Effect of nano-lubrication on the dynamic coefficients of worn journal bearing

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Received 22 Mar. 2017; Received in revised form 26 Apr. 2017; Accepted 28 Apr. 2017; Available online 1 Nov. 2017

Abstract
In this paper, the effect of Nano-lubrication on the dynamic coefficients of worn journal bearing has been studied. The analytical solution of time dependent Reynolds equation for short journal bearing (L/D<1) is presented to determine the dynamic coefficients of fluid film journal bearings. Dufrane wear model has been adopted to include the effect of worn geometry on the oil film thickness. Modified Krieger-Dougherty viscosity model have been employed to predict the relative viscosities of Nano-lubricant. The results show that the cross coupled stiffness coefficients $K_{xy}$ decreases by 34% for a worn journal bearing with wear depth parameter of 0.2 while the damping coefficients $C_{xx}$ and $C_{yy}$ decrease by 9% and 13%.

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Keywords: Nano-lubrication; Journal bearing; Wear; Hydrodynamic journal bearings.

1. Introduction
Hydrodynamic bearings are common components of rotating machinery. They are frequently used in applications involving high loads and/or high speeds between two surfaces that have relative motion. Journal bearings are specific to surfaces that mate cylindrically with the applied load in the radial direction. Some journal-bearing configurations are susceptible to large-amplitude, lateral vibrations due to a “self-excited instability” known as oil whirl.
Lund [1] determined fluid-film bearing dynamic coefficients theoretically and experimentally. Hashimoto et.al [2] investigated the influences of geometric change in the hydrodynamic journal bearings due to wear on the bearings parameters (eccentricity, attitude angle and pressure distribution) theoretically and experimentally under both turbulent and laminar regimes. Hashimoto et al. [3] used the short bearing approximation to analyze the dynamic characteristics problems of turbulent journal bearings. Dufrane et al. [4] established a model of wear geometry for use in the analysis of the worm journal bearings used in steam turbine generators. Nikolakopoulos, et al. [5] developed and presented a numerical method to identify the clearance defect due to wear in a rotating flexible rotor supported on two misaligned journal bearing. Tribological properties of two lubricating oils with CuO, TiO$_2$ and Diamond nanoparticles additives were examined by Wu et.al [6]. Reciprocating sliding tribotester was used to perform friction and wear experiments. The static and dynamic characteristics hydrodynamic journal bearing lubricated with Nano-lubricant was discussed by Nair et.al [7]. The main goal of the present work is to study the effect of adding Nano-particles to the base oil on the dynamic characteristics of the worn journal bearing.
2. Theoretical analysis

2.1 Wear model

The wear model used in present work is that proposed by Dufrane et al. [4], it is shown schematically in Figure 1, and the lubricant thickness $h_w$ in the worn region ($\theta_s \leq \theta \leq \theta_f$) is described by the equation:

$$h_w = d_o + e_o \cos \theta - c \cos(\theta + \Phi_o)$$  \hspace{1cm} (1)

The film thickness $h_o$ in the non worn region ($\theta \leq \theta_s \text{ and } \theta \leq \theta_f$) is given by the equation:

$$h_o = c + e_o \cos \theta$$  \hspace{1cm} (2)

where $c$ is the radial clearance of the journal bearing.

The starting and the end of the worn region ($\theta_s \text{ and } \theta_f$) are given by the solution of equation:

$$\cos(\theta + \Phi_o) = \frac{d_o}{c} - 1$$  \hspace{1cm} (3)

![Figure 1. Worn journal bearing.](image)

2.2 Modified Reynolds equation

The following modified time dependent Reynolds equation for an incompressible, laminar and isothermal fluid flow has been adopted Khonsari, and Booser [8].

$$\frac{\partial}{\partial x} \left( \frac{h^3}{12 \mu} \frac{\partial P}{\partial x} \right) + \frac{\partial}{\partial z} \left( \frac{h^3}{12 \mu} \frac{\partial P}{\partial z} \right) = \frac{\partial h}{\partial t} + \frac{\Omega \partial h}{2 \partial \theta}$$  \hspace{1cm} (4)

