Exergoeconomic performance optimization of an endoreversible intercooled regenerative Brayton combined heat and power plant coupled to variable-temperature heat reservoirs

Bo Yang, Lingen Chen, Fengrui Sun

College of Naval Architecture and Power, Naval University of Engineering, Wuhan 430033, P. R. China.

Abstract
An endoreversible intercooled regenerative Brayton combined heat and power (CHP) plant model coupled to variable-temperature heat reservoirs is established. The exergoeconomic performance of the CHP plant is investigated using finite time thermodynamics. The analytical formulae about dimensionless profit rate and exergy efficiency of the CHP plant with the heat resistance losses in the hot-, cold- and consumer-side heat exchangers, the intercooler and the regenerator are deduced. By taking the maximum profit rate as the objective, the heat conductance allocation among the five heat exchangers and the choice of intercooling pressure ratio are optimized by numerical examples, the characteristic of the optimal dimensionless profit rate versus corresponding exergy efficiency is investigated. When the optimization is performed further with respect to the total pressure ratio, a double-maximum profit rate is obtained. The effects of the design parameters on the double-maximum dimensionless profit rate and corresponding exergy efficiency, optimal total pressure ratio and optimal intercooling pressure ratio are analyzed in detail, and it is found that there exist an optimal consumer-side temperature and an optimal thermal capacitance rate matching between the working fluid and the heat reservoir, respectively, corresponding to a thrice-maximum dimensionless profit rate.

Keywords: Finite time thermodynamics; Intercooled regenerative Brayton combined heat and power plant; Exergoeconomic performance; Profit rate; Optimization.

1. Introduction
Combined heat and power (CHP) plants in which heat and power are produced together are now widely used and are more advantageous in terms of energy and exergy efficiencies than plants which produce heat and power separately [1]. It is important to determine the optimal design parameters of the CHP plants. By using classical thermodynamics, Rosen et al [2] performed energy and exergy analyses for CHP-based district energy systems and exergy methods are employed to evaluated overall and component efficiencies and to identify and assess thermodynamic losses. Khaliq [3] performed the exergy analysis of a gas turbine trigeneration system for combined production of power, heat and refrigeration and investigated the effects of overall pressure ratio, turbine inlet temperature, and pressure drop on the exergy destruction. Reddy and Butcher [4] investigated the exergetic efficiency performance of a natural gas-fired intercooled reheat gas turbine CHP system and analyzed the effects of intercooling,
reheat and total pressure ratio on the performance of the CHP plant. Khaliq and Choudhary [5] evaluated the performance of intercooled reheat regenerative gas turbine CHP plant by using the first law (energetic efficiency) and second law (exergetic efficiency) of thermodynamics and investigated the effects of overall pressure ratio, cycle temperature ratio, pressure losses on the performance of the CHP plant.

Finite-time thermodynamics (FTT) [6-18] is a powerful tool for analyzing and optimizing performance of various thermodynamic cycles and devices. Some authors have performed the performance analysis and optimization for various CHP plants by using finite-time thermodynamics. Bojic [19] investigated the annual worth of an endoreversible Carnot cycle CHP plant with the sole irreversibility of heat resistance. Sahin et al [20] performed exergy output rate optimization for an endoreversible Carnot cycle CHP plant and found that the lower the consumer-side temperature, the better the performance. Erdil et al [21] optimized the exergetic output rate and exergetic efficiency of an irreversible combined Carnot cycle CHP plant under various design and operating conditions and found that the optimal performance stayed approximately constant with consumer-side temperature. Atmaca et al [22] performed the exergetic output rate, energy utilization factor (EUF), artificial thermal efficiency and exergetic efficiency optimization of an irreversible Carnot cycle CHP plant. Ust et al [23] provided a new exergetic performance criterion, exergy density, which includes the consideration of the system sizes, and investigated the general and optimal performances of an irreversible Carnot cycle CHP plant.

In industry, Brayton cycle is widely used. Some authors are interested in the CHP plants composed of various Brayton cycles. Yilmaz [24] optimized the exergy output rate and exergetic efficiency of an endoreversible simple gas turbine closed-cycle CHP plant, investigated the effects of parameters on exergetic performance and found that the lower the consumer-side temperature, the better the performance. Hao and Zhang [25, 26] optimized the total useful-energy rate (including power output and useful heat rate output) and the exergetic output rate of an endoreversible Joule-Brayton CHP cycle by optimizing the pressure ratio and analyzed the effects of design parameters on the optimal performances. Ust et al [27, 28] proposed a new objective function called the exergetic performance coefficient (EPC), optimized an irreversible regenerative gas turbine closed-cycle CHP plant with heat resistance and internal irreversibility [27] and an irreversible Dual cycle CHP plant with heat resistance, heat leakage and internal irreversibility [28], and compared the results with those obtained using the total exergy output as the objective.

