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A suggested analytical solution of buckling investigation for beam with different crack depth and location effect

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Abstract
In this research the buckling load of a cracked beam is investigated analytically by solution the general equation of beam with crack effect and numerically by finite element method using of ANSYS program ver. 14 with different crack depth and location effect and the results is compared. The analytical results of the effect of a crack in a continuous beam by calculating the equivalent stiffness, EI, for a rectangular beam to involve an exponential function with depth and location of crack effect, with solution of assuming equivalent stiffness beam (EI) using of Fourier series method. And, the beam materials studied are different beam materials with different beam length, width and depth beam. A comparison made between analytical results from theoretical solution of general equation of motion of beam with crack effect with numerical by ANSYS results, where the biggest percentage error is about (3.8 %). Also it is found that the buckling load of beam when the crack is in the middle position is less than the buckling with crack near the end position and the buckling load of beam decreasing with increasing of crack depth due to decreasing of beam stiffness at any location of crack in beam.

Keywords: Buckling beam; Crack beam; Health monitoring; Theoretical buckling crack beam; Buckling of beam with crack effect; Crack size effect; Crack location effect.

1. Introduction

The analysis solution of the buckling beam with crack effect is evaluated by solution the general equation of motion of beam with adding the effect of crack size and location with the general equation of buckling beam and solving the equation theoretical analysis to evaluate the buckling load pf beam with crack location and size effect for different dimensions of beam and types materials of simply supported beam. Cracks found in structural elements have various causes. They may be fatigue cracks that take place under service conditions as a result of the limited fatigue strength. They may also be due to mechanical defects, as in the case of turbine blades of jet turbine engines. In these engines the cracks are caused by sand and small stones sucked from the surface of the runway. Another group involves cracks which are inside the material: they are created as a result of manufacturing processes, [1].

Many studies were performed to examine the vibration and dynamic of buckling cracked beams, as, Muhannad Al-Waily [2] and [3]. In this researches the natural frequency of a cracked beam with different supported is presented simply and clamped beam analytically, experimental and numerically by finite element method using ANSYS program ver. 14 with different crack depth and location effect and the results are compared, theoretical with experimental results [2] and theoretical with numerical results

[3]. The analytical results of the effect of a crack in a continuous beam are calculated by the equivalent stiffness, EI, for a rectangular beam to involve an exponential function with depth and location of crack effect, with solution of assuming equivalent stiffness beam (EI) using of Fourier series.

Shi-Rong Li, Romesh C. Batra [4], Analytical relations between the critical buckling load of a functionally graded material (FGM) Timoshenko beam and that of the corresponding homogeneous Euler–Bernoulli beam subjected to axial compressive load have been derived for clamped–clamped (C–C), simply supported–simply supported (S–S) and clamped–free (C–F) edges. For C–S beams, the transcendental equation has been derived to find the critical buckling load for the FGM Timoshenko beam which is similar to that for a homogeneous Euler–Bernoulli beam.

Liao-Liang Ke et al [5], the post-buckling response of beams made of functionally graded materials (FGMs) containing an open edge crack is studied based on Timoshenko beam theory and von Kármán nonlinear kinematics. Ritz method is employed to derive the nonlinear governing equations, which are then solved by using Newton–Raphson method to obtain the post-buckling load-end shortening curves and post-buckling deflection-end shortening curves.

G. Domokos et al [6], consider elastic buckling of an inextensible beam confined to the plane and subject to fixed end displacements, in the presence of rigid, frictionless side-walls which constrain overall lateral displacements. We formulate the geometrically nonlinear (Euler) problem, derive some analytical results for special cases, and develop a numerical shooting scheme for solution. We compare these theoretical and numerical results with experiments on slender steel beams.

To achieve the above objectives, analytical solution is developed for buckling analysis of beam with and without crack effect to evaluate the critical buckling load of beam using the analytical solution of general equation of motion of beam with crack effect, by building a computer program for analytical solution using Fortran power station 4.0 program, and compare the theoretical results with numerical results evaluated by using finite element method with using ANSYS program ver. 14.

2. Theoretical study

Consider the beam shown in Figure 1, having the following geometrical and material characteristics $(l, w, d, d_c, E, I(x), \rho)$, where; E-modulus of elasticity; and ρ-density of beam and other notations as shown in the figure. The beam is supposed to be loaded with a bending moment and to have a uniform transverse surface crack of depth (a) located at a given position xc from the left edge of the beam.

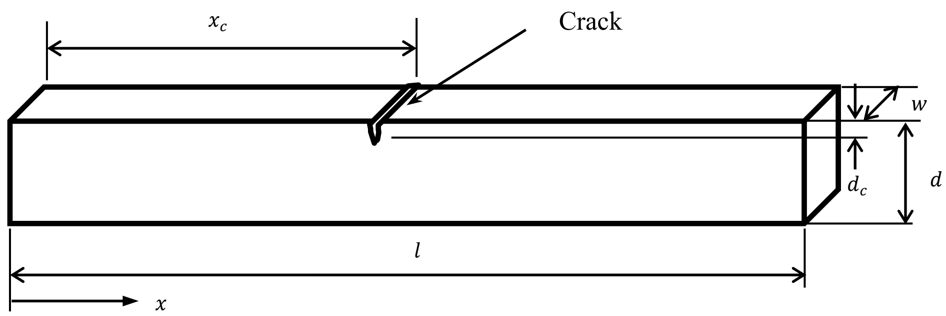

Figure 1. Dimensions of Beam with crack.

The effect of a crack in a continuous beam is done by calculating the stiffness, EI, for a rectangular beam to involve an exponential function and is given by, [7],

$$EI(x) = \frac{EI_0}{1 + C\exp(-2\alpha|x - x_c|/d)} \tag{1}$$

Where, $C = \frac{(I_0 - I_c)}{I_c}$, for, $I_0 = \frac{wd^3}{12}$ and $I_c = \frac{w(d-d_c)^3}{12}$.

w and d are the width and depth of the beam, respectively.

d_c is the crack depth, x is the position along the beam, and x_c the position of the crack.

α is a constant equal to (0.667), [7].

For buckling analysis of the beam having a crack with a finite length, relation Eq. 1 can be expanded as a sum of sine and cosine functions in the domain $0 \leq x \leq L$ by Fourier series, as, [8],

$$EI = \left[A_o + \sum_{n=1}^{\infty} A_n \cos\frac{2n\pi x}{L} + \sum_{n=1}^{\infty} B_n \sin\frac{2n\pi x}{L}\right] \tag{2}$$

Where, $A_o, A_n,$ and B_n are Fourier series constant can be evaluated as,

$$\begin{aligned} A_o &= \frac{1}{L}\int_0^L EI(x)dx = \frac{1}{L}\int_0^L \frac{EI_0}{1+C\exp(-2\alpha|x-x_c|/d)}dx \\ A_n &= \frac{2}{L}\int_0^L EI(x)\cos\frac{2n\pi x}{L}dx = \frac{2}{L}\int_0^L \frac{EI_0}{1+C\exp(-2\alpha|x-x_c|/d)}\cos\frac{2n\pi x}{L}dx \\ B_n &= \frac{2}{L}\int_0^L EI(x)\sin\frac{2n\pi x}{L}dx = \frac{2}{L}\int_0^L \frac{EI_0}{1+C\exp(-2\alpha|x-x_c|/d)}\sin\frac{2n\pi x}{L}dx \end{aligned} \tag{3}$$

By integral Eq. 3 by x, using Simpson's rule integration method, [9], gets the Fourier series constant, as,

$$\int_{x_i}^{x_f} f(x)dx = \frac{1}{3}\left(\frac{x_f-x_i}{\overline{m}_d}\right)\left[f(x_i) + f(x_f) + 4\sum_{s=1,3,5,\dots}^{\overline{m}_d-1} f(x_s) + 2\sum_{s=2,4,6,\dots}^{\overline{m}_d-2} f(x_s) +\right] \tag{4}$$