Which can be rewritten for short bearing approximation as

$$\frac{\partial}{\partial z} \left( \frac{h^3}{12 \mu} \frac{\partial P}{\partial z} \right) = \frac{\partial h}{\partial t} + \frac{\Omega \partial h}{2 \partial \theta}$$  \hspace{1cm} (5)

The fluid (oil) film thickness of non- worn journal bearing can be expressed as in Cameron, A. [9].

$$h(\theta) = c + e \cos \theta$$  \hspace{1cm} (6)

For small amplitude motions around the equilibrium position of the journal center, it can be considered that, $e = e_o + \Delta e$, $\Phi = \Phi_o + \Delta \Phi$, and $\theta = \gamma - \Phi_o$.

Taking into consideration the following boundary conditions $P = 0$ at $z = \pm L/2$. 

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The pressure distribution of non-worn and worn bearings can be obtained by integrating equation (5) in the axial direction of journal bearing Papanikolaou [10].

\[
P(\theta, z, t) = \frac{6\mu}{C' H} \left[ e^\frac{e}{2} \left( \Delta^2 + \frac{\Omega e}{2} \right) \cos \theta \left( e^\frac{e}{2} \right) \sin \theta \left( e^\frac{e}{2} \right) \right] \left( z^2 - \frac{L^2}{4} \right)
\]

\[
P(\theta, z, t) = \frac{6\mu}{C' H} \left[ e^\frac{e}{2} \left( \Delta^2 + \frac{\Omega e}{2} \right) \cos \theta \left( e^\frac{e}{2} \right) \sin \theta \left( e^\frac{e}{2} \right) \right] \left( z^2 - \frac{L^2}{4} \right)
\]

2.3 Effect of nano-lubricants
Modified Krieger-Dougherty viscosity model has been employed to predict the relative viscosities of Nano lubricant for varying nanoparticle concentrations. The following viscosity model has been used to include the effect of Nano-particle concentration on the oil viscosity Binu, et al [11]:

\[
\mu = \mu_o \left(1 - \frac{\Phi_a}{\Phi_{m0}}\right)^{n\Phi_m}
\]

\[
\Phi_a = \Phi \left( \frac{a}{a} \right)^{3-D}
\]

where: \( \Phi_a \) is the effective volume fraction. \( \Phi \) is Nano-particles volume fraction. \( a, \alpha \) are the radii of aggregates and primary particles, respectively. \( D \) is the fractal index having a typical value of 1.8 for Nano-fluids. \( \Phi_{m0} \) is the maximum particle packing fraction, which is approximately 0.605 at high shear rates. \( \eta \) is the intrinsic viscosity, whose typical value for mono disperse suspension of hard spheres is 2.5.

Then the modified Krieger–Dougherty equation in dimensional form can be written as:

\[
\mu = \mu_o \left(1 - \frac{\Phi_a}{\Phi_{m0}} \left( \frac{a}{a} \right)^{1.2} \right)^{-1.5}
\]

2.4 Calculation of dynamic coefficients
Eight dynamic coefficients, four stiffness and four damping \((K_{ij}, C_{ij})\), are usually used to define the dynamic characteristics of the journal bearings. Two of them are direct while the other are cross coupled coefficients. The radial and tangential components of fluid film journal bearing force are Andres [12]:

\[
\begin{align*}
\{F_r\} & = 2 \int_0^{L/2} \int_0^\pi P(\theta, z, t) \left[ \cos \theta \sin \theta \right] R d\theta \, dz \\
\{F_t\} & = \int_0^\pi P(\theta, z, t) \left[ \cos \theta \sin \theta \right] \frac{H C}{e} d\theta
\end{align*}
\]