Exergoeconomic (or thermoeconomic) analysis and optimization [29, 30] is a relatively new method that combines exergy with conventional concepts from long-run engineering economic optimization to evaluate and optimize the design and performance of energy systems. Salamon and Nitzan [31] combined the endoreversible model with exergoeconomic analysis for endoreversible Carnot heat engine with the only loss of heat resistance. It was termed as finite time exergoeconomic analysis [32-38] to distinguish it from the endoreversible analysis with pure thermodynamic objectives and the exergoeconomic analysis with long-run economic optimization. Furthermore, such a method has been extended to universal endoreversible heat engine [39] and generalized irreversible Carnot heat engine [40] and refrigerator [41]. On the basis of Refs. [32-41], Tao et al [42, 43] performed the finite time exergoeconomic performance analysis and optimization for endoreversible simple [42] and regenerative [43] gas turbine closed-cycle CHP plant coupled to constant temperature heat reservoirs by optimizing the heat conductance allocation among the hot-, cold- and consumer-side heat exchangers, the regenerator and the pressure ratio of the compressor. Chen et al [44] and Yang et al [45] analyzed and optimized the finite time exergoeconomic performance of an endoreversible intercooled regenerative Brayton CHP plant coupled to constant-temperature heat reservoirs.

A thermodynamic model of an endoreversible intercooled regenerative Brayton CHP plant coupled to variable-temperature heat reservoirs was established in Ref. [46]. The performance investigation and parametric analysis were performed by using finite time exergoeconomic analysis. The analytical formulae about dimensionless profit rate and exergy efficiency were deduced, respectively [46]. A further step made in this paper is to optimize the heat conductance allocation among the hot-, cold- and consumer-side heat exchangers, the intercooler and the regenerator, the intercooling pressure ratio and the total pressure ratio by taking the maximum dimensionless profit rate as the objective. Effects of design parameters on the optimal performance are analyzed in detail and the thermal capacitance rate matching between the working fluid and the heat reservoir is discussed.
2. Cycle model

A T-s diagram of CHP plant composed of an endoreversible intercooled regenerative Brayton closed-cycle coupled to variable-temperature heat reservoirs is shown in Figure 1. Process 1-2 and 3-4 are isentropic adiabatic compression processes in the low- and high-pressure compressors, while the process 5-6 is isentropic adiabatic expansion process in the turbine. Process 2-3 is an isobaric intercooling process in the intercooler. Process 4-7 is an isobaric absorbed heat process and process 6-8 is an isobaric evolved heat process in the regenerator. Process 7-5 is an isobaric absorbed heat process in the hot-side heat exchanger and process 9-1 is an isobaric evolved heat process in the cold-side heat exchanger. Process 8-9 is an isobaric evolved heat process in the customer-side heat exchanger.

Assuming that the working fluid used in the cycle is an ideal gas with constant thermal capacity rate \( C_{wf} \). The hot-side heat reservoir is considered to have a thermal capacity rate \( H \) and the inlet and the outlet temperatures of the heating fluid are \( T_{inH} \) and \( T_{outH} \), respectively. The cold-side heat reservoir is considered to have a thermal capacity rate \( L \) and the inlet and the outlet temperatures of the cooling fluid are \( T_{inL} \) and \( T_{outL} \), respectively. The cooling fluid in the intercooler is considered to have a thermal capacity rate \( I \) and the inlet and the outlet temperatures of the cooling fluid are \( T_{inI} \) and \( T_{outI} \), respectively. The consumer-side temperature is \( K \). The heat exchangers between the working fluid and the heat reservoir, the regenerator and the intercooler are counter-flow and the heat conductances (heat transfer surface area and heat transfer coefficient product) of the hot-, cold- and consumer-side heat exchangers, the intercooler and the regenerator are \( U_H \), \( U_L \), \( U_I \), \( U_R \) respectively. According to the heat transfer processes, the properties of the heat reservoirs and working fluid, and the theory of heat exchangers, the rate \( (Q_H) \) of heat transfer from heat source to the working fluid, the rate \( (Q_L) \) of heat transfer from the working fluid to the heat sink, the rate \( (Q_K) \) of heat transfer from the working fluid to the heat consuming device, the rate \( (Q_I) \) of heat exchanged in the intercooler, and the rate \( (Q_R) \) of heat regenerated in the regenerator are, respectively, given by:

\[
Q_H = U_H \frac{(T_{inH} - T_1) - (T_{outH} - T_1)}{\ln\left(\frac{T_{inH} - T_1}{T_{outH} - T_1}\right)} = C_{wf}(T_{inH} - T_1) = C_{min} E_{H1}(T_{inH} - T_1)
\]

\[
Q_L = U_L \frac{(T_0 - T_{outL}) - (T_1 - T_{inL})}{\ln\left(\frac{T_0 - T_{outL}}{T_1 - T_{inL}}\right)} = C_L(T_{outL} - T_{inL}) = C_{min} E_{L1}(T_0 - T_{inL})
\]

\[
Q_K = U_K \frac{T_0 - T_k}{\ln\left(\frac{T_0 - T_k}{T_0 - T_k}\right)} = C_{wf}(T_0 - T_k) = C_{min} E_{K1}(T_0 - T_k)
\]

\[
Q_I = U_I \frac{(T_2 - T_{outI}) - (T_3 - T_{inI})}{\ln\left(\frac{T_2 - T_{outI}}{T_3 - T_{inI}}\right)} = C_I(T_{outI} - T_{inI}) = C_{min} E_{I1}(T_2 - T_{inI})
\]

