A suggested analytical investigation of heat generation inducing into vibration beam subjected to harmonic loading

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Abstract
The vibration of structure are cusecs generation heat on the machine parts, the heat generation dependent onto the natural frequency supplied on the structure and the materials types (mechanical and thermal properties). Therefore, in this paper investigation the heat generation in the beam due to vibration of beam under frequency excitation (harmonic load excitation), for the beam supported with various boundary conditions, simply supported; clamped supported; and cantilever supported beam. Also, the results included evaluated the heat generation as a function of time with different frequency applied of harmonic load, less and more than the natural frequency of beam. The heat generation due to vibration of beam with time is evaluated by analytical investigation and comparison with numerical study, by using finite element method with CFD package program. The results evaluated are shown the generation heat is decreasing with increasing the frequency of the applied harmonic load. In addition to, the value of natural frequency of beam, dependent is effect on the heat generation of vibration beam, therefore, the supported types of beam are effect of the heat generation due to harmonic load, since the natural frequency of beam is dependent on supported types of beam. The comparison of results between analytical investigation and numerical work had shown the good agreement between with maximum error about (1.36%).

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1. Introduction
The heat generation in the structure subjected to the high frequency due to mechanical viscoelastic losses in the materials. The thermo-elastic damping “represent between mechanical and thermal energy of the energy conversion” occur due to the slow temperature rise in a vibration structure. Due to internal damping of the materials, the loss of energy occur when structure vibration under frequency effect. And, when the structure subjecting to high frequency and vibration under the high frequency, generation the heat and high displacement due to internal damping characteristic of materials. When a structure is exposed to vibrations of high frequency, an important amount of heat will be generated within the structure because of mechanical losses in the material due to the viscoelastic effects. A second mechanism contributing to the
slow temperature increase in a vibrating structure is called thermo-elastic damping, and represents the energy conversion between mechanical and thermal energy. Vibration damping in solids is the phenomena whereby vibrational energy is converted to heat due to internal mechanisms within the solid. These mechanisms include internal friction between individual atoms, friction between different phases or precipitates, the movement of dislocations within a material. Every material has different damping characteristics due to its unique structure. Thus, in general the crack effect on the mechanical behaviors are investigation by multi researchers with various crack parameters effect, as effect of crack on natural frequency of beam structure, [1-5], crack effect on natural frequency on plate structure, [6-10], crack effect on natural frequency of pipe induce vibration behavior, [11, 12], crack effect on buckling behavior of beam, [13], and other application. Where, the researchers are presented that the crack defect causes decrease the stiffness for structure, in addition to increase responses for structure with various load applied.

The thermal induce vibration of structure are studied from many researchers with different technique as shown in same present review below. In 1980, S. Pantellou and A. Dimarogonas, [14], presented the analysis of heat generation for rotation shaft from materials damping, assuming both lumped mass and continuous system are considered. And, the forced torsional vibration is assumed. The elastic deformation range and an elasto-plastic materials, used hysteretic model for materials damping to yield the heat generation, assumed in the plastic range. From the heat conduction solved for cooling surface cylindrical shaft and the maximum temperatures and the maximum surface temperature obtained, shown that the develop of substantial temperature in shafts undergoing torsional vibration.

At 1997, Michael I. F. et. al, [15], studied the active damping of thermal vibration smart structure, the simply supported beam made of aluminum materials is presented to studied the effect of thermal induce vibration beam. The control system design studied with effect of large temperature changes. The results of paper shown that the smart structure can be used to damping the vibration with thermal effect. In 2013, Jadwiga K. K., [16], presented the analysis of vibration cantilever beam with thermal effect. The periodically time varying heat source acts onto surface of beam are assumed in the paper. The variables of thermal moment effect are add to the Bernoulli-Euler equation to vibration beam. And, Green’s function method used to solving the heat and vibration problem of beam.

After this, at 2014, I. Jafarsadeghi P. et. al, [17], studied the dynamic behaviors for cantilever beam made of graded materials and subjected to the harmonic thermal load. The metal and ceramic materials are used to produce the graded materials of the beam used in this paper. The equation of coupled energy and equation of motion are derived in addition to Euler-Bernoulli beam theory is used. Also, the numerical method presented in the paper.