Where, $x_i = 0$ and $x_f = L$, $\overline{m}_d$ is the subdivisions of interval $[x_i, x_f]$, usually even number,
And, $x_s = x_i + \left(\frac{x_f-x_i}{\overline{m}_d}\right) s$
Then,

$$\begin{aligned} A_o &= \frac{1}{3L}\left(\frac{x_f-x_i}{\overline{m}_d}\right)\begin{pmatrix} \frac{EI_0}{1+C\exp(-2\alpha|x_i-x_c|/d)} + 4\sum_{s=1,3,5,\dots}^{\overline{m}_d-1}\frac{EI_0}{1+C\exp(-2\alpha|x_s-x_c|/d)} + \\ 2\sum_{s=2,4,6,\dots}^{\overline{m}_d-2}\frac{EI_0}{1+C\exp(-2\alpha|x_s-x_c|/d)} + \frac{EI_0}{1+C\exp(-2\alpha|x_f-x_c|/d)} \end{pmatrix} \\ A_n &= \frac{2}{3L}\left(\frac{x_f-x_i}{\overline{m}_d}\right)\begin{pmatrix} \frac{EI_0}{1+C\exp(-2\alpha|x_i-x_c|/d)}\cos\frac{2n\pi x_i}{L} + 4\sum_{s=1,3,5,\dots}^{\overline{m}_d-1}\frac{EI_0}{1+C\exp(-2\alpha|x_s-x_c|/d)}\cos\frac{2n\pi x_s}{L} + \\ 2\sum_{s=2,4,6,\dots}^{\overline{m}_d-2}\frac{EI_0}{1+C\exp(-2\alpha|x_s-x_c|/d)}\cos\frac{2n\pi x_s}{L} + \frac{EI_0}{1+C\exp(-2\alpha|x_f-x_c|/d)}\cos\frac{2n\pi x_f}{L} \end{pmatrix} \\ B_n &= \frac{2}{3L}\left(\frac{x_f-x_i}{\overline{m}_d}\right)\begin{pmatrix} \frac{EI_0}{1+C\exp(-2\alpha|x_i-x_c|/d)}\sin\frac{2n\pi x_i}{L} + 4\sum_{s=1,3,5,\dots}^{\overline{m}_d-1}\frac{EI_0}{1+C\exp(-2\alpha|x_s-x_c|/d)}\sin\frac{2n\pi x_s}{L} + \\ 2\sum_{s=2,4,6,\dots}^{\overline{m}_d-2}\frac{EI_0}{1+C\exp(-2\alpha|x_s-x_c|/d)}\sin\frac{2n\pi x_s}{L} + \frac{EI_0}{1+C\exp(-2\alpha|x_f-x_c|/d)}\sin\frac{2n\pi x_f}{L} \end{pmatrix} \end{aligned} \tag{5}$$

Then, by substation Eq. 5 into Eq. 2, get,

$$EI = \begin{pmatrix} \left[\frac{1}{3L}\left(\frac{x_f-x_i}{\overline{m}_d}\right)\begin{pmatrix} \frac{EI_0}{1+C\exp(-2\alpha|x_i-x_c|/d)} + 4\sum_{s=1,3,5,\dots}^{\overline{m}_d-1}\frac{EI_0}{1+C\exp(-2\alpha|x_s-x_c|/d)} + \\ 2\sum_{s=2,4,6,\dots}^{\overline{m}_d-2}\frac{EI_0}{1+C\exp(-2\alpha|x_s-x_c|/d)} + \frac{EI_0}{1+C\exp(-2\alpha|x_f-x_c|/d)} \end{pmatrix}\right] + \\ \sum_{n=1}^{\infty}\left[\frac{2}{3L}\left(\frac{x_f-x_i}{\overline{m}_d}\right)\begin{pmatrix} \frac{EI_0}{1+C\exp(-2\alpha|x_i-x_c|/d)}\cos\frac{2n\pi x_i}{L} + \\ 4\sum_{s=1,3,5,\dots}^{\overline{m}_d-1}\frac{EI_0}{1+C\exp(-2\alpha|x_s-x_c|/d)}\cos\frac{2n\pi x_s}{L} + \\ 2\sum_{s=2,4,6,\dots}^{\overline{m}_d-2}\frac{EI_0}{1+C\exp(-2\alpha|x_s-x_c|/d)}\cos\frac{2n\pi x_s}{L} + \\ \frac{EI_0}{1+C\exp(-2\alpha|x_f-x_c|/d)}\cos\frac{2n\pi x_f}{L} \end{pmatrix}.\cos\frac{2n\pi x}{L}\right] + \\ \sum_{n=1}^{\infty}\left[\frac{2}{3L}\left(\frac{x_f-x_i}{\overline{m}_d}\right)\begin{pmatrix} \frac{EI_0}{1+C\exp(-2\alpha|x_i-x_c|/d)}\sin\frac{2n\pi x_i}{L} + \\ 4\sum_{s=1,3,5,\dots}^{\overline{m}_d-1}\frac{EI_0}{1+C\exp(-2\alpha|x_s-x_c|/d)}\sin\frac{2n\pi x_s}{L} + \\ 2\sum_{s=2,4,6,\dots}^{\overline{m}_d-2}\frac{EI_0}{1+C\exp(-2\alpha|x_s-x_c|/d)}\sin\frac{2n\pi x_s}{L} + \\ \frac{EI_0}{1+C\exp(-2\alpha|x_f-x_c|/d)}\sin\frac{2n\pi x_f}{L} \end{pmatrix}.\sin\frac{2n\pi x}{L}\right] \end{pmatrix} \tag{6}$$

The effect of crack beam (as shown in Eq. 6) will be added to the general equation of buckling crack beam within driving the general equation of buckling beam from the general differential equation for bucking plate, [10], as,

$$M_{x,xx} - 2M_{xy,xy} + M_{y,yy} = -q \tag{7}$$

Since the behavior of beam is in one dimension (x-direction), then the derivative of deflection in y-direction is equal to zero, then reduce Eq. 7. to,

$$M_{x,xx} = -q \tag{8}$$

Where, [12],

$$M_x = -\,\mathrm{EI}\; w_{,xx} \tag{9}$$

Then, by substation Eq. 9 into Eq. 8, get,

$$(\mathrm{EI}\; w_{,xx}\,)_{xx} = q \tag{10}$$

So the equation of buckling beam will be as, with reduce of equation of plate, [12],

$$\frac{d^2}{dx^2}(\mathrm{EI}\; w_{,xx}\,) = -N_x w_{,xx} \tag{11}$$

Substation Eq. 2 into Eq. 11, get the general equation of buckling beam with crack effect, as,

$$\frac{d^2}{dx^2}\left(\left[A_o + \sum_{n=1}^{\infty} A_n \cos\frac{2n\,\pi\,x}{L} + \sum_{n=1}^{\infty} B_n \sin\frac{2n\,\pi\,x}{L}\right].\frac{d^2w}{dx^2}\right) = -N_x\frac{d^2w}{dx^2} \tag{12}$$

Or,

$$\begin{pmatrix} \frac{d^4w}{dx^4}\left[A_o + \sum_{n=1}^{\infty} A_n \cos\frac{2n\,\pi\,x}{L} + \sum_{n=1}^{\infty} B_n \sin\frac{2n\,\pi\,x}{L}\right] - \\ 2.\frac{d^3w}{dx^3}.\left(\frac{2n\,\pi}{L}\right)\left[\sum_{n=1}^{\infty} A_n.\sin\frac{2n\,\pi\,x}{L} - \sum_{n=1}^{\infty} B_n \cos\frac{2n\,\pi\,x}{L}\right] - \\ \frac{d^2w}{dx^2}.\left(\frac{2n\,\pi}{L}\right)^2\left[\sum_{n=1}^{\infty} A_n \cos\frac{2n\,\pi\,x}{L} + \sum_{n=1}^{\infty} B_n \sin\frac{2n\,\pi\,x}{L}\right] \end{pmatrix} = -N_x\frac{d^2w}{dx^2} \tag{13}$$

Assuming that the effect of crack is small on the deflection of beam, then the behaviors of beam with crack is assumed as the same the behaviors of beam without crack, for simply supported beam, as,

$$w = A\sin\frac{m\pi x}{L} \tag{14}$$

By substitution Eq. 14 into Eq. 13, get,

$$\begin{pmatrix} \left(\frac{m\pi}{L}\right)^4\left[A_o + \sum_{n=1}^{\infty} A_n \cos\frac{2n\,\pi\,x}{L} + \sum_{n=1}^{\infty} B_n \sin\frac{2n\,\pi\,x}{L}\right].\sin\frac{m\pi x}{L} + \\ 2\left(\frac{m\pi}{L}\right)^3\left(\frac{2n\,\pi}{L}\right)\left[\sum_{n=1}^{\infty} A_n.\sin\frac{2n\,\pi\,x}{L} - \sum_{n=1}^{\infty} B_n \cos\frac{2n\,\pi\,x}{L}\right].\cos\frac{m\pi x}{L} + \\ \left(\frac{m\pi}{L}\right)^2\left(\frac{2n\,\pi}{L}\right)^2\left[\sum_{n=1}^{\infty} A_n \cos\frac{2n\,\pi\,x}{L} + \sum_{n=1}^{\infty} B_n \sin\frac{2n\,\pi\,x}{L}\right].\sin\frac{m\pi x}{L} \end{pmatrix} = N_x\left(\frac{m\pi}{L}\right)^2.\sin\frac{m\pi x}{L} \tag{15}$$