\[
Q_R = C_{wf}(T_3 - T_1) = C_{wf}(T_k - T_1) = C_{min} E_{R1}(T_3 - T_1)
\]

where \( E_{H1}, E_{L1}, E_K, E_I, \) and \( E_R \) are the effectivenesses of the hot-, cold-, consumer-side heat exchangers, the intercooler and the regenerator, respectively, and are defined as:

\[
E_{H1} = \frac{1 - \exp\left[-N_{H1} \left(1 - C_{min} / C_{max}\right)\right]}{1 - \left(C_{min} / C_{max}\right) \exp\left[-N_{H1} \left(1 - C_{min} / C_{max}\right)\right]}
\]

\[
E_{L1} = \frac{1 - \exp\left[-N_{L1} \left(1 - C_{min} / C_{max}\right)\right]}{1 - \left(C_{min} / C_{max}\right) \exp\left[-N_{L1} \left(1 - C_{min} / C_{max}\right)\right]}
\]

\[
E_K = 1 - \exp(-N_K)
\]
where \( C_{H_{\text{min}}} \) and \( C_{H_{\text{max}}} \) are the smaller and the larger of the two capacitance rates \( C_H \) and \( C_{\text{of}} \), \( C_{L_{\text{min}}} \) and \( C_{L_{\text{max}}} \) are the smaller and the larger of the two capacitance rates \( C_L \) and \( C_{\text{of}} \), \( N_{H_{\text{in}}} \), \( N_{H_{\text{out}}} \), \( N_{K} \), \( N_{L_{\text{in}}} \) and \( N_{L_{\text{out}}} \) are the numbers of heat transfer units of the hot-side, cold-side, consumer-side heat exchangers, the intercooler and the regenerator, respectively, and are defined as:

\[
N_{H_{\text{in}}} = U_H / C_{H_{\text{min}}} , N_{H_{\text{out}}} = U_L / C_{L_{\text{min}}} , N_{K} = U_K / C_{\text{of}} \ , N_{L_{\text{in}}} = U_L / C_{L_{\text{min}}} , N_{L_{\text{out}}} = U_K / C_{\text{of}}
\]

\[
C_{H_{\text{min}}} = \min\{ C_H , C_{\text{of}} \} , C_{H_{\text{max}}} = \max\{ C_H , C_{\text{of}} \} , C_{L_{\text{min}}} = \min\{ C_L , C_{\text{of}} \} , C_{L_{\text{max}}} = \max\{ C_L , C_{\text{of}} \}
\]

Defining the working fluid isentropic temperature ratios \( x \) and \( y \) for the low-pressure compressor and the total compression process, i.e. \( x = T_2 / T_1 \) and \( y = T_3 / T_0 \). According to the properties of endoreversible process, one has:

\[
x = \pi_i^{(k-1)/k} , \ y = \pi^{(k-1)/k} , \ T_1 = T_3 yx^{-1}
\]

where \( \pi_i \) is the intercooling pressure ratio which satisfies \( \pi_i \geq 1 \), \( \pi \) is the total pressure ratio which satisfies \( \pi \geq \pi_i \), and \( k \) is the specific heat ratio of the working fluid.

![T-s diagram for the cycle process](image)

**Figure 1.** T-s diagram for the cycle process

### 3. The profit rate and exergy efficiency analytical formulae [46]

Assuming the environment temperature is \( T_0 \), the total rate of exergy input of the CHP plant is:

\[
E_{\text{fi}} = \frac{1 - \exp[-N_{H_{\text{in}}}(1 - C_{L_{\text{min}}} / C_{L_{\text{max}}})]}{1 - (C_{L_{\text{min}}} / C_{L_{\text{max}}}) \exp[-N_{H_{\text{in}}}(1 - C_{L_{\text{min}}} / C_{L_{\text{max}}})]}
\]

\[
E_{\text{g}} = N_{g} / (N_{g} + 1)
\]

According to the first law of thermodynamics, the power output (the exergy output rate of power) of the CHP plant is:

\[
e_{\text{H}} = \frac{\int_{T_{\text{in}}}^{T_{\text{out}}}(1 - T_0 / T)C_H dT - \int_{T_2}^{T_3}(1 - T_0 / T)C_L dT - \int_{T_{\text{in}}}^{T_{\text{out}}}(1 - T_0 / T)C_i dT}{Q_H - Q_L - Q_f - T_0 \left[ C_H \ln(T_{\text{in}} / T_{\text{in}}) - C_L \ln(T_{\text{out}} / T_{\text{in}}) - C_i \ln(T_{\text{out}} / T_{\text{in}}) \right]}
\]
The entropy generation rate of the CHP plant is:

$$\sigma = C_1 \ln(T_{\text{in}}/T_{\text{in}}) + C_1 \ln(T_{\text{in}}/T_{\text{in}}) + Q_K / T_K - C_H \ln(T_{\text{in}}/T_{\text{in}})$$  \hfill (15)$$

From the exergy conservation principle for the CHP plant, one has:

$$e_{\text{f}} = P + e_K + T_0 \sigma$$  \hfill (16)$$

where \( e_K \) is thermal exergy output rate, i.e. the exergy output rate of process heat, \( T_0 \sigma \) is the exergy loss rate.