Also, in 2014, Wenbo Zhang et. al, [18], presented the investigation of thermal effect on high frequency of beam vibration, and developed the energy flow analysis to predict frequency response beam. The methods used in the paper are EFA model and numerical simulations to study the effect of temperature effect onto thermal stress and materials properties changed. In addition to other researchers studied the problem with different way to evaluating the heat generation and temperature distribution, but, not presented the analytical solution of problem as in this work. Where, the paper presented analytical solution evaluating the heat generation as a function of time and dependent onto natural frequency subjected onto beam and with different boundary condition of beam and various beam vibration modes. In addition to, comparison the analytical results are evaluated with numerical results production by finite element method, with using CFD program.

2. Analytical investigation
The analytical investigation is techniques used to give exact solution of problem with various parameters effect, [19-23], in addition to, the analytical solution is given agreement results comparison with other technique used, [24-31]. Thus, the analytical part included two section, first evaluated of natural frequency of beam, and then, in the section part derived the equation determine the generation heat due to supplied frequency onto beam and evaluated the equation of temperature as a function of time and temperature as a function of length for beam (distribution of temperature through length of beam).

2.1 Vibration of beam
To evaluated the natural frequency of beam, solution of general equation of motion, [32-35], of free vibration uniform cross section and constant modulus beam, as, [36],

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EI $\frac{d^4w(x,t)}{dx^4} + \rho A \frac{d^2w(x,t)}{dt^2} = 0 \quad (1)$

Where, $E.I, \rho.A$ are modulus of elasticity, moment of inertia, density, and cross section area of beam, respectively, $x$ is beam direction through length of beam, $t$ is time, and $w$ is the deflection of beam through lateral direction of beam.

Then, by using separation of variable as,

$W(x,t) = W_n(x) \times W_n(t) \quad (2)$

Then, by substitution Eq. 2 in to Eq. 1, and solve equation, get,

$W(x) = \left[ C_1 (\cos \beta x + \cosh \beta x) + C_2 (\cos \beta x - \cosh \beta x) + C_3 (\sin \beta x + \sinh \beta x) + C_4 (\sin \beta x - \sinh \beta x) \right] \quad (3)$

Where, $C_1, C_2, C_3, and \ C_4$ are constant. And, $\beta$ can be determine from boundary conditions of beam. And, the natural frequency of beam can be determine from, [36],

$\omega_n = (\beta l)^2 \frac{EI}{\rho Al^4} \quad (4)$

Where, $l$ is the length of beam and $n$ is the number of mode of beam.

Therefore, to evaluated the value of natural frequency must be selected the boundary condition of beam, then, can be selected three types of boundary condition beam as,

1. Cantilever beam, the boundary condition of cantilever beam included first ends fixed and other edges free as shown in Figure 1, as,

$W = 0, \frac{dW(x)}{dx} = 0$ at fixed end, and, $\frac{d^2W(x)}{dx^2} = 0, \frac{d^3W(x)}{dx^3} = 0$ at free end. \quad (5)

Then by substitution Eq. 5 into Eq. 3 and solution, get (first four natural frequencies),

$\beta_1 l = 1.875104, \quad \beta_2 l = 4.694091, \quad \beta_3 l = 7.854757, \quad \beta_4 l = 10.995541 \quad (6)$

And, the normal mode functions of beam, as,

$W_n(x) = \left[ (\sin \beta_n x - \sinh \beta_n x) - \frac{\sin \beta_n l + \sinh \beta_n l}{\cos \beta_n l + \cosh \beta_n l} (\cos \beta_n x - \cosh \beta_n x) \right] \quad (7)$

Figure 1. Boundary condition of cantilever beam supported.

2. Simply supported beam, the boundary condition of simply supported beam are pinned ends for two edges of beam, as shown in Figure 2, as,

$W = 0, \frac{d^2W(x)}{dx^2} = 0$ at two edges of beam. \quad (8)

Also, by substitution Eq. 8 in to Eq. 3, get (first four natural frequencies),
\[
\beta_1 l = 3.141593, \beta_2 l = 6.283185, \beta_3 l = 9.424778, \beta_4 l = 12.566371 \tag{9}
\]

And, the normal mode functions of beam, as,
\[
W_n(x) = (\sin \beta_n x) \tag{10}
\]

And, the normal mode functions of beam, as,
\[
W_n(x) = (\sin \beta_n x)
\]

3. Clamped beam, the boundary condition of clamped supported beam are fixed ends for two edges of beam, as shown in Figure 3, as,

\[
W = 0, \frac{\partial W(x)}{\partial x} = 0 \text{ at two edges of beam.} \tag{11}
\]