Multiplying Eq. (15) by $\sin\left(\frac{m\pi x}{L}\right)$ and integral with x for $0 \le x \le l$, get, the buckling load of beam with crack effect, gives,

$$N_x = \frac{\int_0^L \begin{pmatrix} \left(\frac{m\pi}{L}\right)^4 \left[A_o + \sum_{n=1}^{\infty} A_n \cos\frac{2n\,\pi\,x}{L} + \sum_{n=1}^{\infty} B_n \sin\frac{2n\,\pi\,x}{L}\right] . \left(\sin\frac{m\pi x}{L}\right)^2 + \\ 2\left(\frac{m\pi}{L}\right)^3 \left(\frac{2n\,\pi}{L}\right) \left[\sum_{n=1}^{\infty} A_n . \sin\frac{2n\,\pi\,x}{L} - \sum_{n=1}^{\infty} B_n \cos\frac{2n\,\pi\,x}{L}\right] . \cos\frac{m\pi x}{L} . \sin\frac{m\pi x}{L} + \\ \left(\frac{m\pi}{L}\right)^2 \left(\frac{2n\,\pi}{L}\right)^2 \left[\sum_{n=1}^{\infty} A_n \cos\frac{2n\,\pi\,x}{L} + \sum_{n=1}^{\infty} B_n \sin\frac{2n\,\pi\,x}{L}\right] . \left(\sin\frac{m\pi x}{L}\right)^2 \end{pmatrix} .dx}{\int_0^L \left[\left(\frac{m\pi}{L}\right)^2 . \left(\sin\frac{m\pi x}{L}\right)^2\right] .dx} \tag{16}$$

Integration of Eq. 16 with respect to x, using Simpson's rule integration method, [9], gives the critical buckling load of beam with crack effect, as,

$$\int_{x_i}^{x_f} f(x)dx = \frac{1}{3}\left(\frac{x_f - x_i}{\overline{m}_d}\right)\left[f(x_i) + 4\sum_{s=1,3,5,\dots}^{\overline{m}_d - 1} f(x_s) + 2\sum_{s=2,4,6,\dots}^{\overline{m}_d - 2} f(x_s) + f(x_f)\right] \tag{17}$$

Where, $x_i = 0$ and $x_f = L$, and $\overline{m}_d$ is the subdivisions of interval $[x_i, x_f]$, usually even number, and, $x_s = x_i + \left(\frac{x_f - x_i}{\overline{m}_d}\right) s$

Then, the critical buckling loads (N_x) of beam in (N), evaluated as,

$$N_x = \frac{\frac{1}{3}\left(\frac{x_f - x_i}{\overline{m}_d}\right) \begin{pmatrix} \begin{bmatrix} \left(\frac{m\pi}{L}\right)^4 \left[A_o + \sum_{n=1}^{\infty} A_n \cos\frac{2n\,\pi\,x_i}{L} + \sum_{n=1}^{\infty} B_n \sin\frac{2n\,\pi\,x_i}{L}\right] . \left(\sin\frac{m\pi x_i}{L}\right)^2 + \\ 2.\left(\frac{m\pi}{L}\right)^3 . \left(\frac{2n\,\pi}{L}\right) \left[\sum_{n=1}^{\infty} A_n . \sin\frac{2n\,\pi\,x_i}{L} - \sum_{n=1}^{\infty} B_n \cos\frac{2n\,\pi\,x_i}{L}\right] . \cos\frac{m\pi x_i}{L} . \sin\frac{m\pi x_i}{L} + \\ \left(\frac{m\pi}{L}\right)^2 . \left(\frac{2n\,\pi}{L}\right)^2 \left[\sum_{n=1}^{\infty} A_n \cos\frac{2n\,\pi\,x_i}{L} + \sum_{n=1}^{\infty} B_n \sin\frac{2n\,\pi\,x_i}{L}\right] . \left(\sin\frac{m\pi x_i}{L}\right)^2 \end{bmatrix} + \\ 4\sum_{s=1,3,5,\dots}^{\overline{m}_d - 1} \begin{bmatrix} \left(\frac{m\pi}{L}\right)^4 \left[A_o + \sum_{n=1}^{\infty} A_n \cos\frac{2n\,\pi\,x_s}{L} + \sum_{n=1}^{\infty} B_n \sin\frac{2n\,\pi\,x_s}{L}\right] . \left(\sin\frac{m\pi x_s}{L}\right)^2 + \\ 2.\left(\frac{m\pi}{L}\right)^3 . \left(\frac{2n\,\pi}{L}\right) \left[\sum_{n=1}^{\infty} A_n . \sin\frac{2n\,\pi\,x_s}{L} - \sum_{n=1}^{\infty} B_n \cos\frac{2n\,\pi\,x_s}{L}\right] . \cos\frac{m\pi x_s}{L} . \sin\frac{m\pi x_s}{L} + \\ \left(\frac{m\pi}{L}\right)^2 . \left(\frac{2n\,\pi}{L}\right)^2 \left[\sum_{n=1}^{\infty} A_n \cos\frac{2n\,\pi\,x_s}{L} + \sum_{n=1}^{\infty} B_n \sin\frac{2n\,\pi\,x_s}{L}\right] . \left(\sin\frac{m\pi x_s}{L}\right)^2 \end{bmatrix} + \\ 2\sum_{s=2,4,6,\dots}^{\overline{m}_d - 2} \begin{bmatrix} \left(\frac{m\pi}{L}\right)^4 \left[A_o + \sum_{n=1}^{\infty} A_n \cos\frac{2n\,\pi\,x_s}{L} + \sum_{n=1}^{\infty} B_n \sin\frac{2n\,\pi\,x_s}{L}\right] . \left(\sin\frac{m\pi x_s}{L}\right)^2 + \\ 2.\left(\frac{m\pi}{L}\right)^3 . \left(\frac{2n\,\pi}{L}\right) \left[\sum_{n=1}^{\infty} A_n . \sin\frac{2n\,\pi\,x_s}{L} - \sum_{n=1}^{\infty} B_n \cos\frac{2n\,\pi\,x_s}{L}\right] . \cos\frac{m\pi x_s}{L} . \sin\frac{m\pi x_s}{L} + \\ \left(\frac{m\pi}{L}\right)^2 . \left(\frac{2n\,\pi}{L}\right)^2 \left[\sum_{n=1}^{\infty} A_n \cos\frac{2n\,\pi\,x_s}{L} + \sum_{n=1}^{\infty} B_n \sin\frac{2n\,\pi\,x_s}{L}\right] . \left(\sin\frac{m\pi x_s}{L}\right)^2 \end{bmatrix} + \\ \begin{bmatrix} \left(\frac{m\pi}{L}\right)^4 \left[A_o + \sum_{n=1}^{\infty} A_n \cos\frac{2n\,\pi\,x_f}{L} + \sum_{n=1}^{\infty} B_n \sin\frac{2n\,\pi\,x_f}{L}\right] . \left(\sin\frac{m\pi x_f}{L}\right)^2 + \\ 2.\left(\frac{m\pi}{L}\right)^3 . \left(\frac{2n\,\pi}{L}\right) \left[\sum_{n=1}^{\infty} A_n . \sin\frac{2n\,\pi\,x_f}{L} - \sum_{n=1}^{\infty} B_n \cos\frac{2n\,\pi\,x_f}{L}\right] . \cos\frac{m\pi x_f}{L} . \sin\frac{m\pi x_f}{L} + \\ \left(\frac{m\pi}{L}\right)^2 . \left(\frac{2n\,\pi}{L}\right)^2 \left[\sum_{n=1}^{\infty} A_n \cos\frac{2n\,\pi\,x_f}{L} + \sum_{n=1}^{\infty} B_n \sin\frac{2n\,\pi\,x_f}{L}\right] . \left(\sin\frac{m\pi x_f}{L}\right)^2 \end{bmatrix} \end{pmatrix}}{\frac{1}{3}\left(\frac{x_f - x_i}{\overline{m}_d}\right)\left(\left[\left(\frac{m\pi}{L}\right)^2 . \left(\sin\frac{m\pi x_i}{L}\right)^2\right] + 4\sum_{s=1,3,5,\dots}^{\overline{m}_d - 1}\left[\left(\frac{m\pi}{L}\right)^2 . \left(\sin\frac{m\pi x_s}{L}\right)^2\right] + 2\sum_{s=2,4,6,\dots}^{\overline{m}_d - 2}\left[\left(\frac{m\pi}{L}\right)^2 . \left(\sin\frac{m\pi x_s}{L}\right)^2\right] + \left[\left(\frac{m\pi}{L}\right)^2 . \left(\sin\frac{m\pi x_f}{L}\right)^2\right]\right)} \tag{18}$$

Using of the building a computer program, as shown in flow chart (Figure 2), for analytical solution using Fortran power station 4.0 program, can be results of above equations can be solved to evaluate critical buckling of different dimensions and materials beam with crack depth and location effect.

