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A suggested analytical solution for dynamic behavior of a cracked pipe conveying fluid

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Abstract
This research investigates the effect of cracks into a simply supported pipe conveying fluid on the frequency and the response of the pipe. A plastic pipe with modulus of elasticity 800 N/mm2 and dimensions of 1m length, 2cm internal diameter and 3cm external diameter used as models. A laminar flow established according to Reynold's number calculation between 500 to 1500 with a flow velocity 1 m/sec. to achieve the objectives, two techniques are used I- Analytically using the derived equation of motion for the pipe conveying fluid and the crack model is included in the stiffness of the pipe. Matlab R2016 language, a computer program has been used to solve the derived equations to predict the vibration response and to embrace the theoretical work. II-Numerical investigation is employed using the finite element method (FEM) adopting COMSOL 5.2 program to verify the analytical results. The research conveys the variation of depth and the crack positions (0.25L, 0.375L and 0.5L) where L is the pope length. It was found that the value of frequency of the pipe transmitted fluid with crack is decreased more and more when increasing the size of crack with a maximum discrepancy of (5.53%). Also, it was found that the frequency value becomes smaller when the crack position gets closer to the middle section.

Keywords: Vibration; Crack; Pipe; Flow; Frequency.

1. Introduction

There are many uses for liquid conveying pipes, such as oil transport pipelines and refinery pipes, water transfer pipes, chemical transport pipelines, power plants and others.

The investigation into the presence of crack in the pipeline is important especially if the pipeline carries oil or petrochemical fluids. This reason makes human thinks about the use of different methods of examination to detect the crack in the tube before causing a huge explosion and big losses as example Tesoro's Anacortes Refinery into Washington There was an explosion and fire into heat exchange (April second 2010). The heat exchanger to transfer temperature between low naphtha and crude oil. A scale as like a material forms inside the tube. The high temperature accumulate methylene accumulate into heat exchanger. Stressing in the steel led to large internal crack. One-inch thickness of the heat exchanger shell broken. Generally, it is necessary to predict the shortfall or crack in the pipeline before the disaster.

Housner [1] succeeded in producing a simple mathematical system to study the supporting pipes. After Housner work, many researchers presented several models based on the Euler theory of beam and Newton's fluid laws like Long [2]. Hence, the models were taken by researchers Paidoussis and Li [3] identified and studied important variables for fluid flow in the tube (flow velocity, tube mass, fluid mass, internal pressure of the fluid). The main conclusion of this research was that the natural frequency value increases if the velocity of the fluid is reduced. In addition, there are certain speeds through which pipes lose their stability and depending on the type of tube installation. Curved tubes have been subjected to a large number of research, since the mathematical analysis model for vibration or stability was derived from a number of not a few researchers like Unny et. al [4]. Kochupillai and Ganesan [5] presented an efficient numerical method for examining the stability of linear time varying systems modeled by finite element (FE) methods. In order to solve large size problems, the system matrices are reduced to a smaller size by transforming to modal coordinates using modal vectors. This transformation preserves the stability information and the reduced matrix is used for the evaluation of the monodrama matrix. For dealing with large systems with parametric excitations, it is found that proposed method is numerically efficient.
Lolov and Markova [6] analyzed the fluid-carrying pipes in their curved shape. They developed a numerical solution for the dynamic stability of the tubes at the level; their method was expensive but suitable for fixed-hardened pipes. Through which non-vector variables can be obtained from the speed of flow and frequency. In addition to, the crack effect on natural frequency of structure as beam or plate is presented by many researchers, Muhsin J. Jweeg et. al. [7-10]. The effect of crack on the boundary condition at different crack locations were investigated. They conducted different applications and investigated type modeling of cracks and its effectiveness on the stiffness, frequencies, mode shapes, dynamic stresses and deformations. Through the current research, it is aimed to find an easy and cheap way to find the effect of the presence of the crack in the pipe. For this reason, an analytical solution will be presented and verified numerically using the FEM employing COMSOL programs to prove the results obtained from the research.

