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Natural convection unsteady magnetohydrodynamic mass transfer flow past an infinite vertical porous plate in presence of suction and heat sink

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Abstract
This paper investigates the natural convection unsteady magnetohydrodynamic mass transfer flow of a viscous incompressible electrically conducting fluid past an infinite vertical porous flat plate in presence of constant suction and heat sink. Using multi parameter perturbation technique, the governing equations of the flow field are solved and approximate solutions are obtained. The effects of the flow parameters on the velocity, temperature, concentration distribution and also on the skin friction and rate of heat transfer are discussed with the help of figures and table. It is observed that a growing magnetic parameter or Schmidt number or heat sink parameter leads to retard the transient velocity of the flow field at all points, while the Grashof numbers for heat and mass transfer show the reverse effect. It is further found that a growing Prandtl number or heat sink parameter decreases the transient temperature of the flow field at all points while the heat source parameter reverses the effect. The concentration distribution of the flow field suffers a decrease in boundary layer thickness in presence of heavier diffusive species (growing S_c) at all points of the flow field. The effect of increasing Prandtl number P_r is to decrease the magnitude of skin-friction and to increase the rate of heat transfer at the wall for MHD flow, while the effect of increasing magnetic parameter M is to decrease their values at all points.

Keywords: Natural convection; Magnetohydrodynamic; Mass transfer; Suction; Heat sink.

1. Introduction
The phenomenon of natural convection flow with heat and mass transfer in presence of magnetic field has been given much importance in the recent years in view of its varied applications in science and technology. The study of natural convection flow finds innumerable applications in geothermal and energy related engineering problems. Such phenomena are of great theoretical as well as practical interest in view of their applications in diverse fields such as aerodynamics, extraction of plastic sheets, cooling of infinite metallic plates in a cool bath, liquid film condensation process and in major fields of glass and polymer industries.

In view of the above interests, Hashimoto [1] discussed the boundary layer growth on a flat plate with suction or injection. Sparrow and Cess [2] analyzed the effect of magnetic field on a free convection heat transfer. Gebhart and Pera [3] studied the nature of vertical natural convection flows resulting from the combined buoyancy effects of thermal and mass diffusion. Soundalgekar and Wavre [4] investigated the unsteady free convection flow past an infinite vertical plate with constant suction and mass transfer. Hossain and Begum [5] estimated the effect of mass transfer and free convection on the flow past a vertical plate. Bestman [6] analyzed the natural convection boundary layer flow with suction and mass transfer in a porous medium. Pop *et al.* [7] reported the conjugate MHD flow past a flat plate.

Singh [8] discussed the effect of mass transfer on free convection MHD flow of a viscous fluid. Raptis and Soundalgekar [9] analyzed the steady laminar free convection flow of an electrically conducting fluid along a porous hot vertical plate in presence of heat source/sink. Na and Pop [10] explained the free convection flow past a vertical flat plate embedded in a saturated porous medium. Takhar *et al.* [11] discussed the unsteady flow and heat transfer on a semi-infinite flat plate in presence of magnetic field. Chowdhury and Islam [12] developed the MHD free convection flow of a visco-elastic fluid past an infinite vertical porous plate. Raptis and Kafousias [13] analyzed the heat transfer in flow through a porous medium bounded by an infinite vertical plate under the action of a magnetic field. Sharma and Pareek [14] described the steady free convection MHD flow past a vertical porous moving surface. Das and his co-workers [15] estimated numerically the effect of mass transfer on unsteady flow past an accelerated vertical porous plate with suction. Recently, Das and his associates [16] investigated the hydromagnetic convective flow past a vertical porous plate through a porous medium in presence of suction and heat source.

In the present problem, we analyze the natural convection unsteady magnetohydrodynamic mass transfer flow of a viscous incompressible electrically conducting fluid past an infinite vertical porous flat plate in presence of constant suction and heat sink. Approximate solutions are obtained for the velocity, temperature, concentration distribution, skin friction and the rate of heat transfer using multi parameter perturbation technique and the effects of the important parameters on the flow field are analyzed with the help of figures and a table.

2. Formulation of the problem

Consider the unsteady natural convection mass transfer flow of a viscous incompressible electrically conducting fluid past an infinite vertical porous plate in presence of constant suction and heat sink and a transverse magnetic field B_0. The x'-axis is taken in vertically upward direction along the plate and the y'-axis is chosen normal to it. Neglecting the induced magnetic field and the Joulean heat dissipation and applying Boussinesq's approximation the governing equations of the flow field are given by:

Continuity equation:

$$\frac{\partial v'}{\partial y'} = 0 \quad \Rightarrow \quad v' = -v'_0 \quad \text{(constant)}, \tag{1}$$

Momentum equation:

$$\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y'} = \nu\frac{\partial^2 u'}{\partial y'^2} + g\beta\left(T' - T'_\infty\right) + g\beta^*\left(C' - C'_\infty\right) - \frac{\sigma B_0^2}{\rho}u', \tag{2}$$

Energy equation:

$$\frac{\partial T'}{\partial t'} + v'\frac{\partial T'}{\partial y'} = k\frac{\partial^2 T'}{\partial y'^2} + \frac{\nu}{C_p}\left(\frac{\partial u'}{\partial y'}\right)^2 + S'\left(T' - T'_\infty\right), \tag{3}$$

Concentration equation:

$$\frac{\partial C'}{\partial t'} + v'\frac{\partial C'}{\partial y'} = D\frac{\partial^2 C'}{\partial y'^2}. \tag{4}$$

The initial and boundary conditions of the problem are:

$$u' = 0, v' = -v_0', T' = T_w' + \varepsilon(T_w' - T_\infty')e^{i\omega' t'}, C' = C_w' + \varepsilon(C_w' - C_\infty')e^{i\omega' t'} \text{ at } y' = 0,$$
$$u' \to 0, \quad T' \to T_\infty', \quad C' \to C_\infty' \quad \text{as} \quad y' \to \infty. \tag{5}$$

