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Linear irreversible thermodynamic performance analyses for a generalized irreversible thermal Brownian refrigerator

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Abstract
On the basis of a generalized model of irreversible thermal Brownian refrigerator, the Onsager coefficients and the analytical expressions for maximum coefficient of performance (COP) and the COP at maximum cooling load are derived by using the theory of linear irreversible thermodynamics (LIT). The influences of heat leakage and the heat flow via the kinetic energy change of the particles on the LIT performance of the refrigerator are analyzed. It is shown that when the two kinds of irreversible heat flows are ignored, the Brownian refrigerator is built with the condition of tight coupling between fluxes and forces and it will operate in a reversible regime with zero entropy generation. Moreover, the results obtained by using the LIT theory are compared with those obtained by using the theory of finite time thermodynamics (FTT). It is found that connection between the LIT and FTT performances of the refrigerator can be interpreted by the coupling strength, and the theory of LIT and FTT can be used in a complementary way to analyze in detail the performance of the irreversible thermal Brownian refrigerators. Due to the consideration of several irreversibilities in the model, the results obtained about the Brownian refrigerator are of general significance and can be used to analyze the performance of several different kinds of Brownian refrigerators.

Keywords: Linear irreversible thermodynamics; Generalized model; Irreversible thermal Brownian refrigerator; COP.

1. Introduction

In macroscopic systems, thermal fluctuations are not directly observable and their influences on the system can be ignored. However, when the system is small enough, thermal fluctuations become the major driving force of the system and can no longer be ignored. Brownian motor is a typical device which can rectify thermal fluctuations to produce directed motion [1-4]. Nowadays, people are trying to invent miniature and nanoscale devices which help to utilize energy resources in the microscopic scale. And the Brownian motor systems have attracted much interest due to their importance in achieving microscopic energy conversion. Actually, as to the Brownian motors, there are a variety of nonequilibrium driving forces besides the thermal fluctuations, such as external modulation of an underlying potential [5, 6], external force [7-9], chemical potential differences [10, 11] and so on. So far, thermal Brownian motor is the most extensively studied one among the different kinds of Brownian motors.

In the analyses of thermal Brownian motors, the thermodynamics performance is an important factor which has been analyzed by many authors [1, 12-15]. And the central issues as to the thermodynamics performance are the mechanism and efficiency of energy conversion of the Brownian motor systems. By noting the fact that the strict thermodynamic definition of efficiency is external load-dependent and is not adequate for microscopic energy conversion systems, Derényi *et al*. [16] proposed a load-independent new definition of generalized efficiency for the microscopic engines and analyzed its application to a Brownian heat engine. Meanwhile, many researchers are focusing on the efficiency performance of Brownian motors following the classical thermodynamics theory [13, 17-22].

In the past decades, the theory of finite time thermodynamics (FTT) has made tremendous progresses in the performance analyses of conventional macroscopic and quantum energy conversion systems [23-34]. Optimum performance and the transmission losses between the heat reservoirs in energy conversion systems are two major consideration factors in FTT. Parrondo and de Cisneros [35] pointed out that the strategies and principles developed in FTT theory are also valuable for the studies of Brownian motors. So far, the FTT theory has already been applied to analyze performance of Brownian motor systems, such as thermal Brownian heat engines, refrigerators and heat pumps [14, 36-42], and many significant results have been obtained.

Linear irreversible thermodynamics (LIT) is a powerful tool for studying the performance of linear processes and coupled phenomena, such as thermodiffusion, thermoelectric and thermomagnetic effects [43, 44]. In a long time, the LIT theory is limited to study the performance of isothermal energy conversion systems. Recently, Van den Broeck [45] derived the efficiency at maximum power of a heat engine using the LIT theory, and found that the efficiency at maximum power is equal to Novikov-Chambadal-Curzon-Ahlborn (NCCA) efficiency which is one of the most important results obtained in FTT [46-48]. In the derivation of NCCA efficiency in FTT, an endoreversible approximation was used. However, Van de Broeck had shown that in the frame of LIT theory, NCCA efficiency is a fundamental result obtained without approximation. Van den Broeck's work [45] also paves the way for analyzing the nonisothermal heat engines using the theory of LIT. Later, Jiménez de Cisneros *et al*. [49] extended the proposal of Van den Broeck [45] to refrigeration cycle and derived the coefficient of performance (COP) at maximum cooling load which could be equivalent to the NCCA efficiency by using the theory of LIT. At the same time, some research work has been carried out for conventional energy conversion systems within the realm of LIT, e.g., see Refs. [50-52].