Combining equations (13)-(16), the thermal exergy output rate \( e_K \) can be written as:

$$e_K = Q_L (1 - T_0 / T_K)$$  \hfill (17)$$

Assuming that the prices of exergy input rate, power output and thermal exergy output rate be \( \phi_{\text{f}} \), \( \phi_p \) and \( \phi_K \), respectively, the profit rate of CHP plant is:

$$\Pi = \phi_p P + \phi_K e_K - \phi_{\text{f}} e_{\text{f}}$$  \hfill (18)$$

When \( \phi_p = \phi_K = \phi_{\text{f}} \), equation (18) becomes:

$$\Pi = \phi_p (P + e_K - e_{\text{f}}) = -\phi_p T_0 \sigma$$  \hfill (19)$$

The maximum profit rate objective is equivalent to a minimum entropy generation rate objective in this case.

When \( \phi_p = \phi_K \) and \( \phi_{\text{f}} / \phi_p \rightarrow 0 \), equation (18) becomes:

$$\Pi = \phi_p (P + e_K)$$  \hfill (20)$$

The maximum profit rate objective is equivalent to a maximum total exergy output rate objective in this case.

Combining equations (1)-(5) with (12)-(17) yields the inlet temperature of the low-pressure compressor:

$$I_{\text{in}} = 2xy_c c_4 T_{\text{in}} E_{\text{iii}} C_{\text{in}} E_{\text{in}} (c_1 (1 - 1) + y E_c C_{\text{f}}) + x[2C_{\text{f}} (c_4 T_{\text{in}} + T_{\text{in}} E_{\text{in}} C_{\text{min}})]$$

$$T_1 = \frac{(y C_{\text{f}} - c_2 E_{\text{f}}) + 2c_2 c_4 T_K C_{\text{f}} (c_3 - y C_{\text{f}}) + c_2 c_4 c_6 C_{\text{f}} (T_{\text{in}} E_{\text{in}} C_{\text{in}} - c_3 T_K)]}{2x[C_{\text{f}} (y C_{\text{f}} - c_2 E_{\text{f}}) + x y E_c c_4 (c_3 (1 - c_1) - y E_c C_{\text{f}})]}$$  \hfill (21)$$

where

$$c_1 = 2(1 - E_{\text{f}}), c_2 = 1 - E_{\text{f}}, c_3 = C_{\text{f}} - C_{\text{in}} E_{\text{in}}, c_4 = C_{\text{f}} - C_{\text{in}} E_{\text{in}}, c_5 = C_{\text{f}} - C_{\text{in}} E_{\text{in}}$$  \hfill (22)$$

The power output is:

$$C_{\text{min}} E_{\text{iii}} (x c_2 c_4 C_{\text{f}} T_{\text{in}} (1 - E_{\text{f}}) - x y C_{\text{f}} (T_1 C_{\text{f}} - C_{\text{in}} E_{\text{in}} T_{\text{in}} - c_4 E_{\text{f}} T_K)) + c_2 c_4 E y^2$$

$$= (x c_2 + C_{\text{in}} E_{\text{in}} T_{\text{in}}) + x c_2 (1 - E_{\text{f}}) (c_4 C_{\text{f}} C_{\text{in}} E_{\text{in}} E_{\text{in}} T_{\text{in}} + c_2 E_{\text{f}} E_{\text{in}} (x T_1 - P = \frac{T_{\text{in}} + C_{\text{f}} E_{\text{f}} (T_1 C_{\text{f}} - C_{\text{in}} E_{\text{in}} T_{\text{in}} - c_4 E_{\text{f}} T_K)}{x c_2 c_4 (1 - E_{\text{f}})}$$  \hfill (23)$$

The thermal exergy output rate is:
Defining price ratios \( a = \phi_P / \phi_H \), \( b = \phi_k / \phi_H \), \( \Pi \) can be nondimensionalized by using \( \phi_P C_{\text{of}} T_0 \):

\[
\Pi = \frac{\phi_P P + \phi_k e_k - \phi_H e_H}{\phi_P C_{\text{of}} T_0} = \frac{(a-1)P + (b-1)e_k - T_0 \sigma}{C_{\text{of}} T_0}
\]  

The exergy efficiency \( \eta_{\text{ex}} \) is defined as the ratio of total exergy output rate to total exergy input rate:

\[
\eta_{\text{ex}} = \frac{P + e_k}{P + e_k + T_0 \sigma}
\]

where

\[
\sigma = C_{\text{of}} \ln[1 - C_{\text{of}} C_{\text{of}} T_{\text{fin}} (1 - E_k) - C_{\text{of}} C_{\text{of}} T_{\text{fin}} (1 - E_k)] + \frac{c_k C_{\text{of}} C_{\text{of}} T_{\text{fin}} (1 - E_k)}{C_{\text{of}} C_{\text{of}} T_{\text{fin}} (1 - E_k)} + C_{\text{of}} \ln[1 + C_{\text{of}} C_{\text{of}} T_{\text{fin}} (1 - E_k) + \frac{c_k C_{\text{of}} C_{\text{of}} T_{\text{fin}} (1 - E_k)}{C_{\text{of}} C_{\text{of}} T_{\text{fin}} (1 - E_k)} + c_k C_{\text{of}} C_{\text{of}} T_{\text{fin}} (1 - E_k)]
\]