Also, by substation Eq. 11 in to Eq. 3, get (first four natural frequencies),
\[
\beta_1 l = 4.730041, \beta_2 l = 7.853205, \beta_3 l = 10.995608, \beta_4 l = 14.137165 \tag{12}
\]

And, the normal mode functions of beam, as,
\[
W_n(x) = \left[ (\sinh \beta_n x - \sin \beta_n x) + \frac{\sinh \beta_n l - \sin \beta_n l}{\cos \beta_n l - \cosh \beta_n l} (\cosh \beta_n x - \cos \beta_n x) \right] \tag{13}
\]

Then, applied the frequency on the beam less than the natural frequency of beam and evaluated the heat generation on the beam. And then, applied frequency on the beam equal to natural frequency of beam (resonance of beam occurred) and comparison the different between the two case.

Therefore, the general equation of motion of force vibration beam can be used,
\[
\omega_n^2 W_n(x)W_n(t) + W_n(x) \frac{d^2 W_n(t)}{dt^2} = \frac{1}{\rho A} f(x, t) \tag{14}
\]

Where, \( f(x, t) \) general force applied on the beam at distance \( a \) (N/m), as shown in Figures. 1, 2, and 3. By multiplying Eq. 14 throughout by \( W_n(x) \), and integrating from 0 to \( l \), and using the orthogonally, get,
\[
\frac{d^2 W_n(t)}{dt^2} + \omega_n^2 W_n(t) = \frac{1}{\rho A b} Q_n(t) \tag{15}
\]
Where, $Q_n(t)$ is generalized forced (N),

$$Q_n(t) = \int_0^t f(x, t) W_n(x) \, dx.$$  \hspace{1cm} (16a)

And, $b$ is constant (m), and can be expressed as,

$$b = \int_0^l W_n^2(x) \, dx \, (m)$$  \hspace{1cm} (16b)

Then, by using Duhamel integral, the solve of Eq. 16, for zero initial conditions (displacement and velocity), and substitution the solution into Eq. 2, get,

$$w(x, t) = W_n(x) \left( \frac{1}{\rho A b \omega_n} \int_0^t Q_n(\tau) \sin \omega_n(t - \tau) \, d\tau \right)$$  \hspace{1cm} (17)

Therefore, applied harmonic load $f(x, t) = f_0 \sin \omega t$ with frequency of $\omega$, at $x = a$, and substitution it’s in Eqs. 16 and 17, to evaluating the general equation of beam deflection as a function of $x$ and time.

2.2 Heat generation

The forced vibration of the beams results in heat generated due to material damping. The heat generated is due to higher temperatures that reduce the natural frequency, which may affect the work of the beam. The temperatures can arrive high premiums in some states, so the reliability of some of the components of the device may be affected [37]. To calculate the amount of heat generated, the energy of elastic deformation value must first be calculated. The energy of elastic deformation for a unit length, longitudinal direction, under stress can be written as [38].

The energy of elastic deformation for a unit length, longitudinal direction, under stress is, [38],

$$U = \frac{1}{2} b_w \int_{-h/2}^{h/2} \sigma_{xx} \varepsilon_{xx} \, dz$$  \hspace{1cm} (18)

Where, $b_w$ is the width of beam, $\sigma_{xx}$, $\varepsilon_{xx}$ are stress and strain of beam in $x$-direction, $h$ is the beam thickness, and $z$ is the direction of lateral direction beam. Therefore, the strain of beam in $x$-direction can be evaluated from,

$$\varepsilon_{xx} = \frac{du}{dx}$$  \hspace{1cm} (19)

Where, $u$ is the displacement of beam in $x$-direction, can be evaluated as,

$$u = -z \frac{dw}{dx}$$  \hspace{1cm} (20)

Then, by substitution Eq. 20 in to Eq. 19, get,

$$\varepsilon_{xx} = -z \frac{d^2w}{dx^2}$$  \hspace{1cm} (21)

Therefore, with using of Hook’s low, get,

$$\sigma_{xx} = E \varepsilon_{xx} = -Ez \frac{d^2w}{dx^2}$$  \hspace{1cm} (22)

Then, by substitution Eqs. 22 and 21 into Eq. 18, get,

$$U = \frac{1}{2} b_w E \int_{-h/2}^{h/2} \varepsilon_{xx}^2 \, dz = \frac{1}{2} b_w E \int_{-h/2}^{h/2} z^2 \left( \frac{d^2w(x,t)}{dx^2} \right)^2 \, dz$$  \hspace{1cm} (23)

Then, for symmetrical uniform beam can be integration Eq. 23 and get,
\[ U = \frac{1}{24} b_w E h^3 \left( \frac{d^2 W(x,t)}{dx^2} \right)^2 \] \quad (J/m) \quad (24)

Where, \( W(x,t) \) is the lateral deflection beam evaluated form Eq. 17.