Introducing the following non-dimensional variables and parameters,

$$y = \frac{y' v_0'}{\nu}, t = \frac{t' v_0'^2}{4\nu}, \omega = \frac{4\nu\omega'}{v_0'^2}, u = \frac{u'}{v_0'}, \nu = \frac{\eta_0}{\rho}, T = \frac{T' - T_\infty'}{T_w' - T_\infty'}, C = \frac{C' - C_\infty'}{C_w' - C_\infty'}, M = \left(\frac{\sigma B_0^2}{\rho}\right)\frac{\nu}{v_0'^2},$$
$$P_r = \frac{\nu}{k}, S_c = \frac{\nu}{D}, G_r = \frac{\nu g \beta (T_w' - T_\infty')}{v_0'^3}, G_c = \frac{\nu g \beta^* (C_w' - C_\infty')}{v_0'^3}, S = \frac{4 S' \nu}{v_0'^2}, E_c = \frac{v_0'^2}{C_p (T_w' - T_\infty')}. \tag{6}$$

in Eqs. (2)-(4) under boundary conditions (5), we get

$$\frac{1}{4}\frac{\partial u}{\partial t} - \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + G_r T + G_c C - Mu, \tag{7}$$

$$\frac{1}{4}\frac{\partial T}{\partial t} - \frac{\partial T}{\partial y} = \frac{1}{P_r}\frac{\partial^2 T}{\partial y^2} + \frac{1}{4} ST + E_c \left(\frac{\partial u}{\partial y}\right)^2, \tag{8}$$

$$\frac{1}{4}\frac{\partial C}{\partial t} - \frac{\partial C}{\partial y} = \frac{1}{S_c}\frac{\partial^2 C}{\partial y^2}, \tag{9}$$

where g is the acceleration due to gravity, ρ is the density, σ is the electrical conductivity, ν is the coefficient of kinematic viscosity, β is the volumetric coefficient of expansion for heat transfer, β^* is the volumetric coefficient of expansion for mass transfer, ω is the angular frequency, η_0 is the coefficient of viscosity, k is the thermal diffusivity, T is the temperature, T'_w is the temperature at the plate, T'_∞ is the temperature at infinity, C is the concentration, C'_w is the concentration at the plate, C'_∞ is the concentration at infinity, C_p is the specific heat at constant pressure, D is the molecular mass diffusivity, G_r is the Grashof number for heat transfer, G_c is the Grashof number for mass transfer, M is the magnetic parameter, P_r is the Prandtl number, , S is the heat sink parameter, S_c is the Schmidt number and E_c is the Eckert number.

The corresponding boundary conditions are:

$$u = 0, T = 1 + \varepsilon e^{i\omega t}, C = 1 + \varepsilon e^{i\omega t} \text{ at } y = 0,$$
$$u \to 0, T \to 0, \quad C \to 0 \quad \text{as} \quad y \to \infty. \tag{10}$$

3. Method of solution

To solve Eqs. (7)-(9), we assume ε to be very small and the velocity, temperature and concentration distribution of the flow field in the neighbourhood of the plate as

$$u(y,t) = u_0(y) + \varepsilon e^{i\omega t} u_1(y), \tag{11}$$

$$T(y,t) = T_0(y) + \varepsilon e^{i\omega t} T_1(y), \tag{12}$$

$$C(y,t) = C_0(y) + \varepsilon e^{i\omega t} C_1(y). \tag{13}$$

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Substituting Eqs. (11) - (13) in Eqs. (7) - (9) respectively, equating the harmonic and non-harmonic terms and neglecting the coefficients of ε^2, we get
Zeroth order:

$$u_0'' + u_0' - Mu_0 = -G_r T_0 - G_c C_0, \tag{14}$$

$$T_0'' + P_r T_0' + \frac{P_r S}{4} T_0 = -P_r E_c \left(\frac{\partial u_0}{\partial y} \right)^2, \tag{15}$$

$$C_0'' + S_c C_0' = 0. \tag{16}$$

First order:

$$u_1'' + u_1' - \left(\frac{i\omega}{4} + M \right) u_1 = -G_r T_1 - G_c C_1, \tag{17}$$

$$T_1'' + P_r T_1' - \frac{P_r}{4}(i\omega - S) T_1 = -2 P_r E_c \left(\frac{\partial u_0}{\partial y} \right)\left(\frac{\partial u_1}{\partial y} \right), \tag{18}$$

$$C_1'' + S_c C_1' - \frac{i\omega S_c}{4} C_1 = 0. \tag{19}$$

The corresponding boundary conditions are

$$\begin{aligned} &y = 0 : u_0 = 0, T_0 = 1, C_0 = 1, u_1 = 0, T_1 = 1, C_1 = 1, \\ &y \to \infty : u_0 = 0, T_0 = 0, C_0 = 0, u_1 = 0, T_1 = 0, C_1 = 0. \end{aligned} \tag{20}$$