The program evaluated the buckling load of different materials simply supported beam and different beam dimensions with crack depth and location effect. The requires input of program are the beam dimensions and mechanical properties as modulus of elasticity, and the output results are buckling load of beam with crack depth and location effect.

3. Numerical study

The numerical study of buckling analysis for beam is done using the finite elements method using the ANSYS program (ver.14). The three dimensional model were built and the element (Solid Tet 10 node 187) were used. Solid 187 elements is a higher order 3-D, 10-node element. Solid 187 has a quadratic displacement behavior and is well suited to modelling irregular meshes. The element is defined by 10 nodes having three degrees of freedom at each node: translations in the nodal x, y, and z directions.

The element has plasticity, hyper-elasticity, creep, stress stiffening, large deflection, and large strain capabilities. It also has mixed formulation capability for simulating deformations of nearly incompressible elasto-plastic materials, and fully incompressible hyper-elastic materials. In addition to the nodes, the element input data is includes the orthotropic or anisotropic material properties.

Orthotropic and anisotropic material directions correspond to the element coordinate directions. The geometry, node locations, and the coordinate system for this element are shown in Figure 3. A sample of meshed beam is shown in Figure 4, with crack effect.

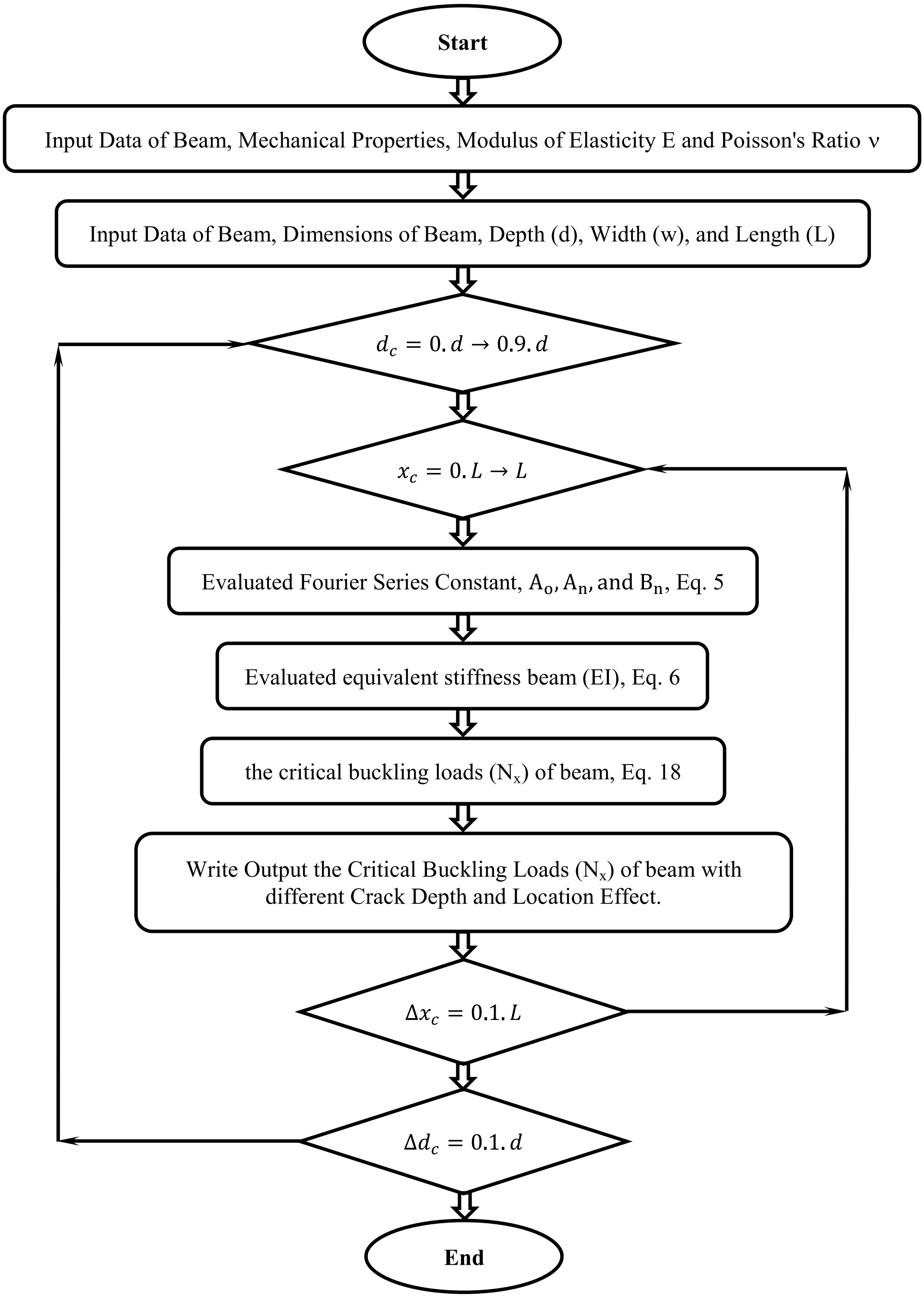

Figure 2. Flow chart of Fortran computer program, for evaluating the buckling load of beam with crack depth and location effect.

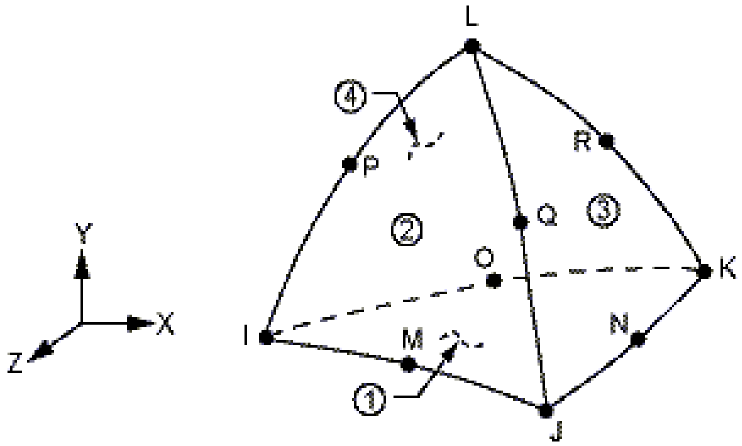

Figure 3. Geometry of solid 187 element.

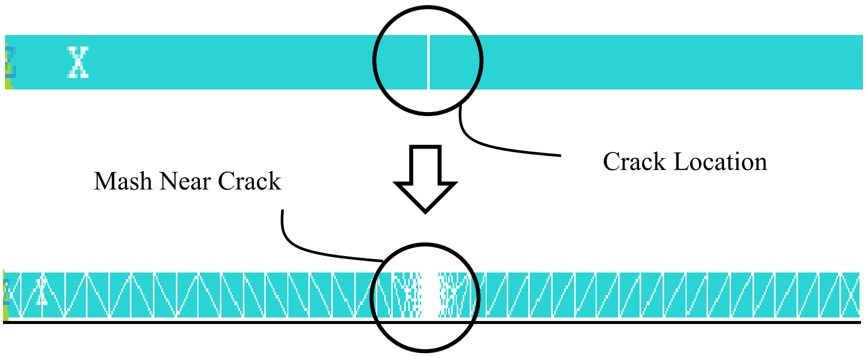

Figure 4. Mash of beam with crack.