Hence, the heat generation rate due to materials damping of beam, [14],

\[ q = \frac{1}{2} \pi * f_n * \eta * U = \frac{\pi}{48} b_w E h^3 \left( \frac{d^2 W(x,t)}{dx^2} \right)^2 f_n \eta \] \quad (W/m) \quad (25)

Where, \( f_n \) is the frequency [Hz], and \( \eta \) is the loss factor \((\eta = 0.0028) [39]\).

Then by substitution Eq. 17 into Eq. 25, get the general heat generation of beam with \( x \)-direction and time for different beam boundary condition, as,

1. Cantilever beam,

\[ q = \frac{\pi}{2} \times f_n \times \eta \times U \]

\[ = \frac{\pi}{48} f_n \eta b_w E h^3 \beta_n^4 \left[ \left( \frac{\sin \beta_n l + \sinh \beta_n l}{\cos \beta_n l + \cosh \beta_n l} \right) \left( \cos \beta_n x + \cosh \beta_n x \right) \left( \frac{1}{\rho A \omega_n} \int_0^\tau Q_n(\tau) \sin \omega_n(t - \tau) \, d\tau \right) \right]^2 \]

\[ q = \frac{\pi}{4} \frac{f_n \eta}{\rho A \beta_n^2} \left[ \left( \frac{\sin \beta_n l + \sinh \beta_n l}{\cos \beta_n l + \cosh \beta_n l} \right) \left( \cos \beta_n x + \cosh \beta_n x \right) \left( \int_0^\tau Q_n(\tau) \sin \omega_n(t - \tau) \, d\tau \right) \right]^2 \quad (26)

2. Simply supported beam,

\[ q = \frac{\pi}{2} \times f_n \times \eta \times U = \frac{\pi}{48} f_n \times \eta \times b_w E h^3 \beta_n^4 \left( \sin \beta_n x \left( \frac{1}{\rho A \omega_n} \int_0^\tau Q_n(\tau) \sin \omega_n(t - \tau) \, d\tau \right) \right)^2 \]

\[ q = \frac{\pi}{4} \frac{f_n \eta}{\rho A \beta_n^2} \left( \sin \beta_n x \left( \int_0^\tau Q_n(\tau) \sin \omega_n(t - \tau) \, d\tau \right) \right)^2 \quad (27)

3. Clamped beam,

\[ q = \frac{\pi}{2} \times f_n \times \eta \times U \]

\[ = \frac{\pi}{48} f_n \eta b_w E h^3 \beta_n^4 \left[ \left( \frac{\sinh \beta_n x + \sin \beta_n x}{\cos \beta_n x + \cosh \beta_n x} \right) \left( \cos \beta_n x + \cosh \beta_n x \right) \left( \frac{1}{\rho A \omega_n} \int_0^\tau Q_n(\tau) \sin \omega_n(t - \tau) \, d\tau \right) \right]^2 \]

\[ q = \frac{\pi}{4} \frac{f_n \eta}{\rho A \beta_n^2} \left[ \left( \frac{\sinh \beta_n x + \sin \beta_n x}{\cos \beta_n x + \cosh \beta_n x} \right) \left( \cos \beta_n x + \cosh \beta_n x \right) \times \left( \int_0^\tau Q_n(\tau) \sin \omega_n(t - \tau) \, d\tau \right) \right]^2 \quad (28)

Therefore, by integrating Eq. 26, 27 or, 28, get the general equation of heat generation due to vibration beam as a function of time and longitudinal beam. Where, and by building computer program by using Matlab program to evaluate the heat generation of beam with different beam boundary condition and as a function of time and \( x \)-direction of beam. But to calculate the amount of heat in the program Matlab for the purpose of giving results in the form of data and also to draw the figures to compare the amount of heat per unit volume, so the following equation is applied: \( \dot{q} = \frac{q}{A} \).