Solving Eqs. (16) and (19) under boundary condition (20), we get

$$C_0 = e^{-S_c y}, \tag{21}$$

$$C_1 = e^{-m_1 y}, \tag{22}$$

Using multi parameter perturbation technique and assuming E_c <<1, we assume

$$u_0 = u_{00} + E_c u_{01}, \tag{23}$$

$$T_0 = T_{00} + E_c T_{01}, \tag{24}$$

$$u_1 = u_{10} + E_c u_{11}, \tag{25}$$

$$T_1 = T_{10} + E_c T_{11}. \tag{26}$$

Now using Eqs. (23)-(26) in Eqs. (14), (15), (17) and (18) and equating the coefficients of like powers of E_c and neglecting those of E_c^2, we get the following set of differential equations:
Zeroth order:

$$u_{00}'' + u_{00}' - Mu_{00} = -G_r T_{00} - G_c C_0, \tag{27}$$

$$u''_{10} + u'_{10} - \left(M + \frac{i\omega}{4} \right) u_{10} = -G_r T_{10} - G_c C_1 , \tag{28}$$

$$T''_{00} + P_r T'_{00} + \frac{P_r S}{4} T_{00} = 0 , \tag{29}$$

$$T''_{10} + P_r T'_{10} - \frac{P_r}{4} (i\omega - S) T_{10} = 0 . \tag{30}$$

The corresponding boundary conditions are,

$$\begin{aligned} &y = 0 : u_{00} = 0, T_{00} = 1, u_{10} = 0, T_{10} = 1 ; \\ &y \to \infty : u_{00} = 0, T_{00} = 0, u_{10} = 0, T_{10} = 0 . \end{aligned} \tag{31}$$

First order:

$$u''_{01} + u'_{01} - M u_{01} = -G_r T_{01} , \tag{32}$$

$$u''_{11} + u'_{11} - \left(M + \frac{i\omega}{4} \right) u_{11} = -G_r T_{11} , \tag{33}$$

$$T''_{01} + P_r T'_{01} + \frac{P_r S}{4} T_{01} = -P_r \left(u'_{00} \right)^2 , \tag{34}$$

$$T''_{11} + P_r T'_{11} - \frac{P_r}{4} (i\omega - S) T_{11} = -2 P_r \left(\frac{\partial u_{00}}{\partial y} \right) \left(\frac{\partial u_{10}}{\partial y} \right) . \tag{35}$$

The corresponding boundary conditions are,

$$\begin{aligned} &y = 0 : u_{01} = 0, T_{01} = 0, u_{11} = 0, T_{11} = 0 ; \\ &y \to \infty : u_{01} = 0, T_{01} = 0, u_{11} = 0, T_{11} = 0 . \end{aligned} \tag{36}$$

Solving Eqs. (27)-(30) subject to boundary condition (31) we get,

$$u_{00} = A_1 e^{-m_3 y} + A_2 e^{-S_c y} - A_3 e^{-n_1 y} , \tag{37}$$

$$T_{00} = e^{-m_3 y} , \tag{38}$$

$$u_{10} = A_4 e^{-m_5 y} + A_5 e^{-m_1 y} - A_6 e^{-n_3 y} , \tag{39}$$

$$T_{10} = e^{-m_5 y} . \tag{40}$$

Solving Eqs. (32)-(35) subject to boundary condition (36) we get,

$$T_{01} = a_1 e^{-2S_c y} + a_2 e^{-2m_3 y} + a_3 e^{-2m_5 y} + a_4 e^{-(m_3+S_c)y} + a_5 e^{-(n_1+S_c)y} + a_6 e^{-(m_3+n_1)y} - a_7 e^{-m_3 y} , \tag{41}$$

$$\begin{aligned} T_{11} = {} & B_1 e^{-(m_3+m_5)y} + B_2 e^{-(m_1+m_3)y} + B_3 e^{-(m_3+n_3)y} + B_4 e^{-(m_5+S_c)y} + B_5 e^{-(m_1+S_c)y} + B_6 e^{-(n_3+S_c)y} \\ & + B_7 e^{-(m_5+n_1)y} + B_8 e^{-(m_1+n_1)y} + B_9 e^{-(n_1+n_3)y} - B_{10} e^{-m_5 y} , \end{aligned} \tag{42}$$

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$$u_{01} = b_1 e^{-2S_c y} + b_2 e^{-2m_3 y} + b_3 e^{-2n_1 y} + b_4 e^{-(m_3+S_c)y} + b_5 e^{-(n_1+S_c)y} + b_6 e^{-(m_3+n_1)y} + b_7 e^{-m_3 y} - b_8 e^{-n_1 y}, \quad (43)$$

$$u_{11} = D_1 e^{-(m_3+m_5)y} + D_2 e^{-(m_1+m_3)y} + D_3 e^{-(m_3+n_3)y} + D_4 e^{-(m_5+S_c)y} + D_5 e^{-(m_1+S_c)y} + D_6 e^{-(n_3+S_c)y} + D_7 e^{-(m_5+n_1)y} + D_8 e^{-(m_1+n_1)y} + D_9 e^{-(n_1+n_3)y} + D_{10} e^{-m_5 y} - D_{11} e^{-n_3 y}. \quad (44)$$

Substituting the values of C_0 and C_1 from Eqs. (21) and (22) in Eq. (13) the solution for concentration distribution of the flow field is given by

$$C = e^{-S_c y} + \varepsilon e^{i\omega t} e^{-m_1 y}. \quad (45)$$

3.1 Skin friction

The skin friction at the wall is given by

$$\tau_w = \left(\frac{\partial u}{\partial y}\right)_{y=0} = -m_3 A_1 - S_c A_2 + n_1 A_3 - E_c\left[2b_1 S_c + 2b_2 m_3 + 2b_3 n_1 + b_4(m_3+S_c) + b_5(n_1+S_c) + b_6(m_3+n_1) + b_7 m_3 - b_8 n_1\right] + \varepsilon e^{i\omega t}\{-m_5 A_4 - m_1 A_5 + n_3 A_6 - E_c[(m_3+m_5)D_1 + (m_1+m_3)D_2 + (m_3+n_3)D_3 + (m_5+S_c)D_4 + (m_1+S_c)D_5 + (n_3+S_c)D_6 + (m_5+n_1)D_7 + (m_1+n_1)D_8 + (n_1+n_3)D_9 + m_5 D_{10} - n_3 D_{11}]\}. \quad (46)$$