2. Analytical investigation
The system initially straight. In this type, the pipe is accepting to obey the law of Euler- Bernoulli beam theory, thus the pipe design has minimum distortion, the fluid is non-viscose, incompressible, neglect inertia damping and gravity effects see Figure 1 to explain the acting force and moment effects on the fluid element, [11].

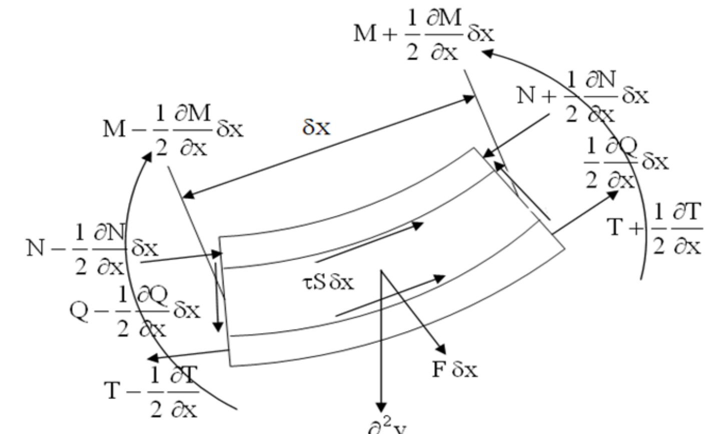


Figure 1. The force and moment effects.

To find equation of motion

$\sum$parallel forces = 0,
$$\frac{\partial Q}{\partial x}+\frac{\partial}{\partial x}\left(T_l\frac{\partial y}{\partial x}\right)+F+\tau_s S\frac{\partial y}{\partial x}=m_p\frac{\partial^2 y}{\partial t^2}$$
$\sum$vertical forces =0,
$$\frac{\partial T}{\partial x}+\tau_s S-F\frac{\partial y}{\partial x}=0 \tag{1a}$$

where, τ_s is internal shear stress, S the internal section circumference of pipe, T_l is longitudinal tension and F is the force between pipe and fluid (per unit length).
The cross shear force (Q) in the pipe is coupled to the bending moment (M) and the pipe distortion by

$$Q = -\frac{\partial M}{\partial x} = -EI\frac{\partial^3 y}{\partial x^3} \tag{1b}$$

The equation of motion for free vibration of pipe conveying fluid is,

$$EI\frac{\partial^2}{\partial x^2}\frac{\partial^2 y}{\partial x^2} + \left(m_f V^2 + p_i A\right)\frac{\partial^2 y}{\partial x^2} + 2m_f V\frac{\partial^2 y}{\partial t \partial x} + m_f\frac{\partial V}{\partial t}\frac{\partial y}{\partial x} + \left(m_f + m_p\right)\frac{\partial^2 y}{\partial t^2} = 0 \tag{2}$$

where, E modulus of elasticity, I second moment of area, L pipe length, m_p pipe mass per unit length, m_f fluid mass per unit length, p_i internal pressure, A cross section area and V velocity of fluid.
Rewrite Eq. (2) in dimensionless form:

$$\frac{\partial^2}{\partial X^2}\bar{\bar{Y}} + (U^2+\gamma)\bar{\bar{Y}} + 2\beta U\dot{\bar{Y}} + \dot{U}\bar{Y} + \ddot{Y} = 0 \tag{3}$$

where: $Y = \frac{y}{L}$, $X = \frac{x}{L}$, $U = VL\sqrt[2]{\frac{m_f}{EI}}$, $\gamma = \frac{p_i A L^2}{EI}$, $\beta = \sqrt[2]{\frac{m_f}{m_p+m_f}}$, $\tau = \frac{t}{L^2}\sqrt[2]{\frac{EI}{m_p+m_f}}$, x: axial coordinate, y lateral coordinate, X, Y are dimensionless coordinate and T is the dimensionless time.
Assuming the effect of crack on the deflection of pipe is small, then it can be assumed the behaviors of pipe with crack is similar to that without crack, thus the common solution for the vibration neutralization of conservative pipes carry fluid follows the following general solution for the vibration,

$$Y(X,T) = \begin{bmatrix} e^{i(\Omega T - aX)}(A\sinh b_1 X + B\cosh b_1 X) + \\ e^{i(\Omega T + aX)}(D\sin b_2 X + E\cos b_2 X) \end{bmatrix} \tag{4}$$

