Scale-up criterion of power consumption for a surface aerator used in wastewater treatment tank

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Abstract
The major part of operation costs in surface aeration basins or tanks is because of power requirements. Therefore, it is always necessary to find a dependable criterion for the predictive scale-up of power consumption measurements obtained at laboratory-scale surface aeration tanks to industrial-scale wastewater treatment surface aeration systems. A scale-up approach was proposed in this work for volumetric power consumption between geometrically similar laboratory-scale and industrial full-scale surface aeration tanks at an invariant Froude number Fr. Scale-up order between the laboratory and industrial sizes was 7.4. A mathematical correlation has been developed to estimate the volumetric power consumption and then compared with a model that already was investigated experimentally. Scale-up criterion involved the evaluation of three similarities; the geometrical, kinematic and dynamics. The scale-up basis that developed in this work led us to achieve a suitable scale-up criterion for volumetric power consumption in aeration tanks at matched surface flow condition. At matched Froude number Fr for the laboratory and industrial scales and at low and moderate turbine rotation speeds for surface aeration than 0.8 rps, complete predictions of volumetric power consumption have been achieved. The prediction by the existing previous model showed higher results than the actual values.

Keywords: Power consumption; Scale-up; Surface aeration; Wastewater treatment tanks.

1. Introduction
Essentially scale-up is applied to figure out the possibility to have an optimal or the most proper geometrical configuration for a system. Furthermore, scale-up helps to determine the operational condition that can be implemented for a full-scale unit in the industrial field to have an identical efficiency and performance with the that obtained from a laboratory-scale model [1]. The scale-up criterion of agitated and aerated tanks based on the following; (1) quantification of mixing time and flow rates [2, 3], (2) power consumption quantification [4, 5], and (3) optimization of operational parameters like impeller rotation speed [6]. The design of surface aeration with mixing processes needs to predict adequately the power consumption [7]. Scale-up of surface aerators in agitated tanks for wastewater treatment depends on the power consumption empirical data obtained from experimental tests. Hence, there are various power consumption correlation models that employed for the aeration process scale-up [8, 9]. The power consumption is usually presented as a dimensionless parameter and used to obtain a correlation that relates it to independent parameters by using dimensional analysis [10, 11]. These correlation models are very indispensable for designing surface aerators in the agitated tank in order to accomplish a sufficient dissolved oxygen concentration in the liquid bulk by varying operational and
geometrical parameters without exceeding any limits of power consumption [12]. If a complex reaction occurs within the agitated tank, it will be difficult to adopt an empirical evaluation to scale-up the unit to larger scales [13]. Different approaches have been followed to scale-up aerated and agitated tanks, many of them aim to predict the occurring process variables in the large size tanks. To achieve this aim some study has kept the volumetric power consumption constant between the tanks [14]. While other approaches assumed the superficial gas velocity could be kept constant in the small and large tanks in order to predict the volumetric power consumption behavior in the larger tank [15].

From an economical point of view, it is expensive to build a full-scale surface aeration system for wastewater treatment in order to investigate the power consumption. So the aim of this work is developing a new approach for the predictive scale-up of volumetric power consumption results at the laboratory-scale surface aeration system to a geometrically similar industrial full-scale system used for wastewater treatment. Moreover, the scale-up criterion for the power consumption based on a matched water surface flow condition between the laboratory and industrial sizes.

2. Materials and methods

2.1 Previously developed models

Issa [16] developed in his work a model for a surface aerator (as presented in Equation 10) in depending on experimental runs that carried out in a lab-scale cylindrical flat bottom tank. The surface aerator consist of two impellers of a turbine and an up pumping propeller. The diameter ratio D/T equals 0.24, the geometrical ratio for the propeller diameter d/T is 0.15.

2.2 Industrial and laboratory scale surface aeration systems

For scale-up purposes, a theoretical surface aeration system was assumed to be geometrically and dynamically similar to industrial scale unit as shown in Figure 1. The actual readings were taken from a low speed turbine type of an aerator (without a lower impeller), and working in sewage plant in Iraq. The tank has a square shape, so T is tank width instead of diameter.