4. Results and discussion

The results at the buckling load of different beam materials and dimensions with crack depth and location effect are evaluated using analytical solution with solution of general equation of simply supported beam with crack effect with suggested solution of equivalent stiffness beam (EI) with crack by Fourier series method. And, suggested analytical solution of beam equation with crack effect is done using orthogonally method and integral of the final equation by Simpson's rule integration method to evaluate the buckling load of beam. And, a comparison is done between the theoretical results with numerical results evaluated with finite element method, using Ansys program.

The dimensions of beam, Length (L), width (w) and depth of beam (d), used are,

$L = 0.25, 0.5, 0.75, 1, 1.25, 1.5$, with different depth and width beam as,

- $Width\ of\ Beam = w = 15\ mm,\ \ with\ Depth\ Beam = d = 5, 15, 25\ mm$
- $Depth\ of\ Beam = d = 15\ mm,\ \ with\ Width\ Beam = w = 5, 15, 25\ mm$

And, the beam types used are, steel, aluminum and copper beam materials, and, the mechanical properties of beam materials used are shown in Table 1, [13].

Table 1. Mechanical properties of beam materials used, [13].

Materials	Density ρ (kg/m^3)	Modulus of Elasicity E (Gpa)	Posiion's Ratio ν
Steel	7800	205	0.3
Aluminum	2800	75	0.3
Copper	8800	125	0.3

The compare between theoretical results, evaluated by solution of general equation of beam with crack effect, and numerical results, evaluated by finite element method, are shown in Figures 5, 6 and 7. For steel and aluminum beam materials with different depth and width beam (d=5, 15 mm and w=5, 15 mm) and various beam length, L=0.5, 1, 1.5 m, respectively, with crack depth and location effect. From the figures show the good agreement between theoretical and numerical results with maximum error about (3.6%).

Figures 8 and 9 show contour of buckling load of simply supported beam for steel beam materials types with crack depth and location effect of different beam length (L=0.25, 0.5, 1, and 1.5 m) and different beam depth (d=15 and 25 mm) with width beam w=15 mm, Figure 8, and different beam width (w=15 and 25 mm) with beam depth d=15 mm, Figure 9. These figures show that the buckling load decreases with increases of crack depth, due to decreasing the stiffness of beam, and the buckling load decreases with crack location near the middle location of beam more than other location of crack (near the ends of beam), since the effect of crack near middle location has more effect on the stiffness and buckling load of beam from the other positions. In addition to, the buckling loads increase with increasing of the modulus of elasticity of beam. And, the buckling load increases with the increasing of the depth or width of beam, due to the increasing of the stiffness of beam, also, the buckling load decreases with increasing of the beam length, due to decreasing the stiffness of beam with increasing of the beam length.

Figure 10 show the effect of beam length on the buckling load of simply supported beam with middle crack location ($x_c = 0.5L$) and crack depth ($d_c = 0.5d$), for depth and width of beam $d = w = 15\ mm$ and different beam materials (steel, aluminum, and copper materials). The figure shows that the buckling load of beam decreases with increasing of beam length due to decreasing of beam stiffness and the length when increases more than (0.75 m) the decreases in buckling load of beam less than the decreasing of buckling load when length beam decreases from 0.25 to 0.75 m.

Figures 11 and 12 show the effect of beam depth and width, respectively, on the buckling load of simply supported beam with middle crack location ($x_c = 0.5L$) and crack depth ($d_c = 0.5d$), for beam length (L=1 m) and beam width $w = 15\ mm$, and beam depth $d = 15\ mm$, (Figure 12), and different beam materials (steel, aluminum, and copper materials). The figure shows that the buckling loads beam increases with increasing of beam depth or width due to the increasing of beam stiffness.

5. Conclusion

The main conclusions for this work with theoretical and numerical investigation study of crack depth and location effect of beam are,

1. The suggested analytical solution is a powerful tool for buckling investigation of beam with crack depth and location effect.
2. The stiffness and buckling load of beam decreases with increasing of the crack depth.
3. The buckling load of beam decreases with middle crack location of simply supported beam more than other location of crack.
4. The buckling load of beam increases with the increasing of beam depth or width, but the increases of buckling load with the increasing of the beam depth more than the increasing of buckling load with increasing of beam width.
5. The crack in beam causes decreasing of the stiffness of beam, and then causes decreasing of the buckling load of beam. And, the position of crack in the beam near the middle of the beam has more effect on the stiffness and buckling load of beam from the other positions, near to the ends beam.
6. A comparison is made between analytical results from solution of general equation of buckling beam and crack effect with numerical results by finite elements method showing a good approximation.

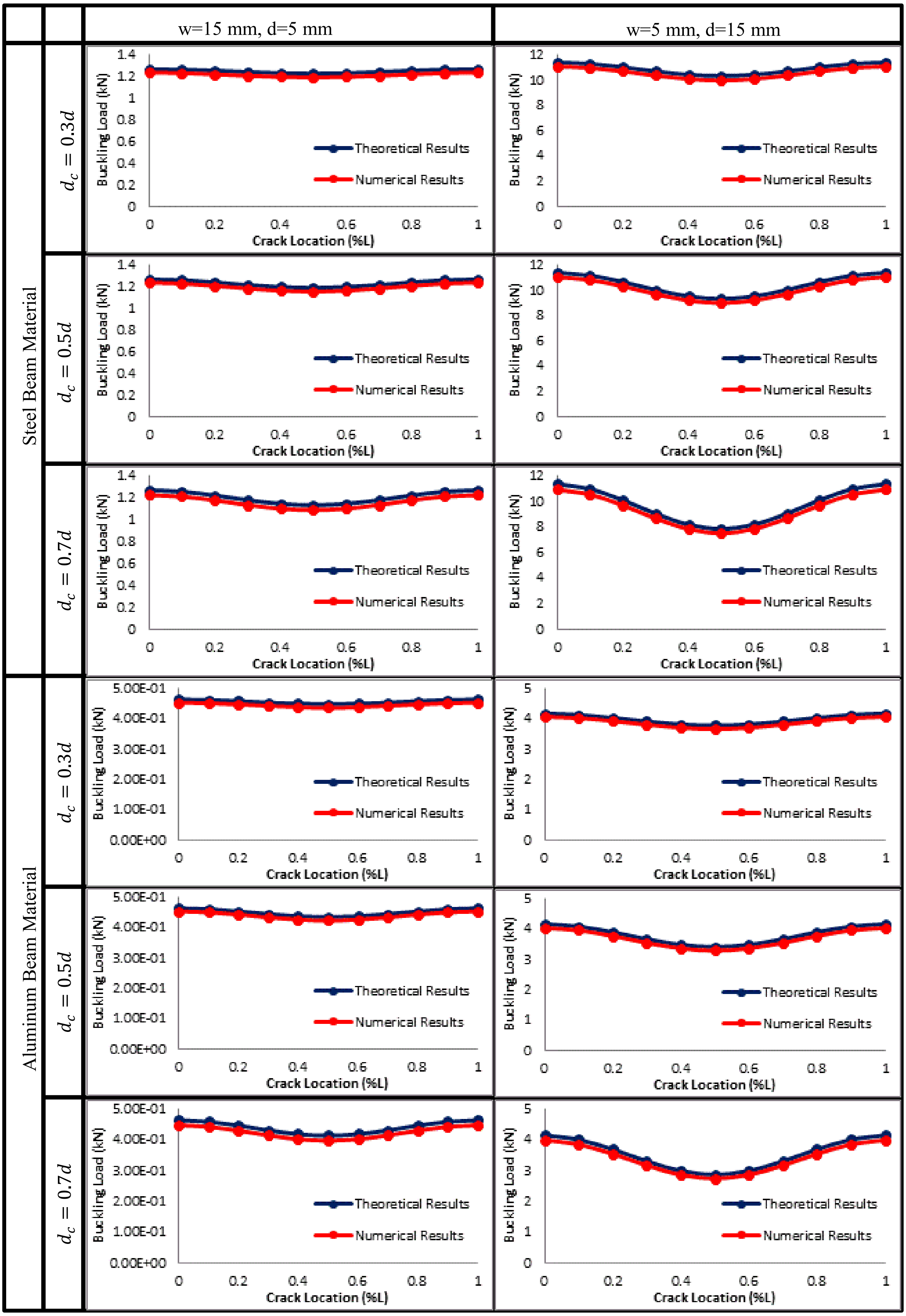

Figure 5. Compare Between Theoretical and numerical Results for Steel and Aluminum Beam with Different Depth and Width Beam and Various Crack Depth and Location Effect, for L=0.5 m.