The boundary condition of the simply supported pipe is:

$$y(0,T) = y''(0,T) = 0, y(l,T) = y''(l,T) = 0 \tag{5}$$

Therefore, subjected boundary conditions, Eq. (5) in to Eq. (4), gives,

$$Y(X.T) = Ae^{i(\Omega T)}\left[\begin{matrix} e^{-aXi}\sinh b_1 X - \frac{2aib_1}{(b_1^2+b_2^2)}\begin{pmatrix} e^{aXi}\cos b_2 X - \\ e^{-aXi}\cosh b_1 X \end{pmatrix} - \\ \dfrac{\left[\left(e^{-ali}\sinh b_1 l - \frac{2aib_1}{(b_1^2+b_2^2)}\begin{pmatrix} e^{ali}\cos b_2 l - \\ e^{-ali}\cosh b_1 l \end{pmatrix}\right)e^{aXi}\sin b_2 X + \frac{2aib_2}{(b_1^2+b_2^2)}(e^{aXi}\cos b_2 X - e^{-aXi}\cosh b_1 X)\right]}{\left[e^{ali}\sin b_2 l + \frac{2aib_2}{(b_1^2+b_2^2)}(e^{ali}\cos b_2 l - e^{-ali}\cosh b_1 l)\right]} \end{matrix}\right] \tag{6}$$

where, A is constant, b_1, b_2 and a are defined as,
$b_1 = \frac{1}{2}\sqrt{\frac{4\beta U\Omega}{\sqrt{\alpha}} + \alpha - 2\kappa}$, $b_2 = \frac{1}{2}\sqrt{\frac{4\beta U\Omega}{\sqrt{\alpha}} - \alpha + 2\kappa}$, $a = \frac{1}{2}\sqrt{\alpha}$

From the boundary condition $\bar{\bar{Y}}\,(l.T) = 0$, the magnitude of Ω can be obtained to find b1 and b2.

Assume that the crack effect in a pipe, as shown in Figure 2, similar to that found in beams which affect the bending stiffness of the beam EI,
To calculate the stiffness for the pipe with crack, [12],

$$\mathrm{EI(X)} = \frac{\mathrm{EI_o}}{1+\mathrm{Cexp}\left(-2\propto\frac{|x-x_c|}{d}\right)} = \frac{\mathrm{EI_o}}{\mathrm{f(X)}} \tag{7}$$

where, $f(X) = 1 + Cexp\,(-2 \propto \frac{|x-x_c|}{d})$, $c = \frac{I_o - I_c}{I_c}$, $I_o = \frac{\pi(d_o^4 - d_i^4)}{64}$, $\mathrm{I_c} = \frac{\pi((d_o - d_c)^4 - d_i^4)}{64}$, d_o, d_i: are external and internal diameter of pipe, L: the pipe length, d_c is crack depth, x: is the position along pipe, x_c is the position of crack and α is constant equal to (0.667).