![Figure 1. The schematic diagram of the surface aerator system.](image)

2.3 Dimensional analysis for power consumption in the agitated surface aerated tank

The dimensional analysis method is applied to minimize the time and expenses spent on experiments and obtain valuable information from the fewest number of experiments possible because the analytical solutions are so complicated for real fluid dynamic systems [17-19]. In surface aeration, power consumption is influenced by many parameters during the operation of physical properties of water and air, operational parameters like rotation speed, flow conditions at the surface and inside the tank, geometrical ratios of the tank and impellers. These parameters are categorized into main dimensionless
groups by the dimensional analysis method. In this work, a part of mentioned parameters were used according to the design objective for deriving a predictive correlation for volumetric power consumption. In surface aeration agitated tanks, each group of variables are governed by a different kind of the system characteristics as shown in Table 1 [10, 20-22]. Key parameters and operating conditions are required to be identified for the surface aeration process over the range of matched Froude number Fr values are shown in Table 1.

Table 1. A set of universally proposed relevant design and operation parameters on volumetric power consumption in surface aeration system.

<table>
<thead>
<tr>
<th>Category</th>
<th>Parameter</th>
<th>Description</th>
<th>SI unit</th>
<th>Dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geometrical variables</td>
<td>D</td>
<td>Turbine diameter</td>
<td>m</td>
<td>L</td>
</tr>
<tr>
<td></td>
<td>d</td>
<td>Impeller diameter</td>
<td>m</td>
<td>L</td>
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<td></td>
<td>H</td>
<td>Water level</td>
<td>m</td>
<td>L</td>
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<td></td>
<td>C</td>
<td>Turbine clearance</td>
<td>m</td>
<td>L</td>
</tr>
<tr>
<td></td>
<td>W</td>
<td>Turbine blades width</td>
<td>m</td>
<td>L</td>
</tr>
<tr>
<td></td>
<td>T</td>
<td>Tank width</td>
<td>m</td>
<td>L</td>
</tr>
<tr>
<td></td>
<td>S</td>
<td>Turbine Blades Submergence</td>
<td>m</td>
<td>L</td>
</tr>
<tr>
<td></td>
<td>V</td>
<td>Tank volume</td>
<td>m$^3$</td>
<td>M$^3$</td>
</tr>
<tr>
<td>Material variables</td>
<td>ρ</td>
<td>Water density</td>
<td>kg/m$^3$</td>
<td>ML$^3$</td>
</tr>
<tr>
<td></td>
<td>µ</td>
<td>Water viscosity</td>
<td>Kg/m s</td>
<td>ML$^-1$T$^-1$</td>
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<tr>
<td>Process variables</td>
<td>g</td>
<td>Acceleration of gravity</td>
<td>m/s$^2$</td>
<td>MT$^-2$</td>
</tr>
<tr>
<td></td>
<td>N</td>
<td>Rotation speed</td>
<td>s$^{-1}$</td>
<td>T$^-1$</td>
</tr>
</tbody>
</table>

Variables illustrated in Table 1 can be expressed in a functional relation with the power consumption

\[ P = f_1(D, C, d, H, S, V, W, T, \rho, \mu, N, g) \]  

(1)

By applying Buckingham \( \pi \) theorem, Equation 1 was replaced and rearranged in terms of selected \( \pi \) independents, for principal length unit (tank width T).

\[ N_p = f_2\left(\frac{D}{T}, \frac{C}{T}, \frac{d}{T}, \frac{H}{T}, \frac{S}{T}, \frac{V}{T}, \frac{W}{T}, R_e, F_r\right) \]  

(2)

where, \( N_p = P/\rho N^3D^5 \)

2.4 Similarity for surface aerators in lab and industrial scales

The geometrical and dynamic (material and process) similarities are needed to be established to predict the process in tanks of different sizes under different dynamic conditions [17, 22]. To apply similarities in surface aeration that related to operational conditions and configurations, there are main guidelines discussed for each as follows.