4. Finite time exergoeconomic performance optimization

According to equations (25) and (26), the dimensionless profit rate \( \Pi \) and exergy efficiency \( \eta_{\text{ex}} \) of the endoreversible intercooled regenerative Brayton CHP plant coupled to variable-temperature heat reservoirs are the functions of the intercooling pressure ratio \( \pi_1 \), the total pressure ratio \( \pi \) and the five heat conductances \( U_H, U_L, U_K, U_J, U_S \) when the other boundary condition parameters \( a, b, T_{\text{fin}}, T_{\text{fin}}, T_{\text{fin}}, T_{\text{fin}}, C_H, C_L, C_J, C_K \) are fixed. In practical design, \( \pi_1, \pi, U_H, U_L, U_K, U_J, U_S \) are changeable and the cost per unit of heat conductance may be different for each heat exchanger because different materials may be used. To simplify the problem, the constraint on total heat exchanger inventory is used for the performance optimization of intercooled regenerated Brayton cycles as Refs. [47-49] by taking power, efficiency and power density as the objectives.

Assuming that the total heat exchanger inventory \( U_T = U_H + U_L + U_K + U_J + U_S \) is fixed, a group of heat conductance allocations are defined as:

\[
u_k = U_H / U_T, u_l = U_L / U_T, u_k = U_K / U_T, u_j = U_J / U_T, u_s = U_S / U_T
\]

Additionally, one has the constraints:

\[
0 < u_k < 1, 0 < u_l < 1, 0 < u_k < 1, 0 < u_j < 1, 0 < u_s < 1, u_h + u_l + u_k + u_j + u_s = 1
\]

For the fixed \( \pi, \pi_1 \) and \( U_T \), the optimization can be performed by searching the optimal heat conductance allocations \( (u_k)_{\Pi_{r1}}, (u_l)_{\Pi_{r1}}, (u_k)_{\Pi_{r1}}, (u_j)_{\Pi_{r1}}, (u_s)_{\Pi_{r1}} \) which lead to the optimal dimensionless profit rate \( \Pi_{r1} \), and one can always obtain \( (u_k)_{\Pi_{r1}} = 0 \). The reason is that regeneration makes the optimal dimensionless profit rate decrease.

When \( u_r, \pi, U_T \) is fixed, the optimization can be performed by searching the other four optimal heat conductance allocations \( (u_k)_{\Pi_{r1}}, (u_l)_{\Pi_{r1}}, (u_k)_{\Pi_{r1}}, (u_s)_{\Pi_{r1}} \) and the optimal intercooling pressure ratio \( (\pi_1)_{\Pi_{r1}} \) which lead to the maximum dimensionless profit rate \( \Pi_{r1} \). If \( \pi \) is changeable, the double-maximum dimensionless profit rate \( (\Pi_{r1})_{\Pi_{r1}} \) and the corresponding exergy efficiency \( (\eta_{\text{ex}})_{\Pi_{r1}} \), optimal total pressure ratio \( (\pi)_{\Pi_{r1}} \) and optimal intercooling pressure ratio \( (\pi_1)_{\Pi_{r1}} \) can be obtained.
To search the optimal values of \( u_h, u_t, u_k, u_i, \pi_i \) and \( \pi \), numerical calculations are provided by using the optimization toolbox of Matlab 7.1. In the calculations, four temperature ratios are defined: 
\[ \tau_i = \frac{T_{in}}{T_0}, \quad \tau_2 = \frac{T_{in}}{T_0}, \quad \tau_3 = T_k/T_0, \quad \text{and} \quad \tau_i = T_k/T_0, \quad \text{and} \quad U_f = 10kW/K, \quad u_r = 0.1, \quad k = 1.4, \]
\[ C_{uf} = 1.0kW/K, \quad C_H = C_t = C_i = 1.2kW/K, \quad \tau_1 = 5, \quad \tau_2 = \tau_3 = 1 \] and \( \tau_4 = 1.2 \) are set. According to analysis in Ref. [50], \( a = 10 \) and \( b = 6 \) are set.

4.1 The optimal dimensionless profit rate

Assuming that \( \pi = 15 (1 \leq \pi_i \leq \pi) \). Figure 2 shows the characteristic of the optimal dimensionless profit rate \( (\Pi_{opt}) \) versus \( \pi_i \) for different \( \tau_i \). It can be seen that there exists an optimal intercooling pressure ratio \( (\pi_i)_{\Pi_{max}} \) which make \( \Pi_{opt} \) reach the maximum dimensionless profit rate \( (\Pi_{max}) \). That is, there exists a sole group of optimal heat conductance allocations \( (u_h)_{\Pi_{max}}, (u_t)_{\Pi_{max}}, (u_k)_{\Pi_{max}}, (u_i)_{\Pi_{max}} \) and an optimal intercooling pressure ratio \( (\pi_i)_{\Pi_{max}} \) which lead to the maximum dimensionless profit rate \( (\Pi_{max}) \). \( \Pi_{opt} \) increases with the increase of \( \tau_i \). The calculation illustrates that when \( \pi_i \) increases to a certain value, one has \( (u_i)_{\Pi_{opt}} = 0 \), and \( \Pi_{opt} \) keeps a constant.