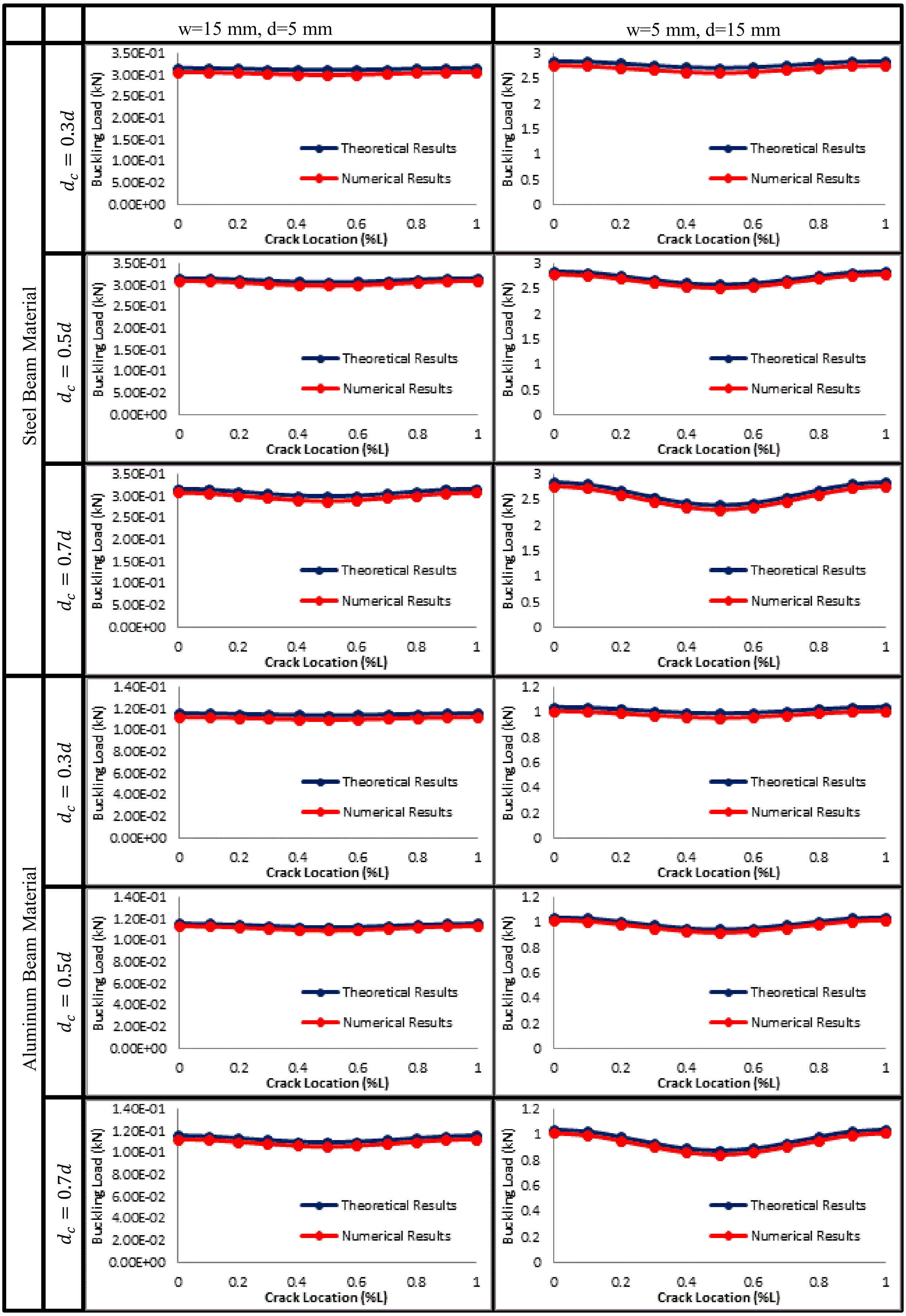

Figure 6. Compare Between Theoretical and numerical Results for Steel and Aluminum Beam with Different Depth and Width Beam and Various Crack Depth and Location Effect, for L=1 m.

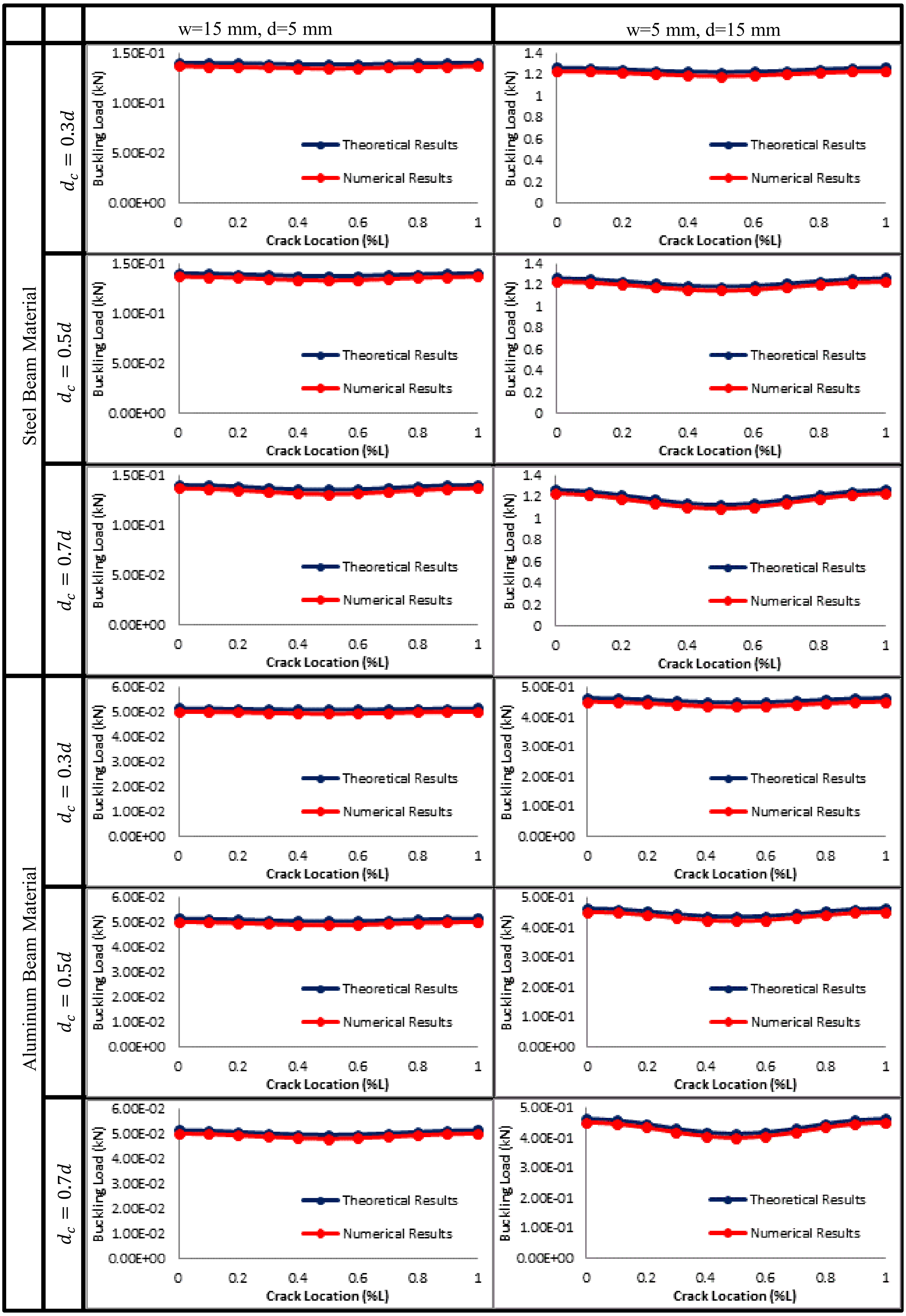

Figure 7. Compare Between Theoretical and numerical Results for Steel and Aluminum Beam with Different Depth and Width Beam and Various Crack Depth and Location Effect, for L=1.5 m.