2.4.1 Geometrical similarity

Geometrical similarity requires that laboratory-scale and industrial-scale systems should be of the same shape, and all the linear dimensions of the laboratory-scale should be related to corresponding dimensions of the industrial-scale by a constant scale factor [17]. In order to achieve an optimal condition of the lowest power consumption prediction, a specific set of dimensionless geometrical parameters were made by applying the second Buckingham theorem as presented in Table 1. The obtained correlation is limited to specific ranges of the independent parameters. By varying one independent dimensionless parameter at a time with observing the response of the dependent parameter with it, a single functional relationship between the dependent parameter and all the independent parameters can be established [17]. The attempt to identify the optimal condition for various independent geometrical may be different for each one of these variables.
2.4.2 Kinematic similarity
Kinematic similarity refers to motion occurred in a system and requires geometrical similarity [23], where the ratio of velocities in the laboratory-scale system and the industrial-scale for corresponding positions should be same. The kinematic similarity has especial importance as the surface aeration is highly related to the velocity distribution in the system.

2.4.3 Dynamic similarity
For similar geometrical and kinematic laboratory-scale and industrial-scale surface aeration systems, the dynamic similarity can be reached for the two scales as all forces acting on the liquid and the spray in the system are same. The inertial, viscous and gravity forces, acting in the surface aeration process, are expressed by two dimensionless parameters, $Fr$, and $Re$ [17]. If values of the dimensionless geometrical parameters are fixed, the dimensionless power consumption $N_p$ will depend on the dynamic parameters, Froude number $Fr$ and Reynolds number $Re$. The predominant parameter in the operation is the water surface agitation condition and the flow regime, which is always kept in turbulent region ($Re > 10^4$), so $Re$ is considered irrelevant to the process objective, so Equation 2 was reduced to

$$N_p = f_3(Fr)$$

(3)

2.5 Invariants identification
The identification of invariants of each process depends on the limiting phenomena that control the quality of the operation or it depends on the operation purpose. The determination of dynamic similarity is not always tied to the identification of invariants where sometimes intermediate solutions are sufficient to figure out the dynamic similarity [1]. In our case, the D/T ratio has been chosen to have geometric similar in both scales of surface aeration systems. The most common invariant dynamic parameters for surface aeration in agitated tank systems can be the Froude number. For aerated systems, other criteria were used for scale-up such as keep the volumetric power ration as an invariant but this criterion didn’t show a good agreement of the results between the tested scales [24]. The dynamic parameter $Fr$ was considered as an invariant in order to achieve total dynamic similitude for the water surface among the geometrically similar systems. Froude number was kept constant by changing the value of $N$. As all the experiments were conducted using water, it is impossible to maintain both $Fr$ and $Re$ as invariants, this agrees with [17], to keep $Fr$ invariant during the surface aeration and water agitation definitively this leads to $Re$ will be variant.

3. Results and discussion
For scale-up purposes, the theoretical laboratory-scale surface aeration system was assumed to be geometrical, kinematical, and dynamical similar to the existed industrial scale. The two processes at the laboratory and industry scales are considered completely similar since same geometrical and dynamic similarities are achieved. The physical properties of the operating fluids and values of the operation related numbers of $Re$ or $Fr$ were already measured for the industrial scale system. For the tested surface aeration, it was found the dynamic similarity often depends on the $Fr$ rather than $Re$ as the surface aeration is mainly controlled by water surface condition. At the laboratory-scale surface aeration system, the same values of $Re$ or $Fr$ were kept constant by manipulating the operation and process conditions. Hence, the surface aeration process takes place in similar geometrical systems and all the developed flow values will have the same numerical values at both scales. For example, Froude numbers that measured at the laboratory-scale surface aeration system for given Reynolds numbers correspond to Froude numbers measured at the industrial-scale. With complete similarity at the both scales of the flow conditions, this will lead to consider the obtained power consumption values will be of the same magnitude since they are a function of $Re$, $Fr$, and geometry.

Surface aeration operation consists of surface flow and hydrodynamics related to power consumption, which is highly dependent on the impellers rotation speed. Also, surface aeration operation consists in water spray discharge flow which is related to power consumption, and it depends on turbine blades submergence. Consequently, the dynamic similarity is governed by $Fr$.

