration control, in addition to, various damper parameters on the vibration behavior for pipe. Also, at same year and same researchers, presented investigation for frequency domain for pipe with various flow and damper parameters by using bode diagram technique, [15]. Therefore, this paper is extension for work presented in [14, 15], which investigation the active damper control for vibration pipe with various flow parameters effect and different damper parameters by using Nyquist's technique. Where, the investigation included drive for vibration pipe equation of motion and using Galerkin’s method to calculate the general behavior for pipe, and then, calculating the stability for pipe with various active damper parameters effect by using Nyquist's technique.

2. Analytical Investigation

A theoretical analysis of the dynamical behavior and vibration of fluid conveying pipes will be offered in this study. For clarity, at the beginning, studies dynamics, and stability of pipe conveying fluid and then studies the active vibration control of pipes fluid conveying. So as to conduct an overall investigation into the dynamic behavior of pipes conveying fluid, a lot of solutions will be attempted. Based on former researchers, the task will focus on the analyses of the stability of the zero balance status of fluid conveying pipes under pulsating flow. Primarily, the motion differential equation of pipes is built, secondly, the differential equation of system state is gained through reduced order treatments, discretization and dimensionless. Then the average method in used to forerun average treatment on the differential equation of system state, based on the averaged independent equation, analyzing the stability of zero balance status. The vibration characteristics for non-conservative and conservative fluid conveying pipes will be investigated. Estimates of the natural frequencies for pipes is the essential purpose of this analysis, [16-23]. For that, there is a needed to gain the linear equations of motion in the district for the position of equilibrium. When the pulsating flow and external force are neglected and the pipe is in the steady state, the linearized equation will be, [14],

\[ \ddot{\eta} + 2M_u \dot{\eta} + [u_\sigma + \Pi] \eta'' + \eta^{(4)} = 0 \]  

(1)

The Eq. (1) is inhomogeneous where the derivative coefficients of \( \eta \) are frank functions of \( \tau \) and \( \xi \) then the discretized equation of motion above, by using the Galerkin’s way let,

\[ \eta(\xi, \tau) = \sum_{i=1}^{\infty} \Phi_i(\xi) \eta_i(\tau) \]  

(2)
Where, $\phi_i(\xi)$ is an comparison function, $q_i(\tau)$ is an generalized coordinate where they satisfy all the boundary conditions. Choose the first three orders to manage researches, that is,

$$
\eta(\xi, \tau) = \sum_{i=1}^{3} \phi_i(\xi) q_i(\tau) = \phi_1(\xi) q_1(\tau) + \phi_2(\xi) q_2(\tau) + \phi_3(\xi) q_3(\tau)
$$

(3)

where, $\phi_i$ are vibration model depending on the pipe supported, then,

I. For pipes pinned at both ends, the function of its vibration model is,

$$
\phi_i = \sqrt{2} \sin(\lambda_i \xi), \quad i = 1, 2, 3
$$

(4)

Where, $\lambda_1 = \pi$, $\lambda_2 = 2\pi$, $\lambda_3 = 3\pi$, which, $\lambda_1$, $\lambda_2$ and $\lambda_3$ are pipe eigenvalues.

II. For pipes fixed at both ends, the function of its vibration model is,

$$
\phi_i = \cosh(\lambda_i \xi) - \cos(\lambda_i \xi) + \frac{\cosh(\lambda_i) - \cos(\lambda_i)}{\sin(\lambda_i) - \sinh(\lambda_i)} [\sin(\lambda_i \xi) - \sinh(\lambda_i \xi)], \quad i = 1, 2, 3
$$

(5)

Where, $\lambda_1 = 4.73$, $\lambda_2 = 7.8532$, $\lambda_3 = 10.9956$

III. For pipes pinned at one end and fixed at another end, the function of its vibration model is,

$$
\phi_i = \cos(\lambda_i \xi) - \cosh(\lambda_i \xi) + \frac{\cos(\lambda_i) - \cosh(\lambda_i)}{\sin(\lambda_i) - \sinh(\lambda_i)} [\sinh(\lambda_i \xi) - \sin(\lambda_i \xi)], \quad i = 1, 2, 3
$$

(6)

Note that $\lambda_1 = 3.9267$, $\lambda_2 = 7.0686$, $\lambda_3 = 10.2102$.