Which can be evaluated for short worn journal bearing to get Andres [12]:

\[
\begin{align*}
\{F_r\} & = -\frac{\mu R L}{H C} \int_0^{\pi} \left[ e^\frac{e}{2} \left( \Delta^2 + \frac{\Omega e}{2} \right) \cos \theta \left( e^\frac{e}{2} \right) \sin \theta \left( e^\frac{e}{2} \right) \right] \cos \theta d\theta - \\
\{F_t\} & = \frac{\mu R L}{H C} \int_0^{\pi} \left[ e^\frac{e}{2} \left( \Delta^2 + \frac{\Omega e}{2} \right) \cos \theta \left( e^\frac{e}{2} \right) \sin \theta \left( e^\frac{e}{2} \right) \right] \cos \theta d\theta
\end{align*}
\]

Equation (13) can be rearranged as:
where $K$ and $C$ represent the direct and cross coupled stiffness and damping coefficients respectively. In order to solve equation (13) the following first order Taylor series expansion for the oil film thickness was adopted:

$$H^{-3} = H_0^{-3} - 3H_0^{-4} H_1$$

where $H_0 = \frac{h_0}{c}$ and $H_{wo} = \frac{h_{wo}}{c}$

The radial and tangential force components in radial and tangential coordinates can be evaluated as; Papanikolaou [10];

$$F_{r0} = \frac{\mu \Delta \Omega L}{2c^2} \left[ \int_0^{\theta_0} \frac{\sin \theta \cos \theta}{H_0^3} \, d\theta + \int_{\theta_0}^\theta \frac{\sin \theta \cos \theta - \cos \theta \sin (\theta + \phi_0)}{H_{wo}^3} \, d\theta \right]$$

$$F_{t0} = \frac{\mu \Delta \Omega L}{2c^2} \left[ \int_0^{\theta_0} \frac{\sin \theta \cos \theta}{H_0^3} \, d\theta + \int_{\theta_0}^\theta \frac{\sin \theta \cos \theta - \cos \theta \sin (\theta + \phi_0)}{H_{wo}^3} \, d\theta \right]$$

Hence the direct and cross coupled stiffness coefficients in radial and tangential coordinates can be evaluated as; Papanikolaou [10];

$$K_r = \frac{\mu \Delta \Omega L}{2c^2} \left[ \int_0^{\theta_0} \frac{\sin \theta \cos \theta}{H_0^3} \, d\theta + 3c \int_0^{\theta_0} \frac{\sin \theta \cos \theta}{H_0^3} \, d\theta - \int_{\theta_0}^\theta \frac{\sin \theta \cos \theta}{H_0^3} \, d\theta + 3c \int_{\theta_0}^\theta \frac{\sin \theta \cos \theta}{H_0^3} \, d\theta \right]$$

$$K_t = \frac{\mu \Delta \Omega L}{2c^2} \left[ \int_0^{\theta_0} \frac{\sin \theta \cos \theta}{H_0^3} \, d\theta + 3c \int_0^{\theta_0} \frac{\sin \theta \cos \theta}{H_0^3} \, d\theta - \int_{\theta_0}^\theta \frac{\sin \theta \cos \theta}{H_0^3} \, d\theta + 3c \int_{\theta_0}^\theta \frac{\sin \theta \cos \theta}{H_0^3} \, d\theta \right]$$

$$K_r = \frac{\mu \Delta \Omega L}{2c^2} \left[ \int_0^{\theta_0} \frac{\sin \theta \cos \theta}{H_0^3} \, d\theta + 3c \int_0^{\theta_0} \frac{\sin \theta \cos \theta}{H_0^3} \, d\theta - \int_{\theta_0}^\theta \frac{\sin \theta \cos \theta}{H_0^3} \, d\theta + 3c \int_{\theta_0}^\theta \frac{\sin \theta \cos \theta}{H_0^3} \, d\theta \right]$$

$$K_t = \frac{\mu \Delta \Omega L}{2c^2} \left[ \int_0^{\theta_0} \frac{\sin \theta \cos \theta}{H_0^3} \, d\theta + 3c \int_0^{\theta_0} \frac{\sin \theta \cos \theta}{H_0^3} \, d\theta - \int_{\theta_0}^\theta \frac{\sin \theta \cos \theta}{H_0^3} \, d\theta + 3c \int_{\theta_0}^\theta \frac{\sin \theta \cos \theta}{H_0^3} \, d\theta \right]$$

