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Semi-analytic solution for stability and free vibration of functionally graded (FG) material micro-pipe conveying fluid

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Abstract
A Micro-scale pipe conveying fluid and the functionally graded (FG) materials have many potential applications. In this article, an analytical solution is offered free vibration for a functionally graded (FG) material micro-pipe conveying fluid. On the basis of the Euler beam model and the modified coupled stress theory. The properties of the material are changed constantly across the micro-pipes thickness and depend on power law distribution. Utilized Hamilton's principle to get an equation of motion for three end boundary conditions (Simply supported, clamped-clamped and cantilever micro-pipes).The differential transformation (DT) method is utilized to obtain the solution for motion's equation and concerned boundary conditions. The effect of fluid flow velocity, the gradient index and parameter of the material length scale on the vibration and stability of fluid conveying FGM micro scale pipes are discussed. The results show that critical velocities and natural frequencies are increased hastily with the increase in the gradient index p.

Keywords: Fluid- conveying FGM micro pipe; DTM; Functionally graded material; Natural frequency; Critical flow velocity.

1. Introduction

Pipe Conveying fluid is very important components for most engineering structures, a nuclear reactor, heat exchanger, marine risers, oil pipelines, microfluidic, Nano fluidic devices and others. Free Vibration analysis of the fluid conveying pipe has been closely studied in past decenniumes.

The vibration problems of the pipe conveying fluid were analyzed through different methods. By reviewing the literature in this field, it was observed that there are many numerical and analytical methods used in solving vibration problems of this structures both in nonlinear and linear dynamics, like finite element (FE) method by Zhang, Gorman, and Reese [1], Galerkin's method by Sarkar, and Païdoussis [2], DQM by Qian and Wang [3] simplistic method by Wang and Liu [4]. Paidoudssis and Issid [5] showed the linear dynamics of the pipe conveying fluid using the Galerkin's method. Yun-dong and Yi-ren [6] developed vibrational iteration (VI) method for analysis free vibration conveying fluid in the pipe, and they are obtained the critical velocity of flow and frequency for fluid conveying pipe with many end conditions.

The (DT) Method was first suggested for solving linear and nonlinear elementary value problems at the analysis of the electrical circuit based on the expansion of Taylor's series by Zhou [7]. This method is an effective and not complicated for solving the differential equations of linear and nonlinear. Zhang and Wang [8] analyzed a free vibration for pipe conveying fluid with many typical boundary conditions by employed the DTM. They have proved that the DT method has computational efficiency and high accuracy for vibration analysis in fluid conveying pipes and this method may be further extensive to the analysis of the response for static and the dynamic nonlinearly for fluid conveying pipes. The free vibration problem with various elastically restrained end conditions of a uniform beam was studied by Agboola [9] using Differential transformation (DT) method to solve the relevant initial boundary value problem. He has compared the vibration frequencies for the present method with those prophesied by Adomian decomposition and variational iteration methods. He has noted that the accuracy of natural frequencies is higher with increasing the term number N. and he has explained that differential transformation (DTM) has computational efficiency and high accuracy in the vibration problem for beam structure. Bozyigit et al [10] analyzed natural frequencies, modes shape and critical fluid velocity of the pipelines based on Timoshenko beam model by using DTM and ADM. They solved the equation of motion with different pipe end conditions using these methods. They were found that efficient and easy mathematical models when using these methods. Also, they found that the results of an analytical method (ANM) have a good agreement when compared with these two methods.

Wang [11] developed the theoretical model for the vibration of microtube based on modified coupled stress theory and he was used (DQM) to solve the equation of motion. He was found that the vibration frequencies decrease with increasing internal flow velocities. Also, He was found that a microtube will be unstable by divergence in a critical flow velocity. Xia and Wang [12] and Ahangar et al. [13] used MCST to study the dynamic conduct of fluid conveying micro-pipes by using Timoshenko and Euler beam theories. They explained that a critical velocity and fundamental frequency would be size dependent when the outside diameter of micro-pipes are compared with the parameter of length scale. Wang et al [14], investigated the effects of microstructure and micro-flow on the flexural vibrations of fluid conveying microscale pipes. The results appeared that the effect of microstructure tends to stiffen the pipe system and hence increased the critical flow velocity; also, the results appeared that the velocity profile of flow tends to decrease a critical mean of flow velocity.

A new material was used for the first time by Yamanouchi [15] these materials are functionally graded (FGM) that are microscopically inhomogeneous composite materials, where the mechanical properties alteration from the surface to another continuously. This is achieved by changes in the composition of the FGM continuously. Synthesis is different continuously with an alteration in the volume fraction of components.