3.2 Heat flux

The heat flux at the wall in terms of Nusselt number is given by

$$N_u = \left(\frac{\partial T}{\partial y}\right)_{y=0} = -m_3 - E_c\left[2a_1 S_c + 2a_2 m_3 + 2a_3 m_5 - a_4(m_3+S_c) + a_5(n_1+S_c) + a_6(m_3+n_1) - a_7 m_3\right] + \varepsilon e^{i\omega t}\{-m_5 - E_c[(m_3+m_5)B_1 + (m_1+m_3)B_2 + (m_3+n_3)B_3 + (m_5+S_c)B_4 + (m_1+S_c)B_5 + (n_3+S_c)B_6 + (m_5+n_1)B_7 + (m_1+n_1)B_8 + (n_1+n_3)B_9 - m_5 B_{10}]\}, \quad (47)$$

where

$$m_1 = \frac{1}{2}\left[S_c + \sqrt{S_c^2 + i\omega S_c}\right], m_2 = \frac{1}{2}\left[-S_c + \sqrt{S_c^2 + i\omega S_c}\right], m_3 = \frac{1}{2}\left[P_r + \sqrt{P_r^2 - SP_r}\right], m_4 = \frac{1}{2}\left[-P_r + \sqrt{P_r^2 - SP_r}\right],$$

$$m_5 = \frac{1}{2}\left[P_r + \sqrt{P_r^2 - P_r(S - i\omega)}\right], m_6 = \frac{1}{2}\left[-P_r + \sqrt{P_r^2 - P_r(S - i\omega)}\right], n_1 = \frac{1}{2}\left[1 + \sqrt{1+4M}\right], n_2 = \frac{1}{2}\left[-1 + \sqrt{1+4M}\right],$$

$$n_3 = \frac{1}{2}\left[1 + \sqrt{1 + 4\left(M + \frac{i\omega}{4}\right)}\right], n_4 = \frac{1}{2}\left[-1 + \sqrt{1 + 4\left(M + \frac{i\omega}{4}\right)}\right], A_1 = \frac{G_r}{(n_1 - m_3)(n_2 + m_3)}, A_2 = \frac{G_c}{(n_1 - S_c)(n_2 + S_c)},$$

$$A_3 = A_1 + A_2, A_4 = \frac{G_r}{(n_3 - m_5)(n_4 + m_5)}, A_5 = \frac{G_c}{(n_3 - m_1)(n_4 + m_1)}, A_6 = A_4 + A_5, a_1 = \frac{P_r S_c^2 A_2^2}{(m_3 - 2S_c)(m_4 + 2S_c)},$$

$$a_2 = \frac{-P_r m_3 A_1^2}{(m_4 + 2m_3)}, a_3 = \frac{P_r n_1^2 A_3^2}{(m_3 - 2n_1)(m_4 + 2n_1)}, a_4 = -\frac{2A_1 A_2 m_3 P_r}{(m_3 + m_4 + S_c)}, a_5 = -\frac{2P_r S_c A_2 A_3 n_1}{(m_3 - n_1 - S_c)(m_4 + n_1 + S_c)},$$

$$a_6 = \frac{2P_r A_1 A_2 m_3}{(m_3 + m_4 + n_1)}, a_7 = a_1 + a_2 + a_3 + a_4 + a_5 + a_6, B_1 = -\frac{2P_r A_1 A_6}{(m_3 + m_5 + m_6)}, B_2 = \frac{2P_r A_1 A_5 m_1 m_3}{(m_5 - m_3 - m_1)(m_6 + m_3 + m_1)},$$

$$B_4 = -\frac{2P_r A_2 A_4 m_5}{(m_6 + m_5 + S_c)}, B_3 = \frac{2P_r A_1 A_6 m_3 n_3}{(n_3 - m_5 + m_3)(n_3 + m_6 + m_3)}, B_5 = \frac{2P_r S_c A_2 A_5 m_1}{(m_5 - m_1 - S_c)(m_6 + m_1 + S_c)},$$

$$B_6 = \frac{2P_r S_c A_2 A_6 n_3}{(n_3 - m_5 + S_c)(n_3 + m_6 + S_c)}, B_7 = \frac{2P_r A_3 A_4 m_5}{(n_1 + m_6 + m_5)}, B_8 = \frac{2P_r A_3 A_5 m_1 n_1}{(n_1 + m_1 - m_5)(n_1 + m_1 + m_6)},$$

$B_9 = \frac{2P_r A_3 A_6 n_1 n_3}{(n_3 + n_1 + m_5)(n_3 + n_1 + m_6)}$, $B_{10} = B_1 + B_2 + B_3 + B_4 + B_5 + B_6 + B_7 + B_8 + B_9$, $b_1 = \frac{G_r a_1}{(n_1 - 2S_c)(n_2 + 2S_c)}$,

$b_2 = \frac{G_r a_2}{(n_1 - 2m_3)(n_2 + 2m_3)}$, $b_3 = \frac{-G_r a_3}{n_1(n_2 + 2n_1)}$, $b_4 = \frac{G_r a_4}{(n_1 - m_3 - S_c)(n_2 + m_3 + S_c)}$, $b_5 = \frac{-G_r a_5}{S_c(n_2 + n_1 + S_c)}$,

$b_6 = \frac{-G_r a_6}{m_3(n_2 + n_1 + m_3)}$, $b_7 = \frac{G_r a_7}{(m_3 - n_1)(n_2 + m_3)}$, $b_8 = b_1 + b_2 + b_3 + b_4 + b_5 + b_6 + b_7$, $D_1 = \frac{G_r B_1}{(n_3 - m_5 - m_3)(n_4 + m_5 + m_3)}$,