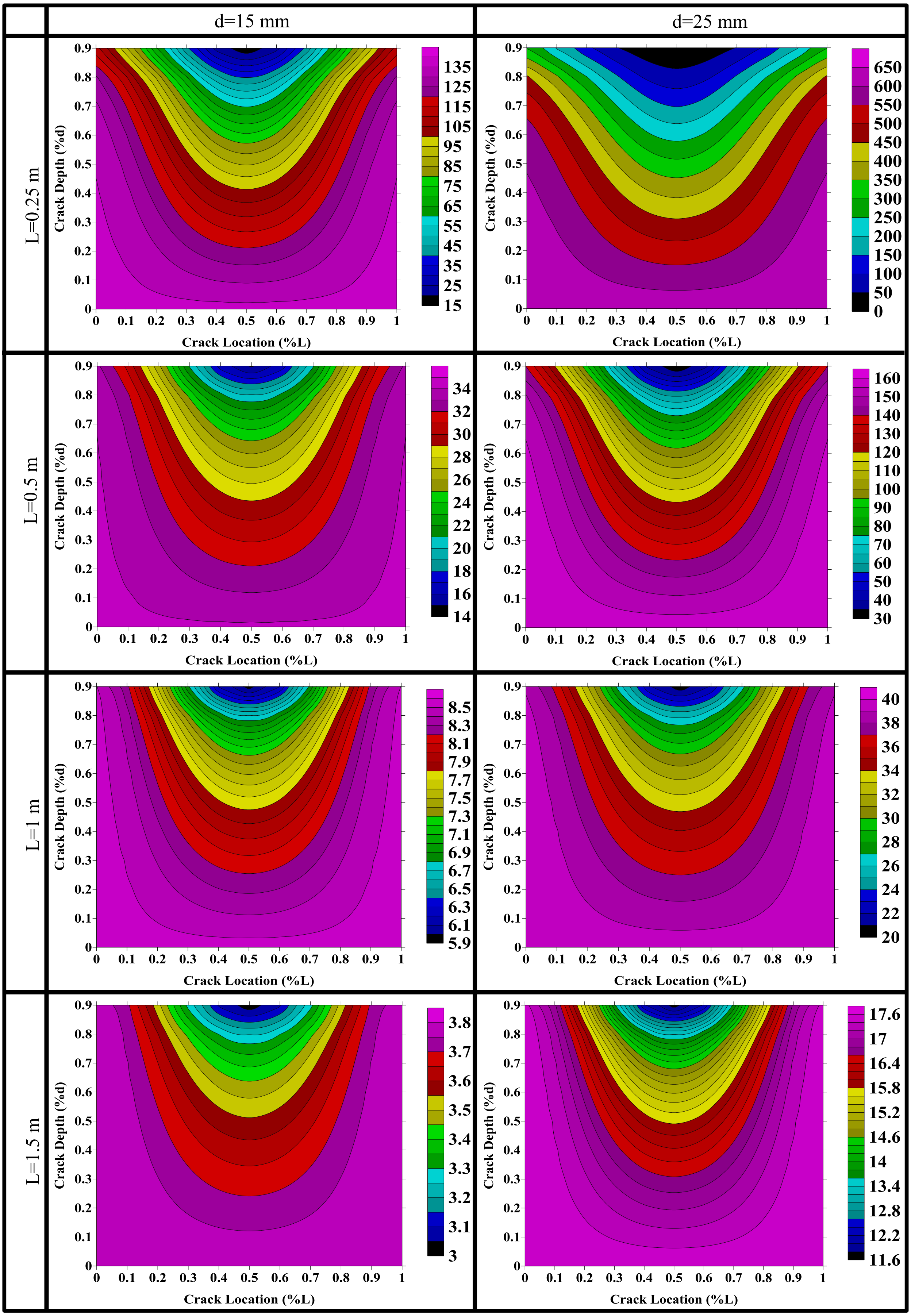

Figure 8. Contour Buckling Load (kN) with Crack Depth and Location Effect for Various Length and Depth Steel Beam, for Beam Width w = 15 mm.

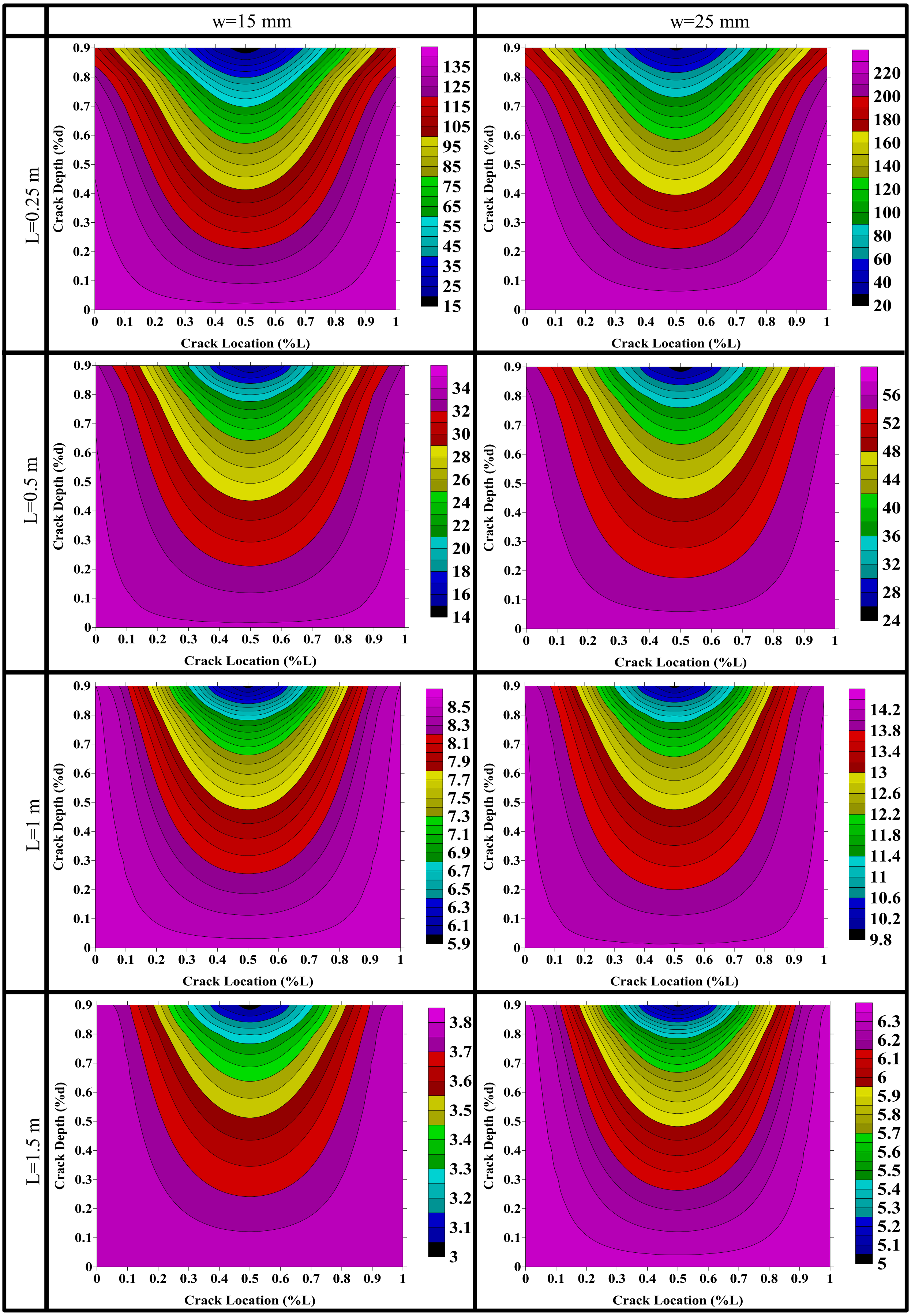

Figure 9. Contour Buckling Load (kN) with Crack Depth and Location Effect for Various Length and Width Steel Beam, for Beam Depth d = 15 mm.