IV. For pipes (cantilever), the function of its vibration model is,

$$
\phi_i = \cosh(\lambda_i \xi) - \cos(\lambda_i \xi) + \frac{\sinh(\lambda_i) - \sin(\lambda_i)}{\cosh(\lambda_i) + \cos(\lambda_i)} \lambda_i [\sinh(\lambda_i \xi) - \sin(\lambda_i \xi)], \quad i = 1, 2, 3
$$

(7)

Note that $\lambda_1 = 1.87512$, $\lambda_2 = 4.6941$, $\lambda_3 = 7.85476$.

Then, Eq. (2) is converted into matrix type, assuming,

$$
\Phi = \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{bmatrix}, \quad Q = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix}
$$

Then,

$$
\eta(\xi, \tau) = \Phi^T Q = \mathbf{q}^T \Phi
$$

(8)

By compensation of Eq. (8) into Eq. (1), and assuming, $H = u_0^2 + \Pi$, then,

$$
\phi^T \dot{Q} + 2M_1 u_0 \phi^T \dot{Q} + \Pi \phi^T \dot{Q} + \phi^{(4)T} Q = 0
$$

(9)

By multiplying $\Phi = \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{bmatrix}$ with two sides of Eq. (9) and then,

$$
\phi \phi^T \dot{Q} + 2M_1 u_0 \phi \phi^T \dot{Q} + \Pi \phi \phi^T \dot{Q} + \phi \phi^{(4)T} Q = 0
$$

(10)

The procedure $\xi$ integral to Eq. (10) within interval $[0, 1]$, then the representation according to orthogonality for the function of trigonometric,

$$
\int_0^1 \phi \phi^T d\xi = I = \begin{bmatrix} \int_0^1 \phi_1 \phi_1^T \\ \int_0^1 \phi_2 \phi_2^T \\ \int_0^1 \phi_3 \phi_3^T \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}
$$

$$
\int_0^1 \phi \phi^T d\xi = B = \begin{bmatrix} \int_0^1 \phi_1 \phi_1^T \\ \int_0^1 \phi_2 \phi_2^T \\ \int_0^1 \phi_3 \phi_3^T \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}
$$

$$
\int_0^1 \phi \phi^T d\xi = C = \begin{bmatrix} \int_0^1 \phi_1 \phi_1^T \\ \int_0^1 \phi_2 \phi_2^T \\ \int_0^1 \phi_3 \phi_3^T \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix}
$$
\[
\int_0^1 \Phi \Phi^T d\xi = \Lambda = \begin{pmatrix}
\int_0^1 \Phi_1 \Phi_1^T & \int_0^1 \Phi_1 \Phi_2^T & \int_0^1 \Phi_1 \Phi_3^T \\
\int_0^1 \Phi_2 \Phi_1^T & \int_0^1 \Phi_2 \Phi_2^T & \int_0^1 \Phi_2 \Phi_3^T \\
\int_0^1 \Phi_3 \Phi_1^T & \int_0^1 \Phi_3 \Phi_2^T & \int_0^1 \Phi_3 \Phi_3^T
\end{pmatrix}
\] (11)

The specific boundary conditions are \( \Phi_1, \Phi_2 \) and \( \Phi_3 \) which are the first three mode functions, then,

I. For pipes pinned at both ends, the B and C matrices are,

\[
B = \begin{pmatrix}
0 & -2.6667 & 0 \\
2.6667 & 0 & -4.8 \\
0 & 4.8 & 0
\end{pmatrix},
C = \begin{pmatrix}
-(\pi^2) & 0 & 0 \\
0 & -(2\pi^2) & 0 \\
0 & 0 & -(3\pi^2)
\end{pmatrix}
\]