Figure 3 shows the characteristic of \( \Pi_{opt} \) versus corresponding exergy efficiency \( (\eta_{ex})_{\Pi_{opt}} \) with \( \tau_1 = 5 \). It can be seen that the characteristic of \( \Pi_{opt} \) versus \( (\eta_{ex})_{\Pi_{opt}} \) is loop-shaped, there exists a maximum dimensionless profit rate \( (\Pi_{max}) \) and the corresponding exergy efficiency \( (\eta_{ex})_{\Pi_{max}} \). The broken line in the curve exists in the case of \( \pi_i < 1 \).

![Figure 2. Effect of \( \tau_i \) on the characteristic of \( \Pi_{opt} \) versus \( \pi_i \)](image)

![Figure 3. The characteristic of \( \Pi_{opt} \) versus \( (\eta_{ex})_{\Pi_{opt}} \)](image)

4.2 The maximum dimensionless profit rate

Figure 4 shows the characteristic of the maximum dimensionless profit rate \( (\Pi_{max}) \) versus \( \pi \) for different \( \tau_i \). Figures 5-9 show the characteristics of the corresponding optimal heat conductance allocations \( (u_h)_{\Pi_{max}}, (u_t)_{\Pi_{max}}, (u_k)_{\Pi_{max}}, (u_i)_{\Pi_{max}} \) and optimal intercooling pressure ratio \( (\pi_i)_{\Pi_{max}} \) versus \( \pi \) for different \( \tau_i \), respectively. Figure 10 shows the characteristic of \( \Pi_{max} \) versus the corresponding exergy efficiency \( (\eta_{ex})_{\Pi_{max}} \) with \( \tau_1 = 5 \).

It can be seen from Figure 4 that there exists an optimal total pressure ratio \( (\pi_{max}) \) which make \( \Pi_{max} \) reach a double-maximum dimensionless profit rate \( (\Pi_{max,2}) \) (the corresponding intercooling pressure

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ratio is $(\pi \Pi_{\text{max}})$. $\Pi_{\text{max}}$ increases with the increase of $\tau_i$. It can be seen from Figures 5-9 that with the increase of $\pi$, $(u_i)_{\Pi_{\text{max}}}$ and $(u_i)_{\Pi_{\text{max}}}$ decrease, $(u_i)_{\Pi_{\text{max}}}$ and $(\pi_i)_{\Pi_{\text{max}}}$ increase. With the increase of $\tau_i$, $(u_i)_{\Pi_{\text{max}}}$ and $(u_i)_{\Pi_{\text{max}}}$ increase, $(u_i)_{\Pi_{\text{max}}}$ and $(u_i)_{\Pi_{\text{max}}}$ decrease, and $(\pi_i)_{\Pi_{\text{max}}}$ decreases slightly. However, when $\pi > 5$, the value of $(u_i)_{\Pi_{\text{max}}}$ is about $0.4 \sim 0.5$, the value of $(u_i)_{\Pi_{\text{max}}}$ is about 0.1, the value of $(u_i)_{\Pi_{\text{max}}}$ is about 0.2, the value of $(u_i)_{\Pi_{\text{max}}}$ is about 0.1 $\sim$ 0.2. It can be seen from Figure 10 that the characteristic of $\Pi_{\text{max}}$ versus $(\eta_{\text{ex}})_{\Pi_{\text{max}}}$ is loop-shaped, there exists a double-maximum dimensionless profit rate $(\Pi_{\text{max}})$ and the corresponding exergy efficiency $(\eta_{\text{ex}})_{\Pi_{\text{max}}}$.

![Figure 4. Effect of $\tau_i$ on the characteristic of $\Pi_{\text{max}}$ versus $\pi$](image1)

![Figure 5. Effect of $\tau_i$ on the characteristic of $(u_i)_{\Pi_{\text{max}}}$ versus $\pi$](image2)

![Figure 6. Effect of $\tau_i$ on the characteristic of $(u_i)_{\Pi_{\text{max}}}$ versus $\pi$](image3)

![Figure 7. Effect of $\tau_i$ on the characteristic of $(u_i)_{\Pi_{\text{max}}}$ versus $\pi$](image4)
Figure 8. Effect of $\tau$ on the characteristic of $(u_i)_{\Pi_{\text{max}}}$ versus $\pi$

Figure 9. Effect of $\tau$ on the characteristic of $(\pi_i)_{\Pi_{\text{max}}}$ versus $\pi$

Figure 10. The characteristic of $\Pi_{\text{max}}$ versus $(\eta_{\text{ex}})_{\Pi_{\text{max}}}$