Loy et al [16] used functionally gradient material (FGM) because it has attracted a lot of attention as a new material and has sophisticated structural materials due to their heat-resistance characteristics. The characteristics are graduated in the thickness direction depending on a volume fraction power-law distribution. The eigenvalue governing equation was obtained using the Rayleigh method and the results appear that the frequency properties are identical to that observed for the homogeneous isotropic of cylindrical shells. The vibration analysis and instability problem of the spinning thin-walled beams with functionally graded (FG) materials was studied by Librescu [17]. A continuously graded change in the composition of the metal and ceramic phases through the beam thickness in expressions of a simple power law was implemented.

The thermomechanical stability of thin-walled conveying fluid of a cantilevered pipe made from functionally graded and it loads through compressive axial force was investigated by Hosseini et al [18]. The pipe is formulated based on Rayleigh's theory and the extended Galerkin's method was used to solve the equations of motion. They investigated the effects of gradient index, compressive axial force, fluid mass ratio, fluid speed, and temperature Variable on the stability of thin-walled FGM pipe. Yang et al [19] evolved the theory of modified coupled stress (MCS) where in just one parameter of length scale appears in an equation, the torsion of the cylindrical bar and a pure bending of the flat plate of unlimited width were analyzed to interpret the effect of the modification. Thereafter Asghari et al. [20], Reddy [21], Nateghi et al [22] and Ansari et al [23] utilized MCST to investigate the mechanical behavior of FGM micro-beams. They investigated the natural frequency, critical buckling load, and the static deflection. They found that the size effect becomes stronger through the decreasing thickness. Akgöz and Civalek [24, 25] studied the free vibration of a single-layered graphene tablets and the axially functionally graded (FG) tapered Bernoulli-Euler microbeams based on a modified couple stress theory, respectively. Setoodeh and Afrahim [26] studied an analytical solution for size-dependent nonlinear vibration analysis for (FGM) microscale

pipe with strain gradient theory. They evidenced by results that the power law index and length scale parameter have a considerable effect on the critical velocity and natural frequency of the FG micro-pipes. Deng et al [27] investigated a free vibration and stability for fluid conveying multi-span FGM micro-pipe. The FGM micro-pipes which has variation continuously through-thickness direction in accordance with the power law. The hybrid method was advanced to find the vibration frequencies and stability. The effect of a number of supports, a parameter of length scale, and an exponent of volume fraction, on dynamic properties, were discussed. The results appeared that the fundamental frequencies determined by the theory of modified couple stress are greater than those acquired by using classical beam theory, also, the results showed that the critical velocities and natural frequencies increase with an increase in the exponent volume fraction (p) when it is lower than 10.
Through the literature mentioned above, it was found that the articles available for study the behavior of FGM micropipe very little, in this paper, the stability and free vibration of micro-pipe made from a functionally graded (FG) material conveying fluid by utilizing differential transformation (DT) method are investigated. The governing equation is derived by using energy Hamilton's principle with modified couple stress theory. Differential transform (DT) method is developed to find the vibration frequencies and mode shapes of the FG micro fluid conveying pipe for different end conditions. The results computed by a differential transformation (DT) method are compared with those in a published literature to verify the current method. The effects of different parameters such as volume fraction n and the parameter of length scale on the stability and free vibration of FGM micro-scale pipes conveying fluid are discussed.

2.2 Model description and governing equations

2.1 Material properties of FGM pipes

In the present investigation, material properties of FG micro-pipes with a Length (L), the inner and outer radii are R_i and R_o respectively and cross-sectional area are presumed to be graduated in the thickness direction (h). U represents fluid flow velocity. The axial and cross displacements on midmost-axis are u and w, respectively as shown in Figures 1 and 2. Material properties are supposed to alteration through a thickness direction continuously. The mechanical materials properties used in this paper are expressed in Table 1 [27].

Table 1. Material properties of FG micro pipe.

Materials	E (GPa)	ρ_p(kg/m^3)	ν
Alumina	380	3800	0.23
Aluminum	70	2700	0.23

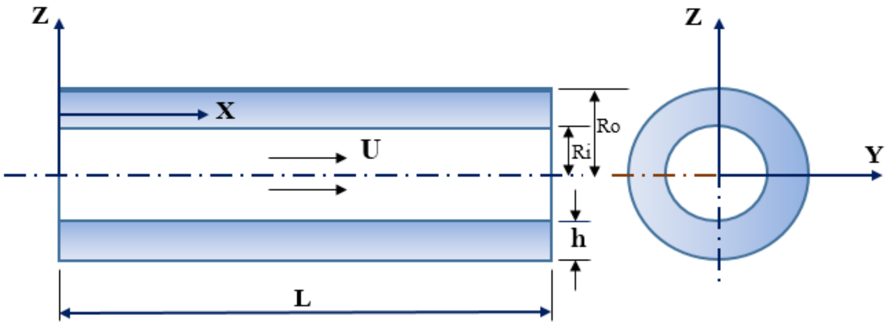


Figure 1. The Geometrical model of a fluid - conveying FGM micro-pipe.