Recently, due to its great significance in revealing the performance characteristic of energy conversion systems, the LIT theory has already been extended to the studies of Brownian motor systems. Van den Broeck and Kawai [53] first calculated the heat flow for an exactly solvable microscopic Brownian refrigerator model by using LIT and compared it with the results of molecular dynamics simulations. Gomez-Marin and Sancho [54] analyzed the tight coupling in a thermal Brownian motor and discussed the model acting as a refrigerator. They calculated the Onsager coefficients and showed how the reciprocity relation holds and that the determinant of the Onsager matrix vanishes. Gao *et al*. [55] calculated the Onsager coefficients and generalized efficiency of a thermal Brownian motor and discussed the influences of the main parameters on the performance of the system. Gao and Chen [56] later derived the Onsager coefficients and calculated the efficiency at maximum power of an irreversible thermally driven Brownian motor.

However, so far the LIT performance analysis for the Brownian motor systems mainly focuses on the system operating as a heat engine and the LIT performance of irreversible thermal Brownian refrigerators have been rarely investigated. Therefore, in this paper, a further step will be taken to analyze in detail the LIT performance of a thermal Brownian refrigerator. On the basis of a generalized irreversible thermal Brownian refrigerator model [42], the Onsager coefficients are derived, and the maximum COP as well as the COP at maximum cooling load of the refrigerator are analytically calculated. It is found that the heat leakage and the heat flow via the kinetic energy change have great influences on the performance of the refrigerator and when the two kinds of heat flows are not considered, the refrigerator becomes a perfectly coupled system. Moreover, the LIT performance of the refrigerator are compared with the FTT performance, and it is shown that theory of LIT and FTT can be used in a complementary way to analyze in detail the performance of the irreversible thermal Brownian refrigerators.

2. Performance characteristics and parametric optimum criteria of a Brownian [42]

A model of a generalized irreversible thermal Brownian refrigerator is shown in Figure 1 [42]. The refrigerator is modeled as moving Brownian particles in a viscous medium which is alternately in contact with a hot heat reservoir (at temperature T_H) and a cold heat reservoir (at temperature T_C) along the space coordinate. Additionally, a periodic sawtooth potential and an external force F are applied to the particles. In the figure, x is the horizontal axis of the coordinate, $\dot{N}_+$ and $\dot{N}_-$ are the numbers of forward and backward jumps per unit time, L_1 and L_2 are the widths of the left and right parts of the potential, and U_0 is the barrier height of the potential.

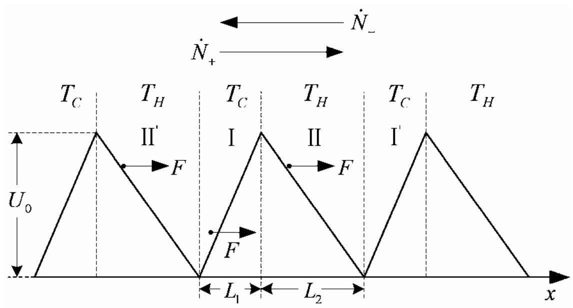

Figure 1. Schematic diagram of a thermal Brownian refrigerator

In the present model, both the irreversibility of heat leakage between two heat reservoirs and the irreversible heat flow via the change of kinetic energy of particles are considered. According to Refs. [42, 57], the rates of total heat absorbed from the cold reservoir ($\dot{Q}_C$) and released to the hot reservoir ($\dot{Q}_H$) of the Brownian refrigerator can be given by