4.3 The effects of design parameters on the optimal performance

Figures 11-18 show the characteristics of $\Pi_{\text{max,2}}$, $(\eta_{\text{ex}})_{\Pi_{\text{max,2}}}$, $(\pi_i)_{\Pi_{\text{max,2}}}$, and $\pi_{\Pi_{\text{max,2}}}$ versus $a$, $b$, $U_r$, and $\tau_4$ with $\tau_1 = 5$, respectively. It can be seen from Figures 11-14 that $\Pi_{\text{max,2}}$ increases with the increases of $a$, $b$ and $U_r$. When $U_r$ is large, $\Pi_{\text{max,2}}$ increases slowly, there exists an optimal consumer-side temperature $((\tau_1)_{\text{opt}})$ which leads to a thrice-maximum dimensionless profit rate $(\Pi_{\text{max,1}})$. The characteristics of $(\eta_{\text{ex}})_{\Pi_{\text{max,2}}}$ versus $a$, $b$ and $\tau_4$ are parabolic-like, but the value of $(\eta_{\text{ex}})_{\Pi_{\text{max,2}}}$ changes slightly with the changes of $a$ and $b$. The characteristic of $(\eta_{\text{ex}})_{\Pi_{\text{max,2}}}$ versus $U_r$ is similar to that of $\Pi_{\text{max,2}}$ versus $U_r$. It can be seen from Figures 15-18 that $(\pi_i)_{\Pi_{\text{max,2}}}$ increases with the increases of $a$ and $U_r$. When $a$ is large, $(\pi_i)_{\Pi_{\text{max,2}}}$ increases slowly, $(\pi_i)_{\Pi_{\text{max,2}}}$ decreases with the increase of $b$. The characteristic of $(\pi_i)_{\Pi_{\text{max,2}}}$ versus $\tau_4$ is parabolic-like. The characteristics of $\pi_{\Pi_{\text{max,2}}}$ versus $a$, $b$, $U_r$ and $\tau_4$ are similar to those of $(\pi_i)_{\Pi_{\text{max,2}}}$ versus $a$, $b$, $U_r$ and $\tau_4$, respectively.
Figure 11. The characteristics of $\Pi_{\text{max}, 2}$ and $(\eta_{ex})_{\Pi_{\text{max}, 2}}$ versus $a$

Figure 12. The characteristics of $\Pi_{\text{max}, 2}$ and $(\eta_{ex})_{\Pi_{\text{max}, 2}}$ versus $b$

Figure 13. The characteristics of $\Pi_{\text{max}, 2}$ and $(\eta_{ex})_{\Pi_{\text{max}, 2}}$ versus $U_T$

Figure 14. The characteristics of $\Pi_{\text{max}, 2}$ and $(\eta_{ex})_{\Pi_{\text{max}, 2}}$ versus $\tau_4$

Figure 15. The characteristics of $(\pi_1)_{\Pi_{\text{max}, 2}}$ and $\pi_{\Pi_{\text{max}, 2}}$ versus $a$

Figure 16. The characteristics of $(\pi_1)_{\Pi_{\text{max}, 2}}$ and $\pi_{\Pi_{\text{max}, 2}}$ versus $b$
5. Thermal capacity rate matching between the working fluid and heat reservoirs

For the variable-temperature heat reservoirs, the thermal capacity rates of working fluid and heat reservoirs have important influence on the performances of intercooled regenerative Brayton power cycles [47, 48]. Figures 19 and 20 shows the characteristic of $\bar{\Pi}_{\text{max}, 2}$ versus the thermal capacity rate matching ($C_{\text{wf}} / C_L$) between the working fluid and the cold-side heat reservoir for different $C_{\text{H}} / C_L$ and $U_T$ with $a=10$, $b=6$, $\tau_1=5.0$, $\tau_4=1.2$ and $C_L=C_I=1.2kW / K$. It can be seen that there exists an optimal thermal capacity rate matching ($C_{\text{wf}} / C_L_{\text{opt}}$) that make $\bar{\Pi}_{\text{max}, 2}$ reach a thrice-maximum dimensionless profit rate ($\bar{\Pi}_{\text{max}, 3}$). When $(C_{\text{wf}} / C_L) > (C_{\text{wf}} / C_L)_{\text{opt}}$, $\bar{\Pi}_{\text{max}, 2}$ decreases rapidly. When $(C_{\text{H}} / C_L) > 1$, the effect of $C_{\text{H}} / C_L$ on the characteristic of $\bar{\Pi}_{\text{max}, 2}$ versus $C_{\text{wf}} / C_L$ is slight. When $(C_{\text{H}} / C_L) < 1$, with the increase of $C_{\text{H}} / C_L$, $(C_{\text{wf}} / C_L)_{\text{opt}}$ increases, and $\bar{\Pi}_{\text{max}, 3}$ decreases slightly. With the increase of $U_T$, $(C_{\text{wf}} / C_L)_{\text{opt}}$ increases, $\bar{\Pi}_{\text{max}, 3}$ increases slightly.
6. Conclusion