$D_2 = \frac{G_r B_2}{(n_3 - m_3 - m_1)(n_4 + m_3 + m_1)}$, $D_3 = \frac{-G_r B_3}{m_3(n_4 + n_3 + m_3)}$, $D_4 = \frac{G_r B_4}{(n_3 - m_5 - S_c)(n_4 + m_5 + S_c)}$,

$D_5 = \frac{G_r B_5}{(n_3 - m_1 - S_c)(n_4 + m_1 + S_c)}$, $D_6 = \frac{-G_r B_6}{S_c(n_4 + n_3 + S_c)}$, $D_7 = \frac{G_r B_7}{(n_3 - n_1 - m_5)(n_4 + n_1 + m_5)}$,

$D_8 = \frac{G_r B_8}{(n_3 - n_1 - m_1)(n_4 + n_1 + m_1)}$, $D_9 = \frac{-G_r B_9}{n_1(n_4 + n_3 + n_1)}$, $D_{10} = \frac{G_r B_{10}}{(n_3 - m_5)(n_4 + m_5)}$,

$D_{11} = D_1 + D_2 + D_3 + D_4 + D_5 + D_6 + D_7 + D_8 + D_9 + D_{10}$.

4. Results and discussions

The problem natural convection unsteady magnetohydrodynamic mass transfer flow of a viscous incompressible electrically conducting fluid past an infinite vertical porous flat plate in presence of constant suction and heat sink has been investigated. The governing equations of the flow field are solved employing multi parameter perturbation technique and the effects of the flow parameters on the velocity, temperature, concentration distribution and also on the skin friction and rate of heat transfer in the flow field are analyzed and discussed with the help of velocity profiles 1-5, temperature profiles 6-7, concentration distribution 8 and Table 1 respectively.

4.1 Velocity field

The velocity of the flow field suffers a substantial change in magnitude with the variation of the flow parameters. The important parameters affecting the velocity of the flow field are magnetic parameter M, Grashof numbers for heat and mass transfer G_r, G_c; heat sink parameter S and Schmidt number S_c. Figures 1-5 discuss the effects of these parameters on the velocity of the flow field.

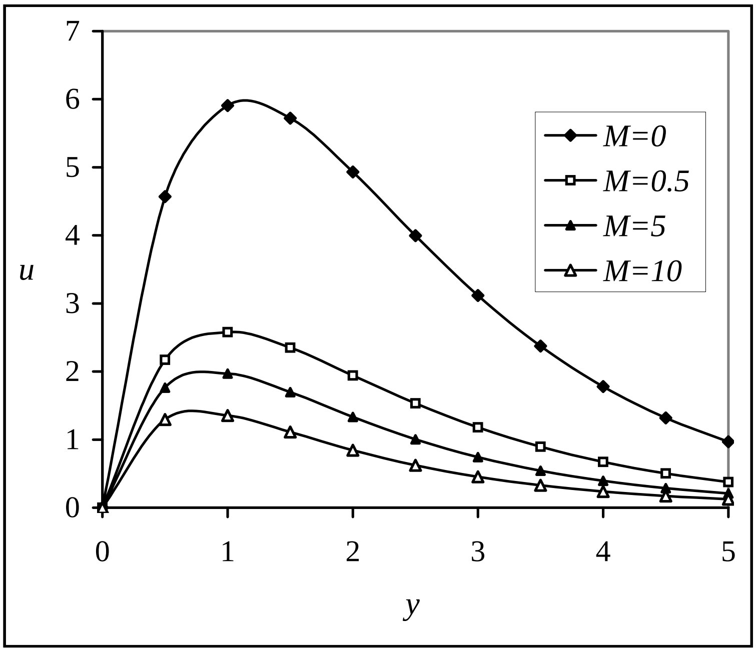

Figure 1. Velocity profiles against y for different values of M with G_r=3, G_c=3, S= -0.1, S_c=0.60, P_r=0.71, E_c=0.002, ω=5.0, ε=0.2, ωt=π/2

The effect of magnetic parameter M on the velocity field is discussed in Figure 1. Curve with M=0 corresponds to the case of non-MHD flow. Comparing the curves of Figure 1, it is observed that a growing magnetic parameter retards the velocity of the flow field at all points due to the dominant effect of the Lorentz force acting on the flow field. In Figures 2 and 3, we observe the effect of Grashof numbers for heat and mass transfer G_r, G_c respectively on the velocity field. Curves with G_r <0 correspond to heating of the plate, while those with G_r >0 correspond to cooling of the plate. Analyzing the curves of Figures 2 and 3, we come to a conclusion that both the parameters G_r and G_c enhance the velocity of the field at all points. Figure 4 elucidates the effect of heat sink/source parameter S on the velocity of the flow field. Curves with S<0 and S>0 correspond to the presence of heat sink and heat source respectively in the flow field. The heat source parameter (S>0) is found to accelerate the velocity of the flow field at all points while the presence of heat sink (S<0) reverses effect. The effect of Schmidt number S_c on the velocity field is discussed in Figure 5. The heavier diffusive species (growing S_c) has a decelerating effect on the velocity of the flow field at all points.