### 3. Numerical investigation

The numerical investigation is technique used to evaluation approximate solution for problem, [40-43], thus, the its technique in more application used to analysis the difficult structure its cannot solution by analytical techniques, [44, 45]. In addition, the numerical technique used to comparison the numerical results by other results are evaluated with other techniques, to given the agreement of results, [46-48]. Therefore, the COMSOL program will be used, which will depend on the finite element method (FEM) for design and analysis of the models, and give the outcomes required by simulations using computer programming to reach the outputs and outcomes of the completion of the research and development of the
mechanism of work, and compared to other outcomes extracted with the analytical solution, and given the percent error of its results.

3.1 COMSOL program
This program is used to find precise numerical solutions and different designs and structures and to find the results of complex mathematical equations that are difficult to solve analytically, [49-53]. In this work, the COMSOL/CFD program was used to find the heat generated values by the vibration of the beam exposed to the harmonic force at different frequencies for different cases of supported. Follow the main steps when using the software in the computer as follows,
1. Design of the models according to properties, dimensions and geometries of the material to be used.
2. Defining the meshing geometry.
3. Applying the boundary conditions, frequency and force.
4. Solving and visualizing your results.

3.2 Modelling and computational domain
The beam design length (84 cm) and cross-section (2.5 cm * 2.5 cm) as in the Figure 4. The material used is carbon steel (carbon=1.5%) with a density of (7750 kg/m$^3$) and modulus of elasticity (200 Gpa). The supported is determined by the type of beam where the use of three types including (cantilever beam, simply supported beam, clamped beam) and each type of supported has special boundary conditions, the required frequency and force are applied to the beam, after which the program is executed and the desired results and figures are included.

The design of the model form on the mesh as in Figure 5 which follows a finite element method (FEM) to give the results the elements, the results will be more accurate as the mesh is accurate if the distance between the nearest mesh lines, the mesh was represented symmetrically for all types of supported, so that the accuracy of the results is uniform for all cases to be used in comparison.

3.3 Meshing solved and processing
Default mesh setups were utilized. The model solution at transient boundaries. It is plotted measured values of displacement, heat generation and the temperature distribution utilizing domain plotting parameters. The Eigen natural frequency analysis outcomes initially then, the resonance are appeared in the distorted shape with sub- domain outcomes. The temperature distribution and heat generation plot outcomes are appeared of normal sub- domain outcomes.

3.4 Work states
The results obtained through the use of the COMSOL program by simulating similar cases in the analytical solution. The results shown in the forms were obtained to determine the relationship between the variables and the behavior of the curves of the model in the cases of assuming that the harmonic force causes vibration very near to the resonance state of each case to show the variables clearly. The heat generated along the beam during a fixed period (60 seconds) as shown in Figure 6 (a, b and c) for different supported beam as shown:
4. Results and discussion

The verification and comparison of the results and figures extracted from the theoretical part of the analytical solution and the numerical solution, for the heat generated by the vibration of the beam exposed to the harmonic force with time. Since the analytical solution can calculate the magnitude of heat generated in the beam through the energy of elastic deformation, so the use of the COMSOL program to calculate the magnitude of heat generated and temperature changes in the beam due to vibration. The numerical solution will be implemented using the COMSOL program by designing models similar to the beam specifications in terms of dimensions, geometry and properties of the material used the same forces exerted during vibration of the beam calculated by analytical solution with compared between them.

Figure 6. The relationship between the heat generated & the length of the beam with constant time.
The magnitude of heat generated is compared between the results calculated in the analytical solution and the results in the numerical solution for the same models for multiple cases of supported.

4.1 Analytical results
The amount of heat generated in the assumed beam (length (L=0.84 m), width, height (w=h=2.5 cm)) is calculated analytically by using the Matlab program during the vibration of the beam under the influence of harmonic force at different frequencies compared to the natural frequency. Sometimes the heat generated and the deflection is calculated for each location along the length of the beam on demonstrated constant time at other times it is calculated with time in a location where the heat generated and deflection the highest value in the beam for several cases of supported, as follows

4.1.1 The heat generated in the cantilever beam
Figure 7 shows the amount of heat generated in the beam changes according to the time and length of the beam when the beam is vibrated at a frequency ($\omega = 0.9 \times \omega_n$).

Now, in Figure 8, the relationship between the heat generated with the length of the beam is explained at constant time for different frequencies ($\omega = (0.5, 0.6, 0.7, 0.8, 0.9$ and 1.2) $\omega_n$) as shown (a), ($\omega = 0.99 \omega_n$) as shown (b) and ($\omega = \omega_n$) as shown (c).