Finite time exergoeconomic analyses is applied to perform the profit rate optimization of the CHP plant composed of an endoreversible intercooled regenerative Brayton closed-cycle coupled to variable-temperature heat reservoirs. The results show that the optimal heat conductance allocation of the regenerator is always zero at the design point of the optimal dimensionless profit rate. When the total pressure ratio, the heat conductance allocation of the regenerator and the total heat conductance are fixed, there exists a sole group of optimal heat conductance allocations among the hot-, cold- and consumer-side heat exchangers and the intercooler, and an optimal intercooling pressure ratio which lead to the maximum dimensionless profit rate. When the total pressure ratio is changeable, there exists an optimal total pressure ratio and an optimal intercooling pressure ratio which lead to a double-maximum dimensionless profit rate, and one can obtain that the value of \( \left( u_h \right)_{\Pi_{\text{max}}} \) is about \( 0.4 \sim 0.5 \), the value of \( \left( u_k \right)_{\Pi_{\text{max}}} \) is about \( 0.1 \), and the value of \( \left( u_l \right)_{\Pi_{\text{max}}} \) is about \( 0.1 \sim 0.2 \), respectively. The characteristic of the maximum dimensionless profit rate versus the corresponding exergy efficiency is studied and the characteristic is loop-shaped. The effects of some design parameters on the double-maximum dimensionless profit rate and the corresponding exergy efficiency, optimal total pressure ratio and optimal intercooling pressure ratio are discussed in detail. It is found that there exists an optimal consumer-side temperature which lead to a thrice-maximum dimensionless profit rate. When the optimization is performed additionally with respect to the thermal capacitance rate matching between the working fluid and the heat reservoirs, a thrice-maximum profit rate is obtained.

Acknowledgements

This paper is supported by The National Natural Science Foundation of P. R. China (Project No. 10905093), The Program for New Century Excellent Talents in University of P. R. China (Project No. NCET-04-1006) and The Foundation for the Author of National Excellent Doctoral Dissertation of P. R. China (Project No. 200136).

Nomenclature

- \( a \) : price ratio of power output to exergy input rate
- \( b \) : price ratio of thermal exergy output rate to exergy input rate
- \( C \) : heat capacity rate (\( kW / K \))
- \( E \) : effectiveness of the heat exchanger
- \( e \) : exergy flow rate (\( kW \))
- \( k \) : ratio of the specific heats
- \( N \) : number of heat transfer units
- \( P \) : power output of the cycle (\( kW \))
- \( Q \) : rate of heat transfer (\( kW \))
- \( s \) : entropy (\( kJ / K \))
- \( T \) : temperature (\( K \))
- \( U \) : heat conductance (\( kW / K \))
- \( u_h \) : hot-side heat conductance allocation
- \( u_i \) : heat conductance allocation of the intercooler
- \( u_k \) : consumer-side heat conductance allocation
- \( u_l \) : cold-side heat conductance allocation
- \( u_r \) : heat conductance allocation of the regenerator
- \( x \) : isentropic temperature ratio for low-pressure compressor
- \( y \) : isentropic temperature ratio for total compression process

Greek symbols

- \( \varphi \) : price of exergy flow rate (\( \text{dollar} / kW \))
- \( \eta \) : efficiency
- \( \Pi \) : profit rate (\( \text{dollar} \))
- \( \pi_i \) : intercooling pressure ratio
- \( \pi \) : total pressure ratio
- \( \sigma \) : entropy generation rate of the cycle (\( kW / K \))
\( \tau_1 \) ratio of the inlet temperature of hot-side heat reservoir to environment temperature

\( \tau_2 \) ratio of the inlet temperature of cold-side heat reservoir to environment temperature

\( \tau_3 \) ratio of the inlet temperature of intercooling fluid to environment temperature

\( \tau_4 \) ratio of the consumer-side temperature to environment temperature

**Subscripts**

- \( \text{ex} \) exergy
- \( H \) hot-side
- \( I \) intercooler
- \( in \) inlet
- \( K \) consumer-side
- \( L \) cold-side
- \( \text{max} \) maximum
- \( \text{min} \) minimum
- \( \text{opt} \) optimal
- \( out \) outlet
- \( R \) regenerator
- \( T \) total
- \( \text{wf} \) working fluid
- \( 0 \) ambient

1, 2, 3, 4, 5, 6, 7, 8, 9 state points of the cycle

- dimensionless

**References**


Bo Yang received his BS Degree in 2008 and MS Degree in 2010 in power engineering and engineering thermophysics from the Naval University of Engineering, P R China. He is pursuing for his PhD Degree in power engineering and engineering thermophysics of Naval University of Engineering, P R China. His work covers topics in finite time thermodynamics and technology support for propulsion plants. Dr Yang is the author or co-author of 11 peer-refereed articles (5 in English journals).

Lingen Chen received all his degrees (BS, 1983; MS, 1986, PhD, 1998) in power engineering and engineering thermophysics from the Naval University of Engineering, P R China. His work covers a diversity of topics in engineering thermodynamics, constructal theory, turbomachinery, reliability engineering, and technology support for propulsion plants. He has been the Director of the Department of Nuclear Energy Science and Engineering, the Director of the Department of Power Engineering and the Superintendent of the Postgraduate School. Now, he is the Dean of the College of Naval Architecture and Power, Naval University of Engineering, P R China. Professor Chen is the author or co-author of over 1200 peer-refereed articles (over 520 in English journals) and nine books (two in English). E-mail address: lchengna@yahoo.com; lingenchen@hotmail.com, Fax: 0086-27-83638709 Tel: 0086-27-83615046

Fengrui Sun received his BS degree in 1958 in Power Engineering from the Harbing University of Technology, P R China. His work covers a diversity of topics in engineering thermodynamics, constructal theory, reliability engineering, and marine nuclear reactor engineering. He is a Professor in the Department of Power Engineering, Naval University of Engineering, P R China. Professor Sun is the author or co-author of over 750 peer-refereed papers (over 340 in English) and two books (one in English).