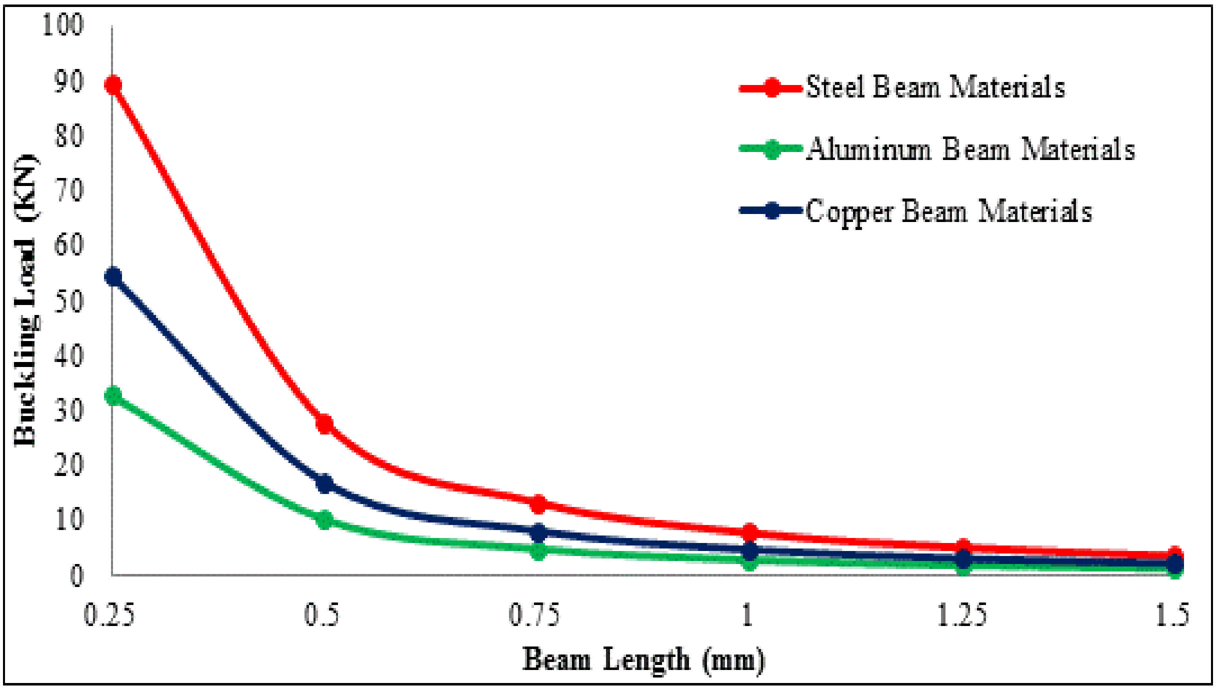

Figure 10. Buckling Beam Load with Various Beam Length and Different Beam Materials for $d = w = 15\ \mathrm{mm}, d_c = 0.5d, x_c = 0.5L$.

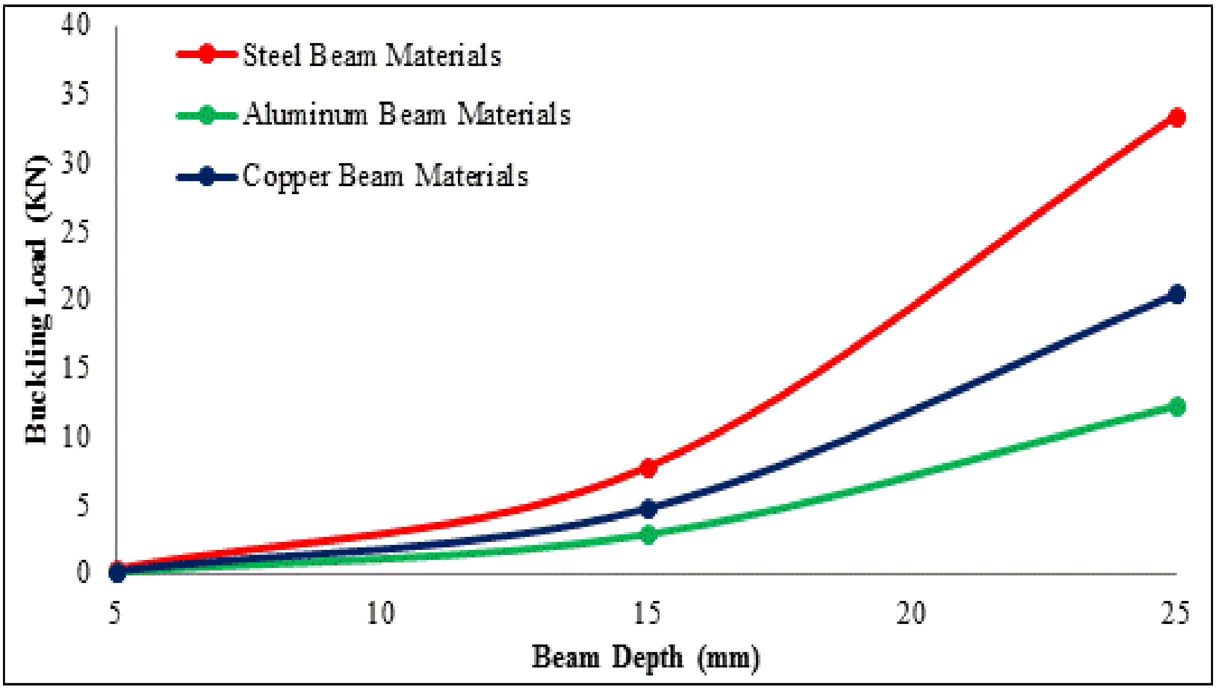

Figure 11. Buckling Beam Load with Various Beam Depth and Different Beam Materials for $L = 1\ \mathrm{m}, w = 15\ \mathrm{mm}, d_c = 0.5d, x_c = 0.5L$.

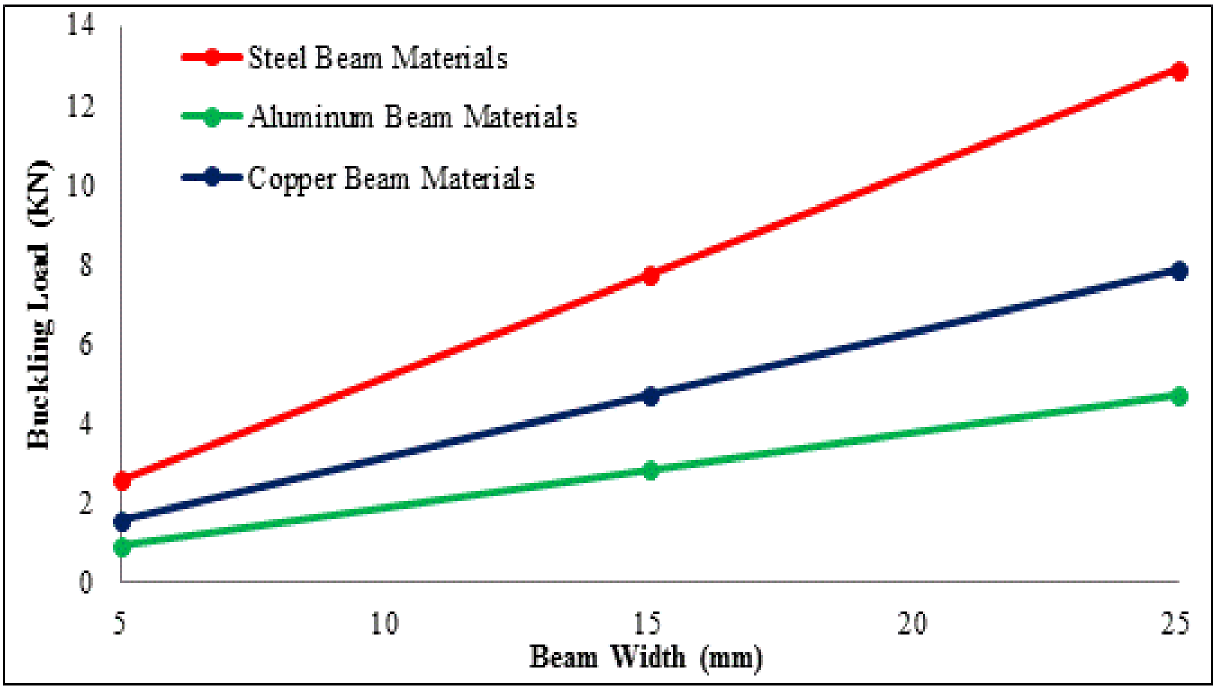

Figure 12. Buckling Beam Load with Various Beam Width and Different Beam Materials for $L = 1\ m, d = 15\ mm, d_c = 0.5d, x_c = 0.5L$.

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