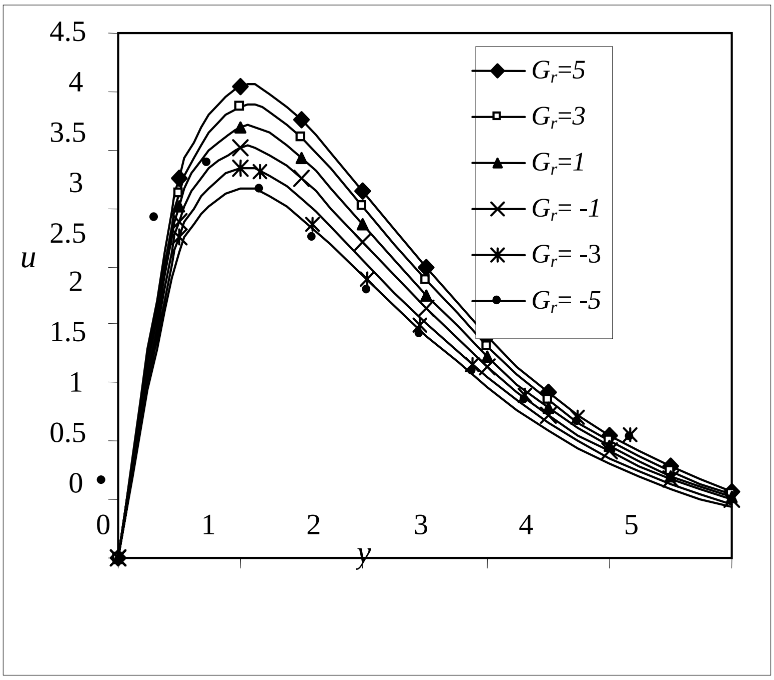

Figure 2. Velocity profiles against y for different values of G_r with G_c=3, M=1, S= -0.1, S_c=0.60, P_r=0.71, E_c=0.002, ω=5.0, ε=0.2, ωt=π/2

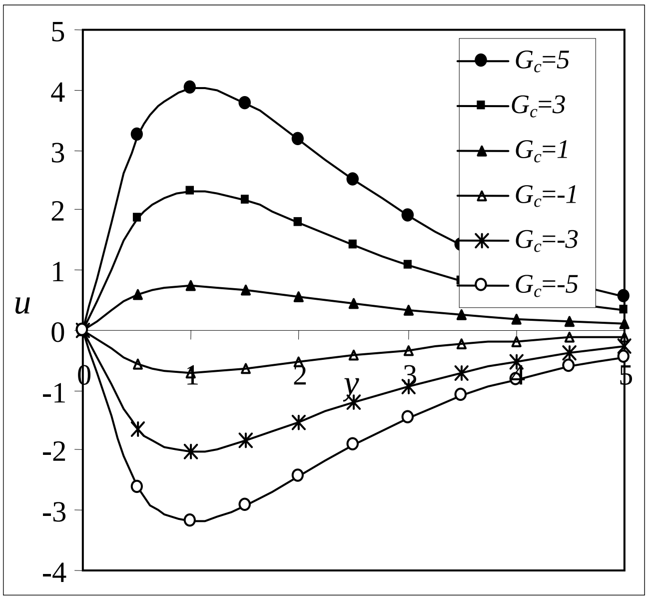

Figure 3. Velocity profiles against y for different values of G_c with G_r=3, M=1, S= -0.1, S_c=0.60, P_r=0.71, E_c=0.002, ω=5.0, ε=0.2, ωt=π/2

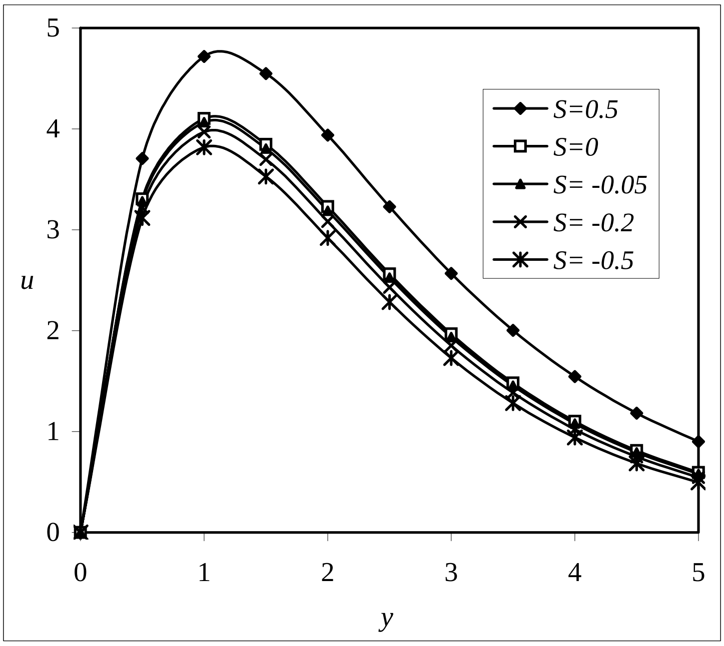

Figure 4. Velocity profiles against y for different values of S with G_r=3, G_c=3, E_c=0.002, M=1, S_c=0.60, P_r=0.71, ω=5.0, ε=0.2, ωt=π/2

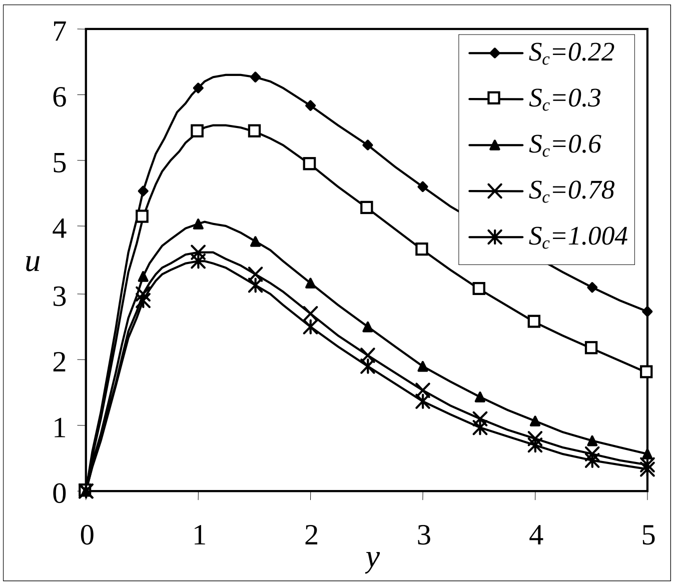

Figure 5. Velocity profiles against y for different values of S_c with G_r=3, G_c=3, E_c=0.002, M=1, S= -0.1, P_r=0.71, ω=5.0, ε=0.2, ωt=π/2

4.2 Temperature field

The temperature field is found to change appreciably with the variation of Prandtl number P_r and heat sink parameter S. These variations have been shown in Figures 6 and 7 respectively. On close observation of the curves of both the figures, we notice that the effect of increasing the magnitude of heat sink parameter and the Prandtl number is to decrease the temperature of the flow field at all points; while the heat source parameter reverses the effect.