II. For fixed pipes at both ends, the matrix B and C are,

\[
B = \begin{pmatrix}
0 & -3.3421 & 0 \\
3.3421 & 0 & -5.5161 \\
0 & 5.5161 & 0
\end{pmatrix},
C = \begin{pmatrix}
-12.3028 & 0 & 9.7315 \\
0 & -46.0501 & 0 \\
9.7315 & 0 & -98.9047
\end{pmatrix}
\]

III. For pipes pinned at one end and fixed at another end, the matrix B and C are,

\[
B = \begin{pmatrix}
0 & -2.9965 & 0.3167 \\
2.9965 & 0 & -5.1468 \\
-0.3167 & 5.1468 & 0
\end{pmatrix},
C = \begin{pmatrix}
-11.5126 & 4.2814 & 3.7993 \\
4.2814 & -42.8964 & 7.8191 \\
3.7993 & 7.8191 & -94.0376
\end{pmatrix}
\]

IV. For pipe (cantilever), the B and C matrices are,

\[
B = \begin{pmatrix}
0.75948 & 2 & 2.22218 \\
2 & -4.75948 & 3.78433 \\
0.21566 & 2.22218 & 2
\end{pmatrix},
C = \begin{pmatrix}
0.8581 & -11.7433 & 27.4531 \\
27.4531 & -9.04205 & -45.9043
\end{pmatrix}
\]

Then, by using Eq. (11), after the reduced order through Eq. (10), the discretized equation is shown below, as,

\[
IQ + 2M_\tau u_0BQ + (CH + \Lambda)Q = 0
\] (12)

Where,

\[
\begin{align*}
\dot{q}_1 &= \begin{pmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \end{pmatrix} \\
\dot{q}_2 &= \begin{pmatrix} \ddot{q}_2 \\ \ddot{q}_2 \\ \ddot{q}_3 \end{pmatrix} \\
\dot{q}_3 &= \begin{pmatrix} \ddot{q}_3 \\ \ddot{q}_3 \\ \ddot{q}_3 \end{pmatrix}
\end{align*}
\]

When, \( \dot{Q} = \Omega i, \ddot{Q} = -\Omega^2 \).

Then, Eq. (12) become,

\[
(-\Omega^2 + 2M_\tau u_0B\Omega i + (CH + \Lambda))Q = 0
\] (13)

Or,

\[
\begin{pmatrix}
\ddot{s}_{11} & \ddot{s}_{12} & \ddot{s}_{13} \\
\ddot{s}_{21} & \ddot{s}_{22} & \ddot{s}_{23} \\
\ddot{s}_{31} & \ddot{s}_{32} & \ddot{s}_{33}
\end{pmatrix} = 0
\] (14)

Where,

\[
\begin{align*}
s_{11} &= \lambda_1^3 + Hc_{11} + 2M_\tau u_0b_{11}\Omega i - \Omega^2, \\
s_{12} &= Hc_{12} + 2M_\tau u_0b_{12}\Omega i, \\
s_{13} &= Hc_{13} + 2M_\tau u_0b_{13}\Omega i, \\
s_{21} &= Hc_{21} + 2M_\tau u_0b_{21}\Omega i, \\
s_{22} &= \lambda_2^3 + Hc_{22} + 2M_\tau u_0b_{22}\Omega i - \Omega^2, \\
s_{23} &= Hc_{23} + 2M_\tau u_0b_{23}\Omega i, \\
s_{31} &= Hc_{31} + 2M_\tau u_0b_{31}\Omega i, \\
s_{32} &= Hc_{32} + 2M_\tau u_0b_{32}\Omega i, \\
s_{33} &= \lambda_3^3 + Hc_{33} + 2M_\tau u_0b_{33}\Omega i - \Omega^2
\end{align*}
\]