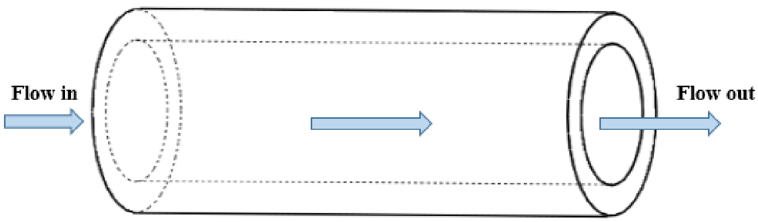


Figure 2. Micro pipe in three dimension.

Most researchers adopted sigmoid law, exponential law and power law to characterize the material properties variation. In this paper, FGM pipes with power law will be used. The volume fraction can be given as [28].

$$Vm = \left(\frac{2z+h}{2h}\right)^{p} \qquad Where\ (0 \le p \le \infty) \tag{1}$$

$$Vc = 1 - Vm \tag{2}$$

Where p is the volume fraction exponent, is a real positive number and which describes the volume fraction profile; also, subscripts m and c indicate the inner and outer layers, respectively. A change of volume fraction V_i with thickness direction for different values of exponents of volume fraction p is depicted in Figure 3, it can be seen that when the exponent p is supposed to be zero, FGM micro-pipe reduces to homogeneous micro pipe [28].

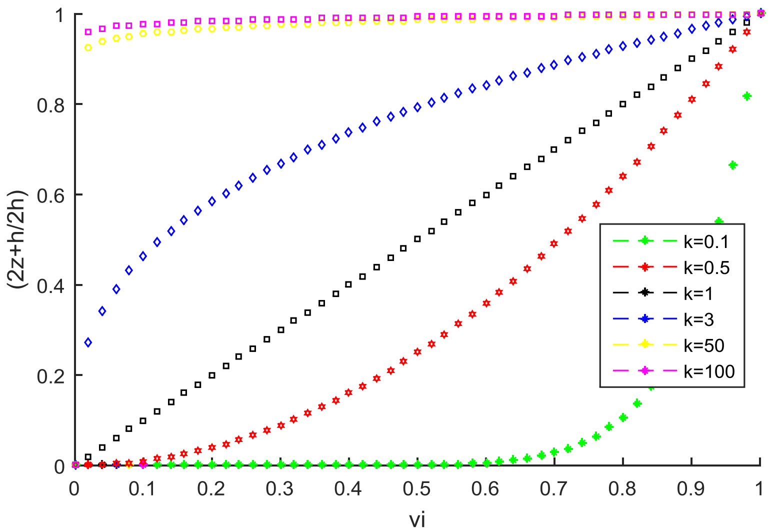


Figure 3. Variation of volume fraction with thickness direction.

$$\rho(z) = V_c\, \rho_c + V_m\, \rho_m \tag{3}$$

$$E(z) = V_c E_c + V_m E_m \tag{4}$$

where ρ and E refer to the density and the Young's modulus respectively.

2.2 Mathematical formulation

Based on Euler–Bernoulli beam theory, the offset field for an arbitrary point along the x and z axes can be written as:

$$u^{-}(x,z,t) = u(x,t) - z\frac{\partial w(x,t)}{\partial x} \tag{5}$$

$$w^{-}(x,z,t) = w(z,t) \tag{6}$$

where (z) is a coordinate measured from the plane of a neutral axis and t denoted time. Assuming that micro-pipe is elastic, the relation of stress–strain is given by:

$$\sigma_{xx} = E\varepsilon_{xx} \tag{7}$$

$$\varepsilon xx = -z\frac{\partial w(x,t)}{\partial x} \tag{8}$$

2.3 Modified couple stress theory

The coupled stress theory is a more public form of the theories of higher order continuum, which looks the both of antisymmetric and uniform parts of higher order deformation gradients. The brief review of this theory was firstly presented by Xia W., and L. Wang and Ahangar Sonia et al [12, 13] respectively. The expression of a strain energy U refer to the linear elastic material occupying zone Ωi with very small deformation is given by:

$$U = \sum_i^s \frac{1}{2} \int (\sigma_{ij}\varepsilon_{ij} + \tau_{xz}\varepsilon_{xz} + m_{xy}\gamma_{xy}) d\Omega i \tag{9}$$

$$U = \frac{1}{2} \int \int_0^L [E(z)z^2 + G(z)l^2\,] \left(\frac{\partial^2 w}{\partial x^2}\right)^2 dx\, dA \tag{10}$$