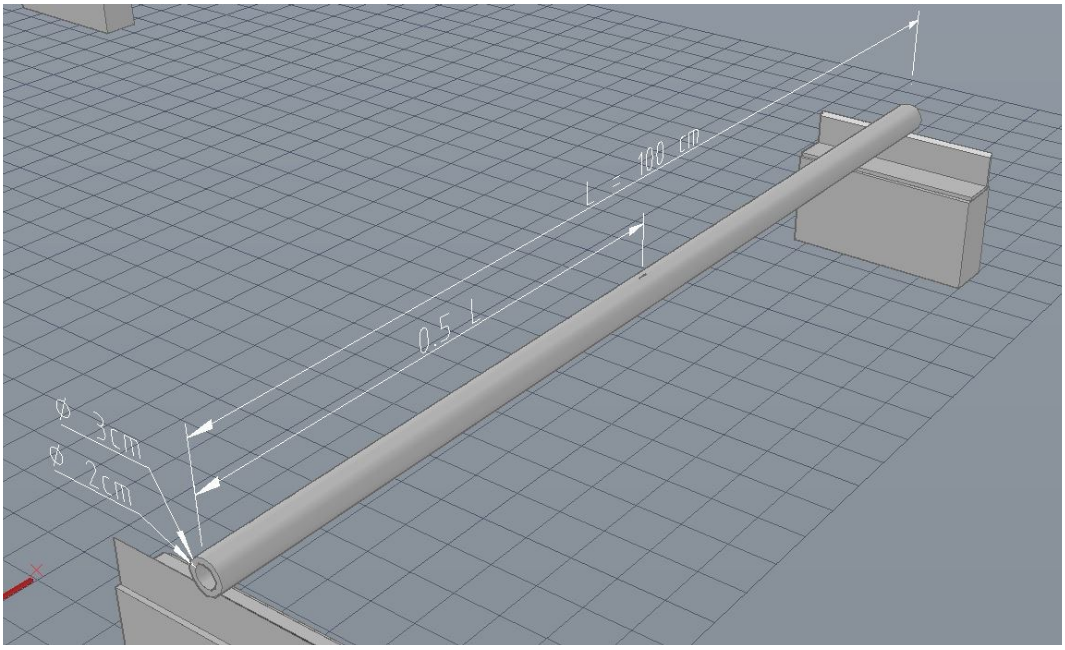


Figure 2. The crack position in pipe.

For vibration analysis of the pipe having a crack with a finite length, relation Eq. (7) can be expanded as a sum of cosine and sine functions in the domain 0 ≤ x ≤ L by Fourier series, as,

$$EI = \frac{\mathrm{EI_o}}{F(X)} = \mathrm{EI_o}\left(A_o + \sum_{n=1}^{\infty} A_n cos\frac{2\pi nx}{L} + \sum_{n=1}^{\infty} B_n sin\frac{2\pi nx}{L}\right) \tag{8}$$

where,

$$\frac{1}{F(X)} = \left(A_o + \sum_{n=1}^{\infty} A_n cos\frac{2\pi nx}{L} + \sum_{n=1}^{\infty} B_n sin\frac{2\pi nx}{L}\right)$$

$$A_o = \frac{1}{L}\int_0^{\mathrm{L}} \frac{1}{\mathrm{f(X)}}\mathrm{dx} = \frac{1}{\mathrm{L}}\int_0^{L} \frac{1}{1 + Cexp\,(-2\propto\frac{|x-x_c|}{d})}dx$$

$$A_n = \frac{2}{L}\int_0^{\mathrm{L}} \frac{1}{\mathrm{f(X)}} cos\frac{2\pi nx}{L}\mathrm{dx} = \frac{2}{\mathrm{L}}\int_0^{L} \frac{1}{1 + Cexp\,(-2\propto\frac{|x-x_c|}{d})} cos\frac{2\pi nx}{L}dx$$

$$B_n = \frac{2}{L}\int_0^{\mathrm{L}} \frac{1}{\mathrm{f(X)}} sin\frac{2\pi nx}{L}\mathrm{dx} = \frac{2}{\mathrm{L}}\int_0^{L} \frac{1}{1 + Cexp\,(-2\propto\frac{|x-x_c|}{d})} sin\frac{2\pi nx}{L}dx$$

Then, with substation Eq. (8) into Eq. 3, the resulted equation is as follows,

$$\frac{\partial^2}{\partial X^2}\frac{1}{F(X)}\bar{\bar{Y}} + (U^2 + \gamma)\bar{\bar{Y}} + 2\beta U\dot{\bar{Y}} + \dot{U}\bar{Y} + \ddot{Y} = 0 \tag{9}$$

where, $U = VL\sqrt[2]{\frac{m_f}{EI_o}}$, $\gamma = \frac{p_i AL^2}{EI_o}$, $\beta = \sqrt[2]{\frac{m_f}{m_p + m_f}}$, $\tau = \frac{t}{L^2}\sqrt[2]{\frac{EI_o}{m_p + m_f}}$

In vibration test, the fluid velocity must be considered steady, so that Eq. (9) is reduced to,

$$\frac{\partial^2}{\partial X^2}\frac{1}{F(X)}\bar{\bar{Y}} + (U^2 + \gamma)\bar{\bar{Y}} + \ddot{Y} = 0 \tag{10}$$