By setting \(|S|\) equal to zero, it is possible to evaluate the natural frequency (\(\Omega\)). The following characteristic equation comes from the expansion of determinant above,

\[
\Omega^6 - k_5\Omega^5 - k_4\Omega^4 - k_3\Omega^3 - k_2\Omega^2 - k_1\Omega - k_0 = 0
\] (15)

Where, the constants \(k_0, k_1, k_2, k_3, k_4, k_5\) depend on the boundary conditions of pipe, and can be using as listed in the Table 1.
Table 1. Parameter Constants for Pipe with Various Boundary Conditions Supported.

<table>
<thead>
<tr>
<th>Parameter Constants</th>
<th>Pinned-Pinned Pipe Supported</th>
<th>Clamped-Clamped Pipe Supported</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_0$</td>
<td>$\left(-34609.9H^3 + 0.478222 \times 10^7 H^2 - \frac{0.165195 \times 10^9 H + 0.119786 \times 10^{10}}{}\right)$</td>
<td>$\left(-51673H^3 + 0.148296 \times 10^8 H^2 - \frac{0.120933 \times 10^{10} H + 0.278330 \times 10^{11}}{}\right)$</td>
</tr>
<tr>
<td>$k_1$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$k_2$</td>
<td>$\left(\frac{+3436.55M^2u_0^2H - 233439M^2u_0^2}{4773.04H^2 + 555683H - 0.132176 \times 10^8}\right)$</td>
<td>$\left(\frac{+4481.05M^2u_0^2H - 714029M^2u_0^2}{6243.22H^2 + 0.134854 \times 10^7 H - 0.648214 \times 10^8}\right)$</td>
</tr>
<tr>
<td>$k_3$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$k_4$</td>
<td>$\left(-138.175H + 9546.1 + 120.608M^2u_0^2\right)$</td>
<td>$\left(-157.258H + 18922 + 166.388M^2u_0^2\right)$</td>
</tr>
<tr>
<td>$k_5$</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter Constants</th>
<th>Clamped-Pinned Pipe Supported</th>
<th>Cantilever Pipe Supported</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_0$</td>
<td>$\left(-43139.2H^3 + 0.877896 \times 10^7 H^2 - \frac{0.478995 \times 10^9 H + 0.645029 \times 10^{10}}{}\right)$</td>
<td>$\left(+441.872H^3 + 8265.4H^2 + \frac{684784 H + 0.228488 \times 10^9}{7604890 + 0.107M_ru_0}\right)$</td>
</tr>
<tr>
<td>$k_1$</td>
<td>$\left(-0.0613238TM_ru_0 + 0.0057^2M_ru_0\right)$</td>
<td>$\left(-154473TM_ru_0 + 1835.27T^2M_ru_0 + \frac{567.72H^2 + 69941.7H}{0.190122 \times 10^7}\right)$</td>
</tr>
<tr>
<td>$k_2$</td>
<td>$\left(\frac{+4030.57M^2u_0^2H - 416514M^2u_0^2}{5516.43H^2 + 887358H - 0.303083 \times 10^8}\right)$</td>
<td>$\left(\frac{+565.864M^2u_0^2H - 123009M^2u_0^2}{567.72H^2 + 69941.7H - 0.190122 \times 10^7}\right)$</td>
</tr>
<tr>
<td>$k_3$</td>
<td>$\left(+0.003TM_ru_0 - 0.001M^3u_0^3\right)$</td>
<td>$\left(+374.357TM_ru_0 - 432.196M^3u_0^3\right)$</td>
</tr>
<tr>
<td>$k_4$</td>
<td>$\left(-148.447H + 13601.8 + 142.275M^2u_0^2\right)$</td>
<td>$\left(-58.3405H + 4304.44\right)$</td>
</tr>
<tr>
<td>$k_5$</td>
<td>0</td>
<td>$\left(\frac{+114.502M^2u_0^2}{+12M_ru_0}\right)$</td>
</tr>
</tbody>
</table>