The practical attempt to scale-up the investigated surface aeration results to larger industrial scale was carried out to explore the power consumption at the industrial scale. As the geometrical ratios are the same in both scales, the turbine diameter to tank width ratios leads to the following relations.
where subscripts 2 and 1 refer to industrial and laboratory scales respectively. For dynamic similarity, the Fr number was kept constant (invariant) as the surface aerator performance is controlled by the water surface condition; the dynamic similarity of Fr number was accomplished by manipulation of the rotation speed as showed. The scale ratio of 7.4 was used on a theoretical basis to compare the developed model in this work with that proposed by Issa [16].

\[
\left( \frac{D}{T} \right)_{\text{lab}} = \left( \frac{D}{T} \right)_{\text{ind}} = 0.2
\]  

\[ D_2 = kD_1 \]  

As explained in the Equation 6, for geometrically similar systems the following consequence was produced.

\[
(N)_{\text{ind}} = k^{-0.5} (N)_{\text{lab}}, k = 7.4
\]

\[
(P)_{\text{ind}} = k^{0.5} (P)_{\text{lab}}, k = 7.4
\]

To relate the surface aerator theoretical values to larger actual industrial scales, four levels of rotational speed (see Table 2). The industrial volume was calculated as an equivalent theoretical cylindrical volume in equations 9 and 10 to determine the corresponding volumetric power in smaller scales. The corresponding rotational speeds in the industrial scale were calculated by Equation 7. Every lab subscript in this work means theoretical.

The two predictive correlations were applied to extrapolate the laboratory-scale results to larger scale. First, the predicted industrial-scale values of power consumption P were obtained by applying Equation 8, the following relation was built when considering Fr is the only controlling parameter with neglecting other parameters. The liquid volume in industrial-scale was calculated by Equations 7 and 8.

\[
V_2 = \frac{\pi}{4} (kT_1) ^2 \times 0.55(kT_1)
\]

\[
P_2 = \frac{\pi}{4} \times 1.55(k^{3.5}T_1^3) \times \left( \frac{P}{V} \right)_1
\]

where subscripts 2 and 1 refer to industrial and laboratory scales respectively.

The second applied predictive correlation for scale translation to industrial full-scale was a power consumption correlation that based on experimental results at laboratory-scale proposed by [16].

\[
P = f(N,h,T,S,g,D,\rho,\mu)
\]

\[
N_P = 38.695(F_r)^{-0.7} \left( \frac{T^2 h}{D^2 S} \right)^{-0.83}
\]

This power correlation is applicable for the ranges P/V (22 - 100 watt/m³), H/D (1.37-1.63) and Fr (0.054 - 0.214), [16].
Equations 10 and 12 enable to predict values of the volumetric power consumption, the predicted values at industrial-scale are plotted with their corresponded rotational speed as shown in Figure 2.

Table 2. The corresponding rotational speed values of the scales for matched Fr (calculated by Equation 7).

<table>
<thead>
<tr>
<th>( N_1 ) [rps]</th>
<th>( N_2 ) [rps]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.67</td>
<td>0.63</td>
</tr>
<tr>
<td>2.08</td>
<td>0.77</td>
</tr>
<tr>
<td>2.50</td>
<td>0.92</td>
</tr>
<tr>
<td>3.33</td>
<td>1.23</td>
</tr>
</tbody>
</table>

The ability of surface aeration scaling predicting approach in this work to provide representative outcomes it was explored by comparing with predicted values calculated by an existing model that based on investigating experimental work developed by Issa [16]. Volumetric power consumptions at the industrial-scale surface aeration system investigated in this work by the developed correlation of Equation 10 and that proposed by [16]. The correlation derived here showed a good agreement with predicted values at both scales for the investigated range of Fr.

The actual value of \( N_p \) at the industrial scale is 2.31, this was slightly higher than that predicted by Equation 10 which was 2.23 and was lower than that predicted by Equation 12 which was 3.07.