While the damping coefficients in radial and tangential coordinates can be evaluated as Papanikolaou [10]
The above dynamic coefficients in x and y coordinate can be obtained by using the suitable transformation.

\[ C_{rr} = \frac{\mu R L^2}{2 c^3} \left[ \int_{\theta_0}^{\theta_2} \frac{\cos^2(\theta)}{H_0^2} \, d\theta + \int_{\theta_3}^{\theta_4} \frac{\cos^2(\theta)}{H_0^2} \, d\theta + \int_{\theta_f}^{\pi} \frac{\cos^2(\theta)}{H_0^2} \, d\theta \right] \]  

\[ C_{rt} = \frac{\mu R L^3}{2 c^3} \left[ \int_{\theta_0}^{\theta_2} \frac{\sin\theta \cos\theta}{H_0^2} \, d\theta + \int_{\theta_3}^{\theta_4} \frac{\sin\theta \cos\theta}{H_0^2} \, d\theta + \int_{\theta_f}^{\pi} \frac{\sin\theta \cos\theta}{H_0^2} \, d\theta \right] \]  

\[ C_{tr} = \frac{\mu R L^3}{2 c^3} \left[ \int_{\theta_0}^{\theta_2} \frac{\sin\theta \cos\theta}{H_0^2} \, d\theta + \int_{\theta_3}^{\theta_4} \frac{\sin\theta \cos\theta}{H_0^2} \, d\theta + \int_{\theta_f}^{\pi} \frac{\sin\theta \cos\theta}{H_0^2} \, d\theta \right] \]  

\[ C_{tt} = \frac{\mu R L^3}{2 c^3} \left[ \int_{\theta_0}^{\theta_2} \frac{\sin\theta \cos\theta}{H_0^2} \, d\theta + \int_{\theta_3}^{\theta_4} \frac{\sin\theta \cos\theta}{H_0^2} \, d\theta + \int_{\theta_f}^{\pi} \frac{\sin\theta \cos\theta}{H_0^2} \, d\theta \right] \]

The above dynamic coefficients in x and y coordinate can be obtained by using the suitable transformation.

\[
\begin{bmatrix}
K_{xx} & K_{xy} \\
K_{yx} & K_{yy}
\end{bmatrix} = \begin{bmatrix}
\sin \phi_0 & \cos \phi_0 \\
-\cos \phi_0 & \sin \phi_0
\end{bmatrix} \begin{bmatrix}
K_{rr} & K_{rt} \\
K_{tr} & K_{tt}
\end{bmatrix} \begin{bmatrix}
\sin \phi_0 & -\cos \phi_0 \\
\cos \phi_0 & \sin \phi_0
\end{bmatrix}
\]

\[
\begin{bmatrix}
C_{xx} & C_{xy} \\
C_{yx} & C_{yy}
\end{bmatrix} = \begin{bmatrix}
\sin \phi_0 & \cos \phi_0 \\
-\cos \phi_0 & \sin \phi_0
\end{bmatrix} \begin{bmatrix}
C_{rr} & C_{rt} \\
C_{tr} & C_{tt}
\end{bmatrix} \begin{bmatrix}
\sin \phi_0 & -\cos \phi_0 \\
\cos \phi_0 & \sin \phi_0
\end{bmatrix}
\]

The dimensionless dynamic coefficients is:

\[ k_{ij} = K_{ij} \frac{c}{F_o} \; ; \; c_{ij} = C_{ij} \frac{c_f}{F_o} \; , \; i,j = x,y \]

A computer program written in MATLAB was used to evaluate the above stiffness and damping coefficients. The flow chart for the program can be seen in Figure a1.