Nyquist developed a standard for the stability of systems in the frequency domain depending on the theory of Cauchy, where this standard creates Nyquist schemes for hydraulic systems. This system can be continuous or discrete, and SISO or MIMO. In the case of MIMO, the Nyquist produces an array of Nyquist plots. These schemes can determine the appropriate frequency within the system frequency range and focus on an appropriate frequency interval by adjusting the vector to the desired frequencies. To encircle the poles and zeros of the system that is located on the right side of the s-plane, is created a Nyquist contour, as shown in Fig. 1. Fig. 2.a, shows the function plane with a gain with two clockwise encirclements. However, if the contour is plotted in function plane, as shown in Fig. 2.b, then one unit moves to the left, and the standard is a critical point or a standard point ($-1, j0$). Note that the number of encirclements is equal to the number of poles. The analysis of the system characteristics in this topic will be discussed in addition to the gain margin of the system and the response, the phase margin of the system as well as the stability for each case of pipe fixation. The analysis of the system characteristics in this topic will be studied in addition to the gain margin of the system and the response, the phase margin of the system as well as the stability for each case of pipe fixation. A frequency domain stability criterion developed by Nyquist (1932) is based upon Cauchy's theorem. If the function $F(s)$ is in fact the characteristic equation of a closed-loop control system, then,

$$F(s) = 1 + G(s)H(s)$$  \(16\)

Note that the roots of the characteristic equation are the closed-loop poles, which are the zeros of $F(s)$. 

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Figure 1. S-Plane Nyquist Contour.

Figure 2. Contours in S-Plane with and without Gain.

The closer the open-loop frequency response locus $G(j\omega)H(j\omega)$ is to the $(1, j0)$ point, the nearer the closed-loop system is to instability, as shown in Fig. 3. Systems possess a margin of stability, generally referred to as gain and phase margins, these are shown in Fig. 4. Which Gain Margin (GM), the gain margin is the increase in open-loop gain required when the open-loop phase is -180 to make the closed-loop system just unstable, can be calculating from,

$$GM = \frac{1}{|G(j\omega)H(j\omega)|_{180}}$$

(17)

And, Phase Margin (PM): The phase margin is the change in open-loop phase, required when the open-loop modulus is unity, to make the closed-loop system just unstable, calculating from,

$$\text{Phase Margin} = 180 - \angle G(j\omega)H(j\omega) \pmod{1}$$

(18)

The Nyquist stability criterion can be stated as, 'A closed-loop control system is stable if, and only if, a contour in the $G(s)H(s)$ plane describes a number of counterclockwise encirclements of the $(-1, j0)$ point, the number of encirclements being equal to the number of poles of $G(s)H(s)$ with positive real parts, as shown in Fig. 5. Hence, because there is a net clockwise encirclement of the $(-1, j0)$ point in Fig. 2.b, the system is unstable. If, however, there had been a net counter-clockwise encirclement, the system would have been stable, and the number of encirclements would have been equal to the number of poles of $G(s)H(s)$ with positive real parts. For the condition $P = 0$, the Nyquist criterion is, 'a closed-loop control system is stable if, and only if, a contour in the $G(s)H(s)$ plane does not encircle the $(-1, j0)$ point when the number of poles of $G(s)H(s)$ in the right-hand s-plane is zero. Only the frequencies $\omega = 0$ to $+\infty$ are of interest and since in the frequency domain $s = j\omega$, a simplified Nyquist stability criterion, as shown
in Fig. 3 is, a closed-loop system is stable if, and only if, the locus of the \( G(j\omega)H(j\omega) \) function does not enclose the \((-1, j0)\) point as \( \omega \) is varied from zero to infinity. Enclosing the \((-1, j0)\) point may be interpreted as passing to the left of the point. The \( G(j\omega)H(j\omega) \) locus is referred to as the Nyquist Diagram.