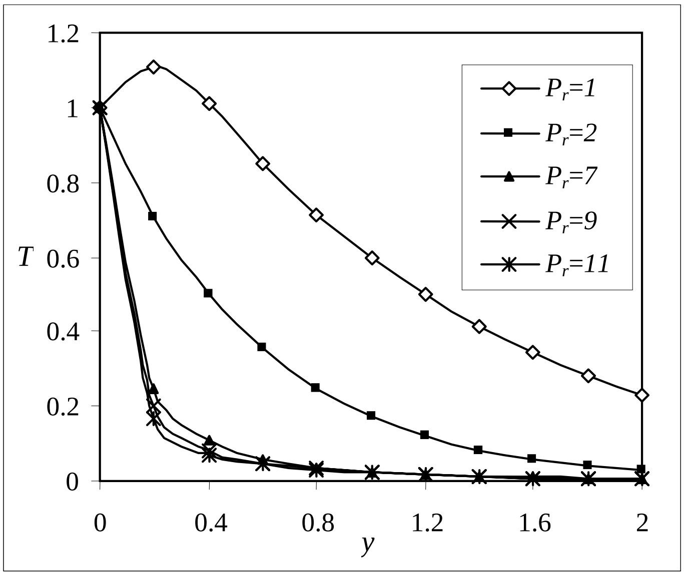

Figure 6. Temperature profiles against y for different values of P_r with G_r=3, G_c=3, M=1, S= -0.1, E_c=0.002, ω=5.0, ε=0.2, ωt=π/2

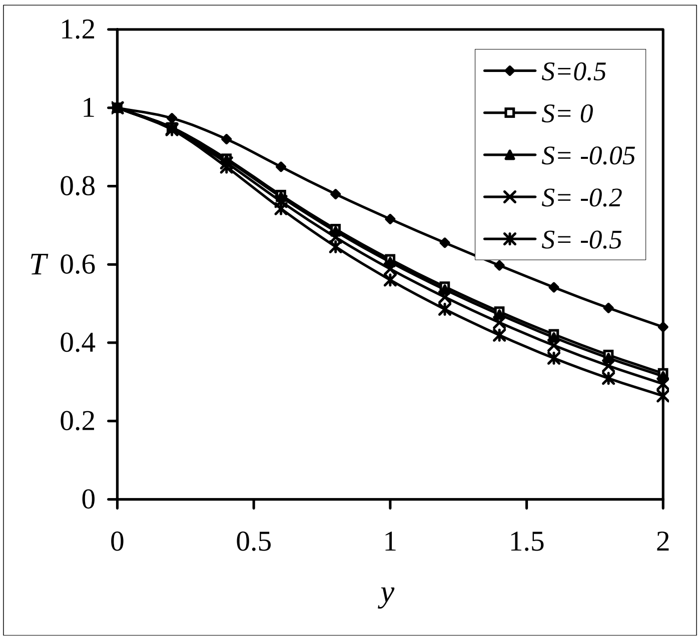

Figure 7. Temperature profiles against y for different values of S with G_r=3, G_c=3, M=1, E_c=0.002, ω=5.0, ε=0.2, ωt=π/2, P_r=0.71

4.3 Concentration distribution

Figure 8 depicts the concentration distribution in presence of foreign species such as H_2, He, H_2O vapour, NH_3 and CO_2 in the flow field with S_c= 0.22, 0.30, 0.60, 0.78 and 1.004 respectively. The concentration distribution of the flow field suffers a decrease in boundary layer thickness in presence of heavier diffusive species (growing S_c) at all points of the flow field. It is further observed that heavier the diffusive species, the sharper is the reduction in the concentration boundary layer thickness of the flow field.

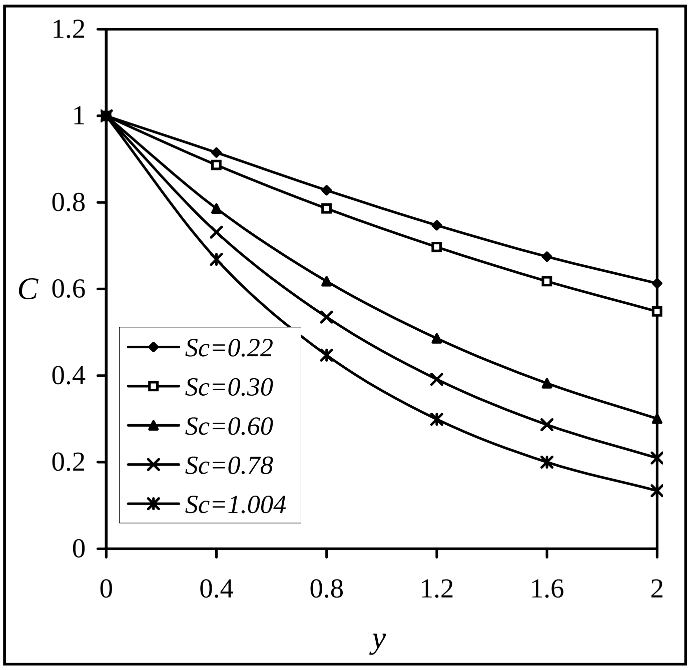

Figure 8. Concentration profiles against y for different values of S_c with ω=5.0, ε=0.2, ωt=π/2