Then, by substation Eq. 8 into Eq. 10, gives,

$$\begin{bmatrix} \frac{\partial^4 y}{\partial x^4}\left(A_o+\sum_{n=1}^{\infty}A_n cos\frac{2\pi nx}{L}+\sum_{n=1}^{\infty}B_n sin\frac{2\pi nx}{L}\right) - \\ \frac{4\pi n}{L}\frac{\partial^3 y}{\partial x^3}\left(\sum_{n=1}^{\infty}A_n sin\frac{2\pi nx}{L}-\sum_{n=1}^{\infty}B_n cos\frac{2\pi nx}{L}\right) - \\ \left(\left(\frac{4\pi n}{L}\right)^2\left(\sum_{n=1}^{\infty}A_n cos\frac{2\pi nx}{L}+\sum_{n=1}^{\infty}B_n sin\frac{2\pi nx}{L}\right)+(U^2+\gamma)\right)\frac{\partial^2 y}{\partial x^2} \end{bmatrix} + \frac{\partial^2 y}{\partial t^2} = 0 \quad (11)$$

Then, by using orthogonally method, multiplying Eq. 11 by Eq. 6, and then, integrating the resulted equation through x direction$(0\ to\ l)$, gives, the general equation of frequency for pipe with crack effect as,

$$\int_0^l (A_1+A_2+A_3) = \int_0^l \Omega_c{}^2 A_4 \quad (12a)$$

where, $I_1\frac{\partial^4 Y(X)}{\partial x^4} * Y(X) = A_1$, $I_2\frac{\partial^3 Y(X)}{\partial x^3} * Y(X) = A_2$, $I_4\frac{\partial^2 Y(X)}{\partial x^2} * Y(X) = A_3$, $\frac{\partial^2 Y(X,t)}{\partial t^2} * Y(X) = A_4$

For,

$$\begin{aligned} I_1 &= \left(A_o+\sum_{n=1}^{\infty}A_n cos\frac{2\pi nx}{L}+\sum_{n=1}^{\infty}B_n sin\frac{2\pi nx}{L}\right) \\ I_2 &= -\left(\sum_{n=1}^{\infty}A_n sin\frac{2\pi nx}{L}-\sum_{n=1}^{\infty}B_n cos\frac{2\pi nx}{L}\right) \\ I_3 &= -\left(A_o+\sum_{n=1}^{\infty}A_n cos\frac{2\pi nx}{L}+\sum_{n=1}^{\infty}B_n sin\frac{2\pi nx}{L}\right) \\ I_4 &= I_3+U^2+\gamma \end{aligned} \quad (12b)$$

Therefore, by using Eq. 12, the frequency of pipe with crack effect will be as follows,

$$\Omega_c{}^2 = \sqrt[2]{\frac{\int_0^l (A_1+A_2+A_3)}{\int_0^l A_4}}\ ,\ \omega_c = \frac{\Omega_c}{L^2\sqrt{(m_f+m_p)/EI}} \quad (13)$$

where ω_c is the frequency of the cracked pipe.

3. Numerical investigation

The numerical investigation is to be conducted using the FEM employing COMSOL program to verify the analytical results. Simulation by computer has be a main part of engineering.

The governing equations were discretized and solved using Finite FEM in a commercial package COMSOL V5.2. Numerical tests were performed to ensure that the solution is independent of the grid size. A computational quadrate mesh consisting of a total of 7902 domain elements, 4902 boundary elements, and 784 edge elements was found to provide sufficient special resolution Figure 3. The coupled set equation is solved iteratively, and the solution is considered to be convergent when the relative error consecutive iterations.

Figure 3. Computational mesh of the computational domain (quadratic).

The Computational mesh of the domain (quadratic) for the case study of Figure 4 for different cracked pipe depths. Tuning of the mesh determine the finite element mesh resolution used to discretize the sample. In this study, the analytical and numerical results of pipe conveying liquid without crack and with crack will be offered.

Discussion of the products and the effects of crack size and the crack position into the frequency of the pipe induces flow. This study includes the evaluation the natural frequency and the mode shape of the pipe with crack effect in addition to the comparison of the results with the developed analytical output for the selected cases.