In equation (15) and Equation (17), by setting \(|S|\) equal to zero, it is possible to evaluate the natural frequency (\( \Omega \)), will be used to find the results of Nyquist diagram, this the analytical solution is exact solution, then, can be dependent on results calculating, [24-30].

![Nyquist Diagram](image1)

**Figure 3.** Nyquist diagram showing stable and unstable contours.

![Gain Margin (GM) and Phase Margin (PM) on the Nyquist diagram](image2)

**Figure 4.** Gain margin (GM) and phase margin (PM) on the Nyquist diagram.

![Closed-loop control system](image3)

**Figure 5.** Closed-loop control system.

### 3. Results and Discussion

The results for this work included investigation for active vibration control for pipe conveying fluid by using active damper, with using Naquist technique. When, the study included derive for general equation of motion for pipe conveying fluid, and then, solving equation of motion, [31-36], by using Naquist theory to calculate the stability for pipe with various parameter effect. Which, the parameters studied were the parameters for active damper, in addition to, investigation for fluid flow pressure effect on pipe stability.

Therefore, the following listing for parameters studied were, effect of the hydraulic damper position, as shown in Fig. 6, effect of the base width of hydraulic damper, as shown in Fig. 7, effect of the damping coefficient, as shown in Fig. 8, and, effect of pressure, as shown in Fig. 9. Then, to agreement for the results calculated, must be comparison with other results evaluated by other technique, [37-45], there, results comparison by results presented by bode diagram and state space techniques, [14, 15], and find a good agreement for results calculated.

Therefore, Fig. 6, shows the s-plane where the x-axis is a real axis and the y-axis is an imaginary axis. From these two axes, the standard for the critical point or the standard point is established so that the standard of the closed-loop active control system can be defined in terms of the number of encirclements in the direction and counterclockwise of the point mentioned above. Different damper locations will be chosen to indicate this, with \( u_0 = 1 \), \( \alpha = 0.01 \), and base width of hydraulic damper (\( \Delta \xi = 0.1 \)). For the
pipe fixed by a pin on both sides, the highest gain at $\xi_a = 0.488$ and this occurs at low frequency and the lowest gain at $\xi_a = 0.647$. Note that the encirclement does not surround the critical point and rotates clockwise and passes the circle to the right of this point, which indicates the stability of the system. The highest stability of the system occurs at $\xi_a = 0.488$ to approach the encirclements of the critical point. As for the fixed pipe on the one hand and the pinned on the other, the minimum gain occurs at $\xi_a = 0.647$. The furthest gain and response is at $\xi_a = 0.488$ and this is almost identical to the fixed pipe on both sides. As for the cantilever pipe, the highest gain at $\xi_a = 0.488$ and the lowest gain and the lowest response at $\xi_a = 0.353$; it is the location of the damper at mid-distance. This makes sense in such pipe fixations. Also, note that all the encirclements rotate clockwise and do not surround the critical point or standard point ($-1, j0$). This shows the stability of all the damper positions mentioned.
c. Clamped-Clamped Pipes.

d. Cantilever Pipes.

Figure 6. Nyquist Diagram of Pipes at Different Damper Positions.

Fig. 7, shows the s-plane where the x-axis is a real axis and the y-axis is an imaginary axis. From these two axes, the standard for the critical point or the standard point is established so that the standard of the closed-loop active control system can be defined in terms of the number of encirclements in the direction and counterclockwise of the point mentioned above. The different width of the base of the damper will be chosen ($\Delta \xi$), with the same survival locations previously selected for the damper, with $u_o = 1$, $\alpha = 0.01$, and damper location $\xi_a = 0.5$.