For the applied similarities, the industrial-scale surface aeration system at a rotation speed of 0.63 rps, predicted volumetric power consumption values \( P/V \) were; by equation 10 was 31.33 kwatt/m³, and by equation 12 it was 43.12 kwatt/m³, as shown in Figure 2. The actual volumetric power consumption that measured at the industrial-scale was 33.8 kwatt/m³.

Figure 2 summarizes prediction results of volumetric power consumption values \( P/V \) by the proposed two correlations at matched Fr values and same power number for different rotation speeds. In Figure 2, performances of the two estimation correlations were compared with the actually obtained volumetric power consumption values at the industrial-scale surface aeration system. At low rotation speeds, the volumetric power consumption at the industrial-scale shows a significant matching with that predicted by Equation 10. This is likely to be a result of the operation at reduced agitation rates and therefore, low
power was consumed at both of the industrial and laboratory scales. The variation between actual and predicted values at moderate rotation speed is expected as the actual scale of the industrial is considerably large compared with laboratory scale. At high rotation speed, there is no reading for P/V, as in actual wastewater treatment conditions surface aeration doesn’t operate at high speeds because of relatively high cost, and generating of undesired noise, emission and odor problems.

4. Conclusion
Predictive scale-up of P/V values for surface aeration tanks used for wastewater treatment has been future examined in terms of various similarities for both laboratory and industrial scales. The scale-up criterion based on three similarity evaluations; the geometrical, kinematic and dynamics for both scales. Several geometrical parameters were tested to reach the optimal geometrical similarity. The scale-up criterion was built for an invariant Froude number between the laboratory and industrial scales. Two different correlations were used to predict experimental volumetric power consumption P/V values. The first one was built in this work (Equation 10) and the second correlation was proposed by [16]. Investigations for the scale-up showed reliable estimations of yielded volumetric power consumptions by direct estimation with Equation 10 for matched Fr. These estimations were adequate at low and moderated rotation speeds (0.6-0.8 rps). At matched surface flow condition for laboratory-scale and industrial-scale units, the prediction of the volumetric power consumed by the correlation proposed by [16] has relatively higher estimated results. The cause for this difference is probably due to the impact of the difference in shapes between the two systems. The scale-up criterion presented in this work of invariant Fr at both laboratory and industrial scales shows that reliable predictions of P/V values could be obtained and this will facilitate further investigations for the power reducing of surface aeration process at wastewater treatment tanks.

List of symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>propeller clearance, [m]</td>
</tr>
<tr>
<td>D_1</td>
<td>turbine diameter in lab scale, [m]</td>
</tr>
<tr>
<td>D_2</td>
<td>turbine diameter in industrial scale, [m]</td>
</tr>
<tr>
<td>F_r</td>
<td>Froude number, [N^2D/g]</td>
</tr>
<tr>
<td>g</td>
<td>acceleration of gravity (9.80665), [m^2 s^{-1}]</td>
</tr>
<tr>
<td>N</td>
<td>rotation speed, [s^{-1}]</td>
</tr>
<tr>
<td>N_p</td>
<td>power number, [P/ρN^3 D^5]</td>
</tr>
<tr>
<td>P</td>
<td>power consumed, [watt]</td>
</tr>
<tr>
<td>Re</td>
<td>Reynolds number, [ρND^2/μ]</td>
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<tr>
<td>S</td>
<td>submergence of the turbine blades, [m]</td>
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<tr>
<td>T_1</td>
<td>tank width in lab scale, [m]</td>
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<tr>
<td>T_2</td>
<td>tank width in industrial scale, [m]</td>
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<tr>
<td>V</td>
<td>water volume, [m^3]</td>
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<tr>
<td>V_2</td>
<td>water volume in industrial scale, [m^3]</td>
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<tr>
<td>W</td>
<td>turbine blade width, [m]</td>
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Greek Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
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<tbody>
<tr>
<td>ρ</td>
<td>water density, [kg m^{-3}]</td>
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<tr>
<td>μ</td>
<td>water dynamic viscosity, [kg m^{-1} s^{-1}]</td>
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References