$$\dot{Q}_C = (\dot{N}_+ - \dot{N}_-)(U_0 - FL_1) - k_B(\dot{N}_+ + \dot{N}_-)(T_H - T_C)/2 - C_i(T_H - T_C) \quad (1)$$

$$\dot{Q}_H = (\dot{N}_+ - \dot{N}_-)(U_0 + FL_2) - k_B(\dot{N}_+ + \dot{N}_-)(T_H - T_C)/2 - C_i(T_H - T_C) \quad (2)$$

where k_B is the Boltzmann's constant and it is taken to be unity for simplicity in the following calculations, C_i is the coefficient of heat leakage, $\dot{N}_+ = (1/t)\exp[-(U_0 - FL_1)/(k_B T_C)]$ and $\dot{N}_- = (1/t)\exp[-(U_0 + FL_2)/(k_B T_H)]$ are the numbers of forward and backward jumps for the Brownian particles per unit time with t a proportionality constant. The derivation of $\dot{N}_+$ and $\dot{N}_-$ is based on the assumption that the system is in a stable flow state and the rates of both forward and backward jumps are proportional to the corresponding Arrhenius' factor [57].

The heat flows between the two heat reservoirs defined by Eqs. (1) and (2) consist of three parts, respectively. The first is the heat flow caused by the particles' moving through the potential barrier, as shown by the first item in the right hand side of Eqs. (1) and(2). The second is the heat flow via the change of kinetic energy due to the particles' recrossing the boundary between the two regions, as shown by the second item in the right hand side of Eqs. (1) and (2). The influence of this kind of heat flow on the performance of Brownian motor systems was first considered by Derényi and Astumian [17] and Hondou and Sekimoto [18], and was later analyzed by many authors [14, 20-22, 38-41]. The last kind is the heat leakage between the two reservoirs, which is similar to the bypass heat leakage in conventional macroscopic heat engines and refrigerators [58, 59]. Velasco *et al*. [57] first considered the heat leakage in a Feynman's ratchet. Later this factor was extended to the studies of several kinds of thermal Brownian motors [41, 42, 55, 56].

In order to show more clearly the configuration of the system, the thermodynamic representation for the generalized model of irreversible thermal Brownian refrigerator is shown Figure 2, where P is the power input into the system, $\dot{Q}_L$ is the heat leakage between the two heat reservoirs, $\dot{Q}_1$ and $\dot{Q}_2$ are, respectively, the rates of heat absorbed from the cold reservoir and released to the hot reservoir by the system and are defined as $\dot{Q}_1 = (\dot{N}_+ - \dot{N}_-)(U_0 + FL_2) - k_B(\dot{N}_+ + \dot{N}_-)(T_H - T_C)/2$ and $\dot{Q}_2 = (\dot{N}_+ - \dot{N}_-)(U_0 - FL_1) - k_B(\dot{N}_+ + \dot{N}_-)(T_H - T_C)/2$.

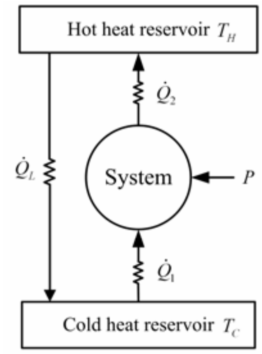

Figure 2. Thermodynamic representation for the generalized irreversible thermal Brownian refrigerator model

3. Onsager coefficients for the thermal Brownian refrigerator

According to the second law of thermodynamics, the entropy generation rate of the system can be expressed as

$$\sigma = \dot{Q}_H / T_H - \dot{Q}_C / T_C \tag{3}$$

Under the external force F, the Brownian particles will move from the cold part to the hot part and the refrigerator provides a cooling load $\dot{Q}_C$ with the power input P. In LIT theory, the sole requirement for the definition of thermodynamic forces and associated fluxes is $\sigma \geq 0$. And in the system, the external force F is the source of input power. Thus, one can consider a driving force $X_1 = F/T_H$ and a thermodynamic flux $J_1 = \dot{x}$, where x is a thermodynamically conjugate variable and the dot refers to the time derivative [45, 49], so that the power input is $P = F\dot{x} = J_1 X_1 T_H$. In the cold reservoir, the rate of heat $\dot{Q}_C$ is pumped at the cost of the input power P. Thus the thermodynamic force can be chosen as $X_2 = 1/T_H - 1/T_C$ with the corresponding flux $J_2 = \dot{Q}_C$. In the system, it is assumed that the temperature difference $\Delta T = T_H - T_C$ is small compared to T_H or T_C so that the driven force X_2 can be written as $X_2 = -\Delta T / T_H^2$.

In linear response regime, by following the LIT theory, the entropy generation rate can be expressed as

$$\sigma = \left(X_1, X_2 \right) \begin{pmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} \tag{4}$$

where L_{ij} $(i, j = 1, 2)$ are the Onsager coefficients. Substituting Eqs. (1) and (2) into Eq. (3) and making some simplification following the rules in steady state ($F \to 0$ and $\Delta T \to 0$) gives

$$\begin{aligned} \sigma = {} & e^{-U_0/(k_B T_H)} (F/T_H)^2 (L_1 + L_2)^2 / (k_B t) + (\Delta T / T_H^2)^2 [e^{-U_0/(k_B T_H)} U_0^2 / (k_B t) + e^{-U_0/(k_B T_H)} k_B T_H^2 / t \\ & + C_i T_H^2] - 2 e^{-U_0/(k_B T_H)} (F/T_H)(\Delta T / T_H^2)(L_1 + L_2) U_0 / (k_B t) \end{aligned} \tag{5}$$

One can obtain the Onsager coefficients for the Brownian refrigerator by comparing Eq. (5) with Eq. (4)

$$L_{11} = e^{-U_0/(k_B T_H)} (L_1 + L_2)^2 / (k_B t) \tag{6}$$

$$L_{22} = e^{-U_0/(k_B T_H)} U_0^2 / (k_B t) + e^{-U_0/(k_B T_H)} k_B T_H^2 / t + C_i T_H^2 \tag{7}$$

$$L_{12} = L_{21} = -e^{-U_0/(k_B T_H)} (L_1 + L_2) U_0 / (k_B t) \tag{8}$$

The Onsager coefficients offer a lot of information about the non-equilibrium thermodynamic properties of the Brownian refrigerator. It is easily found from Eqs. (6)-(8) that the reciprocity relation $L_{12} = L_{21}$ is fulfilled and the diagonal coefficients L_{11} and L_{22} are always positive. The coefficients L_{11} and $L_{12} = L_{21}$ are independent of the heat leakage and the heat flow via the kinetic energy change of the particle; while

L_{22} is closely dependent on the two kinds of irreversible heat flows. Especially, one can find that the relation $L_{11}L_{22} > L_{12}^2$ holds. This implies that the Brownian refrigerator model is inherently irreversible and there exists an entropy generation due to the existence of heat leakage and the heat flow via the kinetic energy change. Similar analyses for Brownian heat engine have been carried out in Refs. [54-56].

A dimensionless coupling strength q defined by Van den Broeck [45] can be introduced to analyze the non-equilibrium thermodynamics performance of the refrigerator

$$q = \frac{L_{12}}{\sqrt{L_{11}L_{22}}} \tag{9}$$

By substituting Eqs. (6)-(8) into Eq.(9), one can find that the absolute value of coupling strength $|q|$ is always smaller than unity.