The verification of data includes comparison the value of frequency results for simply supported pipe. Three positions (0.25L, 0.375L and 0.5L) and four depths of crack positions (1.5mm, 2.5mm, 3.5mm and 4.5mm) by using the two methods (analytical and numerical). For the analytical work, a computer program was written in Matlab R2016a. The numerical investigation is conducted using the FEM employing COMSOL program to verify the analytical and experimental results.

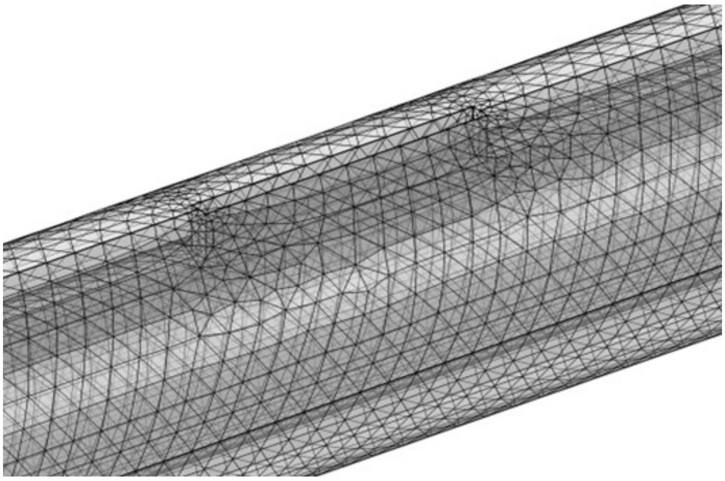


Figure 4. Mesh of pipe with crack depth.

4. Results and discussion

The results include the evaluation the frequency of pipe with effect of different crack locations and depth effect. The results evaluated by analytically by using the derived the general equation of motion for pipe conveying fluid with crack effect together with the percentage of discrepancy. This study is considered to one pipe with 1meter length, 0.02m and 0.03m inner and outer diameter respectively. The properties of the pipe material used in this study are shown in Table 1. The mechanical properties of the pipe are shown in Table 1. However, the comparisons of the results are shown in Table 2.

Figure 5 shows a decrease in the frequency of the pipeline in the event of increased crack. Certainly, the explanation goes back to the stiffness reduction.

As noted from the previous tables that the highest value of discrepancy percent between the numerical and analytical parts is (5.53%).

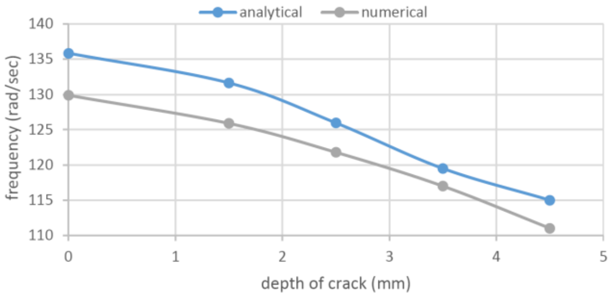
To explain the effect of crack position into the frequency of the pipe, it has been found that the frequency starts downward the closer to the central region as shown in Figure 6 (a, b). This explains the decrease that has a direct relationship with the stiffness. the case of plastic pipe with crack position is 0.25L, using cracked pipe with 4.5mm depth, the frequency was calculating the analytical solution is 115.0039 rad/sec, and for 0.5L and same velocity and same depth of crack the frequency is 108.7724 i.e. the reduction percent is 5.41%. The numerical results agree well with the analytical solution.

Table 1. Mechanical properties of pipe.

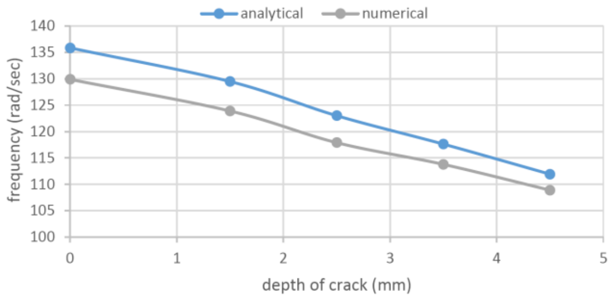
Mechanical	Test value
Density	900 Kg/ m^3
Ultimate strength	40 N/mm^3
Ultimate elongation	800%
Modulus of elasticity	800 N/mm^2

Table 2. Frequency (rad/sec) of analytical and numerical methods.