Or

$$U = \frac{1}{2} \int_0^L (EI_{eq} + GA_{eq} l^2) dx \tag{11}$$

where

$$EI_{eq} = \int_0^{2\pi R_m} \int_{-h/2}^{h/2} E(z) z^2 dzdr \tag{12}$$

$$GA_{eq} = \int_0^{2\pi R_m} \int_{-h/2}^{h/2} \frac{E(z)}{2(1+\upsilon)} dzdr \tag{13}$$

The mass of the fluid and pipe per unit length, m_f and m_p respectively. Their expressions can be given as:

$$m_p = \int_0^{2\pi R_m} \int_{-h/2}^{h/2} \rho(z) dzdr, \quad m_f = \rho_f\, A_f \tag{14}$$

where ρ_f is the density of fluid in FGM micro-pipe and A_f is the flow cross-sectional area.
The kinetic energy of FGM micro-pipe and fluid [27] is known as follows:

$$T_p = \frac{1}{2} \int_0^L \rho A_{eq} \left(\frac{\partial w}{\partial t}\right)^2 dx \tag{15}$$

$$T_f = \frac{1}{2} \int_0^L m_f \left(\frac{\partial w}{\partial t} + u_f \frac{\partial w}{\partial x}\right)^2 dx \tag{16}$$

2.4 Governing equations

The governing equations for FGM micro-pipe conveying fluid are derived by extended Hamilton's principles as [25]:

$$\delta \int_{t1}^{t2} \left(T_p + T_f - U\right) dt = 0 \tag{17}$$

After substitute equations (11), (15) and (16) into equation (17) (see Appendix for the derivation). The equation of motion for a FGM micro pipe convening fluid is:

$$\left(EI_{eq} + GA_{eq} L^2\right) \frac{\partial^4 w}{\partial x^4} + \alpha m_f u_f^2 \frac{\partial^2 w}{\partial x^2} + 2 m_f u_f \frac{\partial^2 w}{\partial x \partial t} + \left(m_f + m_p\right) \frac{\partial^2 w}{\partial x^2} = 0 \tag{18}$$

where α is a factor referred to the effect of micro-flow velocity [14], the non-dimensional form for the equation of motion for FGM micro pipe convening fluid is:

$$(\gamma + \mu) \frac{\partial^4 w_d}{\partial \xi^4} + (\alpha U^2) \frac{\partial^2 w_d}{\partial \xi^2} + 2UMr^{0.5} \frac{\partial^2 w_d}{\partial \xi \partial \tau} + \frac{\partial^2 w_d}{\partial \xi^2} = 0 \tag{19}$$

where:

$$\xi = \frac{x}{L}, \quad w_d = \frac{w}{L}, \quad \lambda = \frac{l}{D_o}$$
$$Mr = \frac{m_f}{m_f + m_p}, \quad \tau = \sqrt{\frac{E_c I_o}{(m_f + m_p)}} \frac{t}{L^2} \tag{20}$$
$$U = \sqrt{\frac{m_f}{E_c I_o}} uL, \quad \mu = \frac{G A_{eq} L^2}{E_c I_o}, \quad \gamma = \frac{E I_{eq}}{E_c I_o}$$

In this work, three cases of ends boundary conditions are considered as shown in Figure 4.
Thus, the BCs at the ends of FG micro-pipe are:
1- Simply support (pined- pined):

$$At \ \xi = 0 \rightarrow w_d(\xi) = 0 \, , \frac{\partial^2 w_d(\xi)}{\partial \xi^2} = 0$$
$$At \ \xi = 1 \rightarrow w_d(\xi) = 0 \, , \frac{\partial^2 w_d(\xi)}{\partial \xi^2} = 0 \tag{21}$$

2- Clamped-Clamped:

$$At \ \xi = 0 \rightarrow w_d(\xi) = 0 \ , \frac{\partial w_d(\xi)}{\partial \xi} = 0$$
$$At \ \xi = 1 \rightarrow w_d(\xi) = 0 \ , \frac{\partial w_d(\xi)}{\partial \xi} = 0 \tag{22}$$

3- Clamped- free (cantilevered):

$$At \ \xi = 0 \rightarrow w_d(\xi) = 0 \ , \frac{\partial w_d(\xi)}{\partial \xi} = 0$$
$$At \ \xi = 1 \rightarrow \frac{\partial^2 w_d(\xi)}{\partial \xi^2} = 0 \, , \frac{\partial^3 w_d(\xi)}{\partial \xi^3} = 0 \tag{23}$$