For the (p-p) pipe, the highest gain at $\Delta \xi = 0.2$ at low frequency and the lowest gain at $\Delta \xi = 0.05$. Note that the encirclement does not surround the critical point and rotates clockwise and passes the circle to the right of this point. Which indicates the stability of the system. The highest stability of the system occurs at $\Delta \xi = 0.2$ to approach the encirclements of the critical point. As for the (c-p) pipe, the highest gain occurs at $\Delta \xi = 0.2$ as. The furthest gain also occurs at $\Delta \xi = 0.05$. In the case of the fixed pipe from both sides, note that the encirclements are expanding. This indicates an increase in gain with the approach of the circle at $\Delta \xi = 0.2$ to the critical point. This is an indicator of increased stability compared to other cases. As for the cantilever pipe, the highest gain occurs at $\Delta \xi = 0.2$ and the lowest gain at $\Delta \xi = 0.05$, with a significant difference between the two gain. This is an indication of the significant impact of the cantilever pipe in the amount of $\Delta \xi$. 
a. Pinned-Pinned Pipes.

b. Clamped-Pinned Pipes.

c. Clamped-Clamped Pipes.
The effect for different coefficient of damping will be studied and discussed, with $u_0 = 1$, $\xi_a = 0.5$, and base width of hydraulic damper ($\Delta \xi = 0.15$), as shown in Fig. 8. For the pipe fixed by a pin on both sides, the highest gain occurs at $\alpha = 0.1$ and the lowest gain at $\alpha = 0.3$. As for the fixed pipe on the one side and the pinned on the other, the highest gain occurs at $\alpha = 0.1$. The furthest gain is at $\alpha = 0.1$, and this is similar to fixing the fixed pipe from both ends except that the gain for the last fixation is wider. As for the cantilever pipe, the highest gain occurs at $\alpha = 0.1$ and the lowest gain and the lowest response at $\alpha = 0.3$. As for all pipe fixation cases it is found that the highest gain at $\alpha = 0.1$ and the gain was significantly reduced when the damping increased. The effect for different pressure values will be discussed, Fig. 9, with $\alpha = 0.01$, and base width of hydraulic damper ($\Delta \xi = 0.1$), $\xi_a = 0.5$. For the (p-p) pipe, the highest gain occurs at $\Pi = 4.5$ when low frequency and the lowest gain at $\Pi = 1$. Note that the encirclement does not surround the critical point and rotates clockwise and passes the circle to the right of this point, which indicates the stability of the system. As for the (c-p) pipe, the highest gain occurs at $\Pi = 4.5$. The furthest gain occurs at $\Pi = 1$. In the case of the fixed pipe from both sides, note that the encirclements are expanding. This indicates an increase in gain with the approach of the circle at $\Pi = 4.5$ to the critical point and it is an indicator of the decrease in stability compared to other cases. As for the cantilever pipe, the highest gain occurs at $\Pi = 4.5$ and the lowest gain at $\Pi = 1$. It is observed that the approximate amount of peak gain at pressure equal to 1 with pressure being equal to 3, with higher gain height at pressure being equal to 4.5 for this type of fixation of pipes.
b. Clamped-Pinned Pipes.

c. Clamped-Clamped Pipes.

d. Cantilever Pipes.

Figure 8. Nyquist Diagram for Pipes at Different $\alpha$. 
a. Pinned-Pinned Pipes.

b. Clamped-Pinned Pipes.

c. Clamped-Clamped Pipes.
4. Conclusion
From the analytical solution of general equation of motion for pipe induced vibration, by using Nyquist’s technique can be listing the following important point for results calculated,
1. Increase in the proportion of mass less critical speeds and all types of fixations for pipes is seen.
2. When the speed is increased, the control performance decreases due to the increased force of Coriolis.
3. Increase in pressure leads to the failure of pipe systems at certain limits if the results are not calculated with high accuracy.
4. There is a very large convergence in the results of control theories used. The results of the change of the parameters of each fixations are compared with each control theory used and a match is found in stability and response.

References


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