If the heat flow via the kinetic energy change is ignored, the coefficient L_{22} becomes

$$L_{22} = e^{-U_0/(k_B T_H)} U_0^2 \big/ (k_B t) + C_i T_H^2 \tag{10}$$

Eqs.(6), (8) and (10) can be used to study the performance of a Brownian refrigerator only considering the heat leakage. It is similar to the Brownian heat engine model where only heat leakage is considered [57]. In this condition, the coupling strength $|q|$ is smaller than unity. Similarly, if the heat leakage is ignored, the coefficient L_{22} becomes

$$L_{22} = e^{-U_0/(k_B T_H)} U_0^2 \big/ (k_B t) + e^{-U_0/(k_B T_H)} k_B T_H^2 \big/ t \tag{11}$$

Eqs.(6), (8) and (11) can be used to study the performance of a Brownian refrigerator only considering the heat flow via the kinetic energy change of the particle, which is just the model studied by Lin and Chen [38] and Ai *et al*. [20]. In this condition, the coupling strength $|q|$ is also smaller than unity.

Moreover, if both the heat leakage and the heat flow via the kinetic energy change of the particles are ignored, the coefficient L_{22} becomes

$$L_{22} = e^{-U_0/(k_B T_H)} U_0^2 \big/ (k_B t) \tag{12}$$

Eqs.(6), (8) and (12) can be used to study the performance of a Brownian refrigerator considering neither the heat leakage nor the heat flow via the kinetic energy change of the particle, which is just the model considered by Asfaw and Bekele [19] and Gomez-Marin [54]. In this condition, the coupling strength $|q| = 1$, which implies that the relevant relation $L_{11}L_{22} = L_{12}^2$ is fulfilled and the refrigerator is built with the condition of tight coupling between fluxes and forces. The refrigerator operates in a reversible regime with zero entropy generation.

4. COP performance analyses

The LIT theory is based on the assumption of local equilibrium with the following linear relation between the fluxes and forces [43, 45]

$$\begin{aligned} J_1 &= L_{11}X_1 + L_{12}X_2 \\ &= [e^{-U_0/(k_B T_H)}(L_1+L_2)^2 F/T_H + e^{-U_0/(k_B T_H)}(L_1+L_2)U_0 \Delta T / T_H^2] \big/ (k_B t) \\ &= e^{-U_0/(k_B T_H)}(L_1+L_2)[(L_1+L_2)F/T_H + U_0 \Delta T / T_H^2] \big/ (k_B t) \end{aligned} \tag{13}$$

$$\begin{aligned} J_2 &= L_{21}X_1 + L_{22}X_2 \\ &= -e^{-U_0/(k_B T_H)}(L_1+L_2)U_0 F \big/ (T_H k_B t) - [e^{-U_0/(k_B T_H)} U_0^2 \big/ (k_B t) \\ &\quad + e^{-U_0/(k_B T_H)} k_B T_H^2 \big/ t + C_i T_H^2] \Delta T \big/ T_H^2 \end{aligned} \tag{14}$$

The physical meaning of the diagonal coefficients can be obtained from the above two equations [45]. For $X_2 = 0$, i.e., $T_C = T_H$ and $\Delta T = 0$, one can find that $\dot{x} = J_1 = L_{11}F/T_H \;\; = e^{-U_0/(k_B T_H)}(L_1+L_2)^2 F \big/ (T_H k_B t)$, so that

L_{11}/T_H is the mobility of the refrigerator system in response to the external force F. For $X_1 = 0$, i.e., $F = 0$, one obtains $\dot{Q}_C = J_2 = -L_{22}\Delta T/T_H^2 = -[e^{-U_0/(k_B T_H)} U_0^2/(k_B t) + e^{-U_0/(k_B T_H)} k_B T_H^2/t + C_i T_H^2]\Delta T/T_H^2$, so that L_{22}/T_H^2 is a coefficient of thermal conductivity. The reciprocity relation $L_{12} = L_{21}$ describes the cross coupling of the system, which has been analyzed in many well-documented cases, such as the Seebeck, Peltier, and Thomson effects [43, 44].

If the motion of the system halts, i.e., $J_1 = \dot{x} = 0$, one has

$$X_1 = -\frac{L_{12}X_2}{L_{11}} = -\frac{e^{-U_0/(k_B T_H)}(L_1 + L_2)U_0\Delta T/(T_H^2 k_B t)}{e^{-U_0/(k_B T_H)}(L_1 + L_2)^2/(k_B t)} = \frac{U_0\Delta T}{T_H^2(L_1 + L_2)} \equiv X_1^{stop} \tag{15}$$

where X_1^{stop} is the stopping force. The external force F^{stop} corresponding to the stopping force is then

$$F^{stop} = X_1^{stop} T_H = U_0\Delta T/[T_H(L_1 + L_2)] \tag{16}$$

Using Eqs. (13) and (14), the power input (P) and COP (ε) of the Brownian refrigerator can be given, repectively by

$$P = J_1 X_1 T_H = L_{11} X_1^2 T_H + L_{12} X_1 X_2 T_H \tag{17}$$

$$\varepsilon = \frac{\dot{Q}_C}{P} = -\frac{T_C}{T_H - T_C}\frac{L_{12}X_1X_2 + L_{22}X_2^2}{L_{11}X_1^2 + L_{12}X_1X_2} \tag{18}$$

One may note that the expressions for the power input and COP for the Brownian refrigerator are the same as those for a conventional microscopic refrigerator [49]. Therefore, it is concluded that in the frame of LIT theory, the COP of different refrigerators have a unified expression while the expressions for the Onsager coefficients of the refrigerators may be different from each other.

4.1 Maximum COP

In Eq. (18), for fixed X_2, maximizing the COP with respect to X_1 by setting $d\varepsilon/dX_1 = 0$ yields [49]

$$X_1 = -X_2\sqrt{L_{22}/L_{11}}\,(1 + \sqrt{1 - q^2})/q \tag{19}$$

And the maximum COP is

$$\varepsilon_{max} = \frac{T_C}{T_H - T_C}\frac{q^2}{2\sqrt{1-q^2} - q^2 + 2} = \frac{\varepsilon_C q^2}{2\sqrt{1-q^2} - q^2 + 2} \tag{20}$$

Substituting Eqs. (6)-(8) into Eq. (20) yields the analytical expression of maximum COP for the generalized irreversible thermal Brownian refrigerator. One may note that when $|q| \to 0$, $\varepsilon_{max} \to 0$; and when $|q| \to 1$, i.e., both the heat leakage and the heat flow via the kinetic energy change of the particles are ignored, the maximum COP ε_{max} can attain the Carnot value ε_C.

4.2 COP at maximum cooling load

The efficiency at maximum power (for a heat engine), or the COP at maximum cooling load (for a refrigerator), is the most important parameter considered in FTT theory. The parameters can also be derived using the LIT. The efficiency at maximum power output for a Brownian heat engine in LIT has been analyzed in Ref. [56]. The COP at maximum cooling load for the irreversible Brownian refrigerator will be discussed in this section. Jiménez de Cisneros *et al*. [49] showed that in LIT theory the function $J_2 X_2$ for a refrigerator is equivalent to $J_1 X_1 \propto P$ for a heat engine. Therefore, the COP at maximum cooling load is equivalent to the COP when $J_2 X_2$ is maximized.

For the Brownian refrigerator, maximizing $J_2 X_2 = L_{12} X_1 X_2 + L_{22} X_2^2$ with respect to X_2 for fixed X_1 by setting $d(J_2 X_2)/dX_2 = 0$ gives

$$X_2 = -L_{12}X_1/(2L_{22}) \tag{21}$$

Substituting Eq. (21) into Eq. (18) yields the corresponding COP at maximum cooling load

$$\varepsilon_{J_2X_2} = \frac{T_C}{T_H - T_C}\frac{q^2}{2(2-q^2)} = \frac{\varepsilon_C q^2}{2(2-q^2)} \tag{22}$$

$\varepsilon_{J_2X_2}$ is equal to half of the Carnot COP times a q-dependent factor $q^2/(2-q^2)$. One may further note that Eq. (22) shares the same form as the COP at maximum cooling load for a conventional cascade refrigerator [49]; and the factor is the same as that for a Brownian heat engine optimized at maximum power condition. In the case of tight coupling, i.e., $|q| \to 1$, the COP at maximum cooling load is exactly equal to half of the Carnot COP.

4.3 Discussions

Comparing Eq. (22) with Eq.(20), one can find that the COP ($\varepsilon_{J_2X_2}$) at maximum cooling load is always smaller than the maximum COP (ε_{max}).

The theory of LIT and FTT can be used in a complementary way to analyze in detail the performance of the irreversible thermal Brownian refrigerators. The FTT performance of the generalized irreversible thermal Brownian refrigerator has already been extensively analyzed in Ref. [42]. The connection between the LIT performance and the FTT performance of the thermal refrigerator can be interpreted by the coupling strength q.

For conventional macroscopic refrigerator, if $|q| = 1$, the refrigerator becomes a perfectly coupled system in LIT, and meanwhile the coupled system corresponds to an endoreversible refrigerator in FTT where the sole irreversibility comes from the finite rate heat transfer [23-27, 60-62]; while for the thermal Brownian refrigerator, the perfectly coupled system with $|q| = 1$ corresponds to an refrigerator without considering the heat leakage and the heat flow via kinetic energy change in FTT where the sole irreversibility comes from the particle transport process.

For conventional macroscopic refrigerator, if $|q| < 1$, the refrigerator in LIT corresponds to an irreversible refrigerator with internal irreversibility and heat leakage besides the irreversibility of finite rate heat transfer; while for the thermal Brownian system, the Brownian refrigerator in LIT with $|q| < 1$ corresponds to an refrigerator considering the heat leakage or the heat flow via kinetic energy change, or both of them in FTT besides the irreversibility in the process of particle transport.

5. Conclusions

Based on a generalized irreversible thermally driven Brownian refrigerator model built in Ref. [42], the Onsager coefficients and the analytical expressions for maximum COP and the COP at maximum cooling load are derived by using the theory of linear irreversible thermodynamics in this paper. The COP performance of the refrigerator are analyzed and it is found that in the frame of LIT, the expressions of cooling load and COP of the refrigerator share the same forms as those for a conventional macroscopic irreversible refrigerator. The influences of heat leakage and the heat flow via the kinetic energy change on the LIT performance of the refrigerator are further analyzed and it is shown that they affect not only the COP performance but also the Onsager coefficients of the refrigerator. When the two kinds of irreversible heat flow are ignored, the Brownian refrigerator becomes a perfectly coupled system. Moreover, the results obtained by LIT theory are compared with those obtained by using the FTT theory. It is found that connection of the LIT and FTT performances for the refrigerator can be interpreted by the defined parameter, i.e., the coupling strength, and the theory of LIT and FTT can be used in a complementary way to analyze in detail the performance of the irreversible thermal Brownian refrigerators. The results obtained about the irreversible model are general and can be used to analyze the performance of several different kinds of Brownian refrigerators.

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Nomenclature

C_i	coefficient of heat leakage ($\mathrm{W/K}$)	x	direction of the coordinate
F	external force	*Greek symbols*	
J_1, J_2	thermodynamic fluxes	ΔT	temperature difference (K)
k_B	Boltzmann's constant ($\mathrm{J/K}$)	ε	coefficient of performance (COP)
L_1, L_2	lengths of the left and right part of the potential	ε_C	Carnot COP
L_{ij} ($i, j = 1, 2$)	Onsager coefficients	σ	entropy generation rate ($\mathrm{W/K}$)
P	power input (W)	I, I'	cold regions of the ratchet
$\dot{Q}$	rate of heat flow (W)	II, II'	hot regions of the ratchet
q	dimensionless coupling strength	*Superscripts*	
R	cooling load (W)	$stop$	stopping force
T	temperature (K)	C	cold electron reservoir
t	a proportionality constant with a time dimension	H	hot electron reservoir
U_0	barrier height of the potential	max	maximum value
X_1, X_2	thermodynamic forces	$+, -$	forward and backward jumps of Brownian particle

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