3. Results and discussion

Figure 2 shows the stiffness and damping coefficients in terms of Sommerfeld number for intact journal bearing of the present work. These results are well agreed with that obtained by Papanikolaou et al. [10]. The maximum deviation between the results has been calculated and found to equal to (2.4%). This represents a suitable validation for the computer program used in the present work. Figure (3a and b) shows the effect of wear depth of the worn journal bearing on the stiffness and the damping dynamic coefficients. Figure 3a shows that the stiffness coefficients \( K_{xy} \) and \( K_{yy} \) are slightly affected by the wear depth parameter while \( K_{xy} \) decreases for the worn journal bearing with higher wear depth parameter. The percentage decrease in \( K_{xy} \) with the wear depth has been calculated and found to be 34% for a worn journal bearing with the wear depth of 0.2 in comparison with that of intact bearing while it becomes 24% for the worn bearing with wear depth of 0.1. This can be attributed to the increase in the component of load which affected perpendicular the journal center motion. Figure 3b shows that both the direct damping coefficients \( C_{xx} \) and \( C_{yy} \) decrease for the worn journal bearing that has higher wear depth parameter. The calculated percentage decrease shows that \( C_{xx} \) decreases by 9% for a worn journal bearing that has wear depth parameter of 0.2 in comparison with that of intact bearing while \( C_{yy} \) decreases by 13% for the same above operating conditions the increase in clearance gap of the worn bearing. This can be attributed to the decrease in lubricant damping effect due to increase of clearance gap. The variation of the dynamic stiffness and damping coefficients with rotor spin speed for a worn journal bearing is presented in Figures 4a and 4b. Figure 4a shows that the cross coupled stiffness coefficients \( k_{xy} \) decreases for a small range of rotor speeds (below 1000rpm) after which it increases while the stiffness coefficient \( k_{xy} \) always increases when the bearing works at higher speeds. Also it can be seen from this figure that the stiffness coefficient \( (k_{xx} \) and \( k_{yy} \)) decrease for a small range of rotor speed (less than 1000rpm) after which it became nearly constant. Figure 4b shows the variation of damping coefficients with the rotational speed of the bearing rotor. It is clear from this figure that the damping coefficients \( C_{xy}=C_{yx} \) for all rotor speed. Also this figure depicts that the cross coupled damping coefficients decreases with increasing rotor rotational speed. The direct damping coefficients \( C_{xx} \) and \( C_{yy} \) show a decrease for a shaft rotational speed less than 5000rpm after which it became constant. Figure 5 depicts the effect of lubricating the worn journal bearing with oil containing TiO2 Nano-
particles with different particle concentrations. The dynamic coefficients have been presented against the bearing wear depth parameter. It is obvious from Figure 5a that the direct stiffness coefficient ($k_{xx}$) is slightly affected when the worn journal bearing that has low values of wear depth ($\delta<0.25$) lubricated with Nano-lubricant that contains different particle concentrations. After that the coefficient $K_{xx}$ becomes lower when the bearing lubricated with oil containing higher particle concentration. This indicates that the effect of increasing the oil viscosity due to the existence of the nanoparticles is less than the effect of increasing bearing wear depth which leads to the decrease in oil film load components. Figure 5b shows that the effect of Nano-particles added to the base oil on the direct stiffness $k_{yy}$. It is clear from this figure that $k_{yy}$ slightly increases for a worn journal bearing that has higher wear depth. This can be attributed to the increase in oil viscosity in this case. A decrease in cross coupled stiffness $k_{xy}$ has been noticed for a worn journal bearing lubricated with oil containing TiO$_2$ Nano-particles as can be shown in Figure 5c. The value of $k_{xy}$ is slightly increased when the bearing lubricated with Nano lubricant that has higher particle concentration. The cross coupled stiffness $k_{yx}$ is slightly affected by the addition of the Nano-particles to the base oil as shown in Figure 5d. The coefficient $k_{yx}$ has a tendency to a higher decrease when the bearing lubricated with oil containing higher particle concentration due to the increase of the oil film thickness in this case. The most important effect for the Nano-lubrication can be seen in Figure 6. This figure depicts the effect of addition nanoparticles to the base oil on the dimensionless damping coefficients for a bearing works at different wear depth parameter: $(d_0/c)$. It is clear from this figure that a considerable increase in damping coefficients has been obtained when the bearing lubricated with lubricant contains higher particle concentration of the Nano-particles. This can be attributed to the increase in oil viscosity in this case. The percentage increase in $C_{xx}$ and $C_{yy}$ have been calculated for a worn journal bearing that has wear depth parameter of 0.2 and it was found to be 22% and 7% respectively when the bearing lubricated with oil containing TiO$_2$ Nano-Particles with 0.5% particle concentration. These percentages increase to 39% and 22% respectively when the worn journal bearing lubricated with oil containing nanoparticles with particle concentration of 1%.