4.4 Skin friction and rate of heat transfer

Variations in the values of skin friction τ and the heat flux i. e. rate of heat transfer N_u against the Prandtl number P_r for different values of magnetic parameter M are entered in Table 1 keeping other parameters of the flow field constant. A growing Prandtl number P_r increases the skin friction for non-MHD flow and decreases it at the wall in case of MHD flow. On the other hand, a growing magnetic parameter M decreases the effect at all points. The effect of increasing Prandtl number P_r is to increase the rate of heat transfer at the wall, while a growing magnetic parameter M leads to decrease its value at all points.

Table 1. Variation in the values of skin friction τ and the rate of heat transfer N_u against P_r for different values of M with S= -0.1, G_r=3, G_c =3, S_c=0.60, E_c=0.002, ω=5.0, ε=0.2, $\omega t=\pi/2$

P_r	M =0		M =0.1		M =5.0		M =20.0	
	τ	N_u	τ	N_u	τ	N_u	τ	N_u
0.71	11.6271	1.6423	11.3191	1.4287	6.8552	-0.3046	4.1016	-0.2363
2	12.1139	3.2345	8.1317	2.3879	5.4092	1.7626	3.5516	-1.5804
7	16.1056	-9.1989	5.9856	-8.9066	4.2561	-5.4226	2.8680	-4.9101
9	18.1481	-10.812	5.5672	-10.508	4.0844	-6.8703	2.7593	-6.2451

8. Conclusion

We present below the following results of physical interest on the velocity, temperature, concentration distribution, skin friction and the rate of heat transfer at the wall of the flow field.

1. A growing magnetic parameter M or Schmidt number S_c or heat sink parameter S leads to retard the transient velocity of the flow field at all points.
2. The effect of increasing Grashof number for heat transfer G_r and mass transfer G_c is to enhance the transient velocity of the flow field at all points.
3. An increase in Prandtl number P_r decreases the transient temperature of the flow field at all points while a growing heat sink parameter S reverses the effect.
4. A heavier diffusive species (growing S_c) has a sharper reduction in the concentration boundary layer thickness at all points of the flow field.
5. A growing Prandtl number P_r increases the skin friction for non-MHD flow and decreases it at the wall in case of MHD flow. On the other hand, a growing magnetic parameter M decreases the effect at all points.
6. The effect of increasing Prandtl number P_r is to enhance the magnitude of rate of heat transfer at the wall, while a growing magnetic parameter M leads to decrease its value at all points.

References

[1] Hashimoto H. Boundary layer growth on a flat plate with suction or injection. J. Phys Soc. Japan. 1957, 22, 7-21.

[2] Sparrow, E. M., Cess R. D. The effect of a magnetic field on a free convection heat transfer. Int. J. Heat Mass Trans. 1961, 4, 267-274.

[3] Gebhart B., Pera L. The nature of vertical natural convection flows resulting from the combined buoyancy effects of thermal and mass diffusion. Int. J. Heat Mass Trans. 1971, 14 (12), 2025-2050.

[4] Soundalgekar V. M., Wavre P. D. Unsteady free convection flow past an infinite vertical plate with constant suction and mass transfer. Int. J. Heat Mass Trans.1977, 20, 1363-1373.

[5] Hossain M. A., Begum R. A. Effect of mass transfer and free convection on the flow past a vertical plate. ASME J. Heat Trans. 1984, 106, 664-668.

[6] Bestman A.R. Natural convection boundary layer with suction and mass transfer in a porous medium. Int. J. Ener. Res. 1990, 14(4), 389-396.

[7] Pop I., Kumari M., Nath G. Conjugate MHD flow past a flat plate. Acta Mech. 1994, 106 (3-4), 215-220.

[8] Singh N. P. Mass transfer effects on free convection in MHD flow of a viscous fluid. Proc. Math. Soc. 1994, 10, 59-62.

[9] Raptis A. A, Soundalgekar V.M. Steady laminar free convection flow of an electrically conducting fluid along a porous hot vertical plate in presence of heat source/sink. ZAMM, 1996, 10, 59-62.

[10] Na T. Y., Pop I. Free convection flow past a vertical flat plate embedded in a saturated porous medium. Int. J. Engng. Sci. 1998, 21(3), 541-553.

[11] Takhar H.S., Chamkha A.J., Nath G. Unsteady flow and heat transfer on a semi-infinite flat plate with an aligned magnetic field. Int. J. Engng. Sci. 1999, 37 (13), 1723-1736.

[12] Chowdhury M.K., Islam M.N. MHD free convection flow of visco elastic fluid past an infinite vertical porous plate. Heat Mass Trans. 2000, 36, 439-447.

[13] Raptis A.A., Kafousias N. Heat transfer in flow through a porous medium bounded by an infinite vertical plate under the action of a magnetic field. Int. J. Ener. Res. 2000, 6, 241-245.

[14] Sharma P. R., Pareek D. Steady free convection MHD flow past a vertical porous moving surface. Ind. J. Theo. Phys. 2002, 50, 5-13.

[15] Das S. S., Sahoo S. K., Dash G. C. Numerical solution of mass transfer effects on unsteady flow past an accelerated vertical porous plate with suction. Bull. Malays. Math. Sci. Soc. 2006, 29(1), 33-42.

[16] Das S. S., Tripathy U. K., Das J. K. Hydromagnetic convective flow past a vertical porous plate through a porous medium with suction and heat source. Int. J. Ener. Env. 2010, 1(3), 467-478.

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