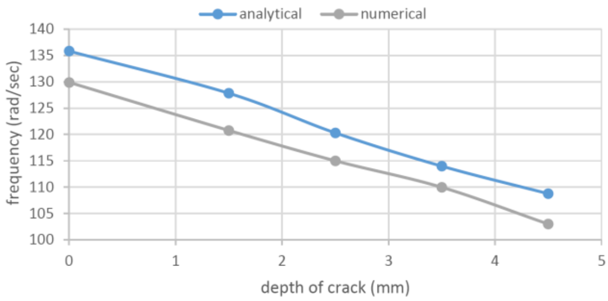
Pipe without crack				
Sequence		Analytical	Numerical	Discrepancy (%)
1		135.844	129.8907	4.38
Pipe with crack (position of crack is 0.25L)				
Sequence	d_c (mm)	Analytical	Numerical	Discrepancy (%)
2	1.5mm	131.6310	125.9021	4.35
3	2.5mm	125.9631	121.7982	3.31
4	3.5mm	119.4981	117.0002	2.08
5	4.5mm	115.0039	111.0206	3.47
Pipe with crack (position of crack is 0.375L)				
Sequence	d_c (mm)	Analytical	Numerical	Discrepancy (%)
6	1.5mm	129.4720	123.9023	4.3
7	2.5mm	123.001	117.8991	4.14
8	3.5mm	117.6191	113.7829	3.26
9	4.5mm	111.9205	108.8921	2.7
Pipe with crack (position of crack is 0.5L)				
Sequence	d_c (mm)	Analytical	Numerical	Discrepancy (%)
10	1.5mm	127.8420	120.7672	5.53
11	2.5mm	120.3201	115.0047	4.2
12	3.5mm	114.0037	109.9820	3.59
13	4.5mm	108.7724	103.0024	5.3



(a) Crack effect (position of crack 0.25L).

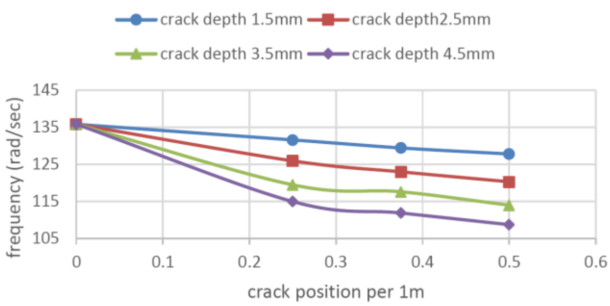


(b) Crack effect (position of crack 0.375L).

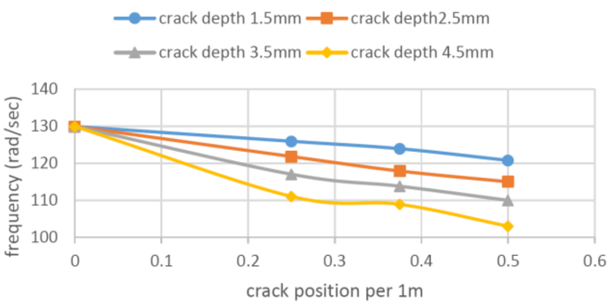


(c) Crack effect (position of crack 0.5L).

Figure 5. Analytical and numerical results of cracked pipes.



(a) Analytical data (crack position effect).



(b) Numerical data (crack position effect).

Figure 6. Crack position effect.

5. Conclusions

From the results, the following conclusions are listed,

1- The increase in the size of crack decrease the frequency.
2- The position of the crack into pipe has an effect on the frequency value of the pipe. This value becomes smaller when the crack position gets closer to the middle position.
3- The developed analytical solution is powerful and cheap solution in dealing with the prediction of frequency for cracked and un-cracked pipes conveying fluid.
4- The comparison of the results has shown a maximum percentage of discrepancy between the developed solution and the numerical prediction is (5.53%).

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