Figure 2. Dimensionless dynamic coefficients against modified Sommerfeld number.

Figure 3. Dimensionless dynamic coefficients against the wear depth parameter $(d_0/c)$.
4. Conclusions
Referring to the discussion of the results mentioned above the following conclusions can be drawn:
1. The direct stiffness coefficients are slightly affected by wear depth parameter.
2. The cross coupled stiffness coefficients $K_{xy}$ decreases by 34% for a worn journal bearing with wear depth parameter of 0.2.
3. The damping coefficients $C_{xx}$ and $C_{yy}$ decrease by 9% and 13% for a bearing that has a wear depth of 0.2 in comparison with that of intact bearing.
4. The stiffness coefficients are slightly affected by the addition of the TiO$_2$ nanoparticles to the base oil.
5. The damping coefficients $C_{xx}$ and $C_{yy}$ increase by 22% and 7% for worn journal bearing lubricated with oil containing 0.5% particle concentration of nanoparticles.

![Figure 6. Dimensionless dynamic coefficients with the wear depth parameter (do/c).](image)

**Nomenclature**

- $c$: Radial clearance (mm)
- $d_0$: Maximum wear depth (mm)
- $e$: Eccentricity (mm)
- $e_0$: Eccentricity at equilibrium position (mm)
- $F_0$: Static fluid film force (N)
- $F_R$ & $F_T$: Radial and tangential components of fluid film force respectively (N)
- $F_{R0}$ & $F_{T0}$: Radial and tangential components of fluid film force at the equilibrium position (N)
- $k_{ij}$: Dimensionless Stiffness Coefficients
- $K_{ij}$: Stiffness Coefficients (N/mm)
- $h_0$, $h_{w0}$: Thickness of Fluid film thickness in the non-worn and worn region (at the equilibrium position) respectively (mm)
- $h$ & $h_w$: Thickness of Fluid film in the non-worn and worn regions respectively (mm)
- $H$ & $H_w$: Dimensionless Fluid film thickness in the non-worn and worn region ($h/c$, $h_w/c$) respectively (mm)
- $L$: Bearing length (mm)
- $P$ & $P_w$: Pressure of oil film in the non-worn and worn region respectively (N/mm²)
- $R_b$: Bearing Radius (mm)
- $\gamma$: Angular displacement (rad)
- $\delta$: Wear depth parameter (do/c)
- $\varepsilon$: Eccentricity ratio
θ: Circumferential direction of bearing (rad)
θ₁, θ₂: Starting and final angle of worn region in journal bearing (rad)
μ: Dynamic Lubricant viscosity (p.a.s)
Φ₀: Attitude angle at equilibrium position (rad)
Ω: Rotor spin speed (rpm)
Δε: small displacement at equilibrium position
ΔΦ: small variation of attitude angle

Appendix A.

Figure A1. Computer program flow chart.

References