The Figure 9, the relationship between the heat generated with the time is explained at the fixed end (x = 0) for the appearance of the highest heat generated as shown in the previous Figure 9 for different frequencies ($\omega = (0.8, 0.9, 1.1$ and 1.2) $\omega_n$) as shown (a) and ($\omega = (0.99, 1 \omega_n$) as shown (b).

4.1.2 The heat generated in the simply supported beam
Figure 10 shows the amount of heat generated in the beam changes according to the time and length of the beam when the beam is vibrated at a frequency ($\omega = 0.9 \times \omega_n$). Now, in Figure 11, the relationship between the heat generated with the length of the beam is explained at constant time for different frequencies ($\omega = \omega_n$) as shown (a) and ($\omega = (0.85, 0.9, 0.95, 0.99, 1.05, 1.1, 1.15$ and 1.2) $\omega_n$) as shown (b). The Figure 12, the relationship between the heat generated with the time is explained at the middle of the beam (x = 0.42 m) for the appearance of the highest heat generated as shown in the previous Figure for different frequencies ($\omega = (0.85,$ to 1.2 $\omega_n$) as shown in Fig. 12(a) and ($\omega = (0.99$ and 1) $\omega_n$) as shown in Figure 12 (b).

![Figure 7. Heat generated with time and length of the beam, (Three dimension).](image-url)
Figure 8. The heat generated with length of the beam.

Figure 9. The heat generated with time with various natural force applied.
4.1.3 The heat generated in the clamped beam

Figure 13 shows the amount of heat generated in the beam changes according to the time and length of the beam when the beam is vibrated at a frequency $(\omega = 0.9 \times \omega_n)$. Now, in Figure 14, the relationship between the heat generated with the length of the beam is explained at constant time for different frequencies $(\omega = \omega_n)$ as shown (a), $(\omega = 0.99\omega_n)$ as shown (b) and $(\omega = (0.85, 0.9, 0.95, 1.05, 1.1 \text{ and } 1.15) \omega_n)$ as shown (c). The Figure 15, the relationship between the heat generated with the time is explained at...
the fixed end (x = 0) for the appearance of the highest heat generated as shown in the previous Figure 14 for different frequencies (ω = (0.85, 0.9, 0.95, 1.05, 1.1, 1.15 and 1.2) ωn) as shown (a) and (ω = (0.99 and 1) ωn) as shown (b, c).

4.2 The comparison the analytical solution and the numerical solution

The comparison the relationship between the heat generated with the length of the beam is explained for the analytical solution and the numerical solution at constant time and constant frequency for different supported beam, as, Figure 16a for cantilever beam, Figure 16b for simply supported beam, and, Figure 16c for clamped beam, with ω = 0.9ωn, t = 60 sec.

Figure 13. Heat generated with time and length of the beam.

Figure 14. The heat generated with length of the beam.
Figure 15. The heat generated with time.

Figure 18. The heat generated (analytical & numerical), \( \omega = 0.9\omega_n, t = 60\ sec.\)
5. Conclusions
From previous results, the following conclusions can be shown briefly as follows:
1. The theoretical method was followed by accurate and logical results to calculate the amount of heat generated in the beam for the different types of supported according to the change of time or location in the beam and frequencies different according to the natural frequency. Also calculate the change in the natural frequency.
2. In the cantilever beam, the higher of heat generated at the fixed end. In the simply supported beam, the higher of heat generated in the middle of the beam. In the clamped beam, the higher of heat generated in the fixed ends and starts decreasing until it reaches approximately zero at (x = (0.25, 0.75) L) and then rise up to the middle of the beam, but less than the ends.
3. To compare the amount of heat generated in the beam for the three types of supported the maximum heat generated in the fixed ends of the clamped beam is less than the heat generated in the middle of the beam for the simply supported beam and the lower is at the fixed end of the cantilever beam.
4. In general and for any type of supported it was observed that the heat generated increases as the frequency supplied is approaching to the natural frequency, and the generated heat is reduced as the frequency supplied goes away the natural frequency significantly.
5. The analytical solution is a good tool technique used to evaluation the heat generation for vibration beam subjected to harmonic load. In addition, the analytical solution was given a good agreement of heat generation results to calculate the amount of heat with compassion by numerical technique used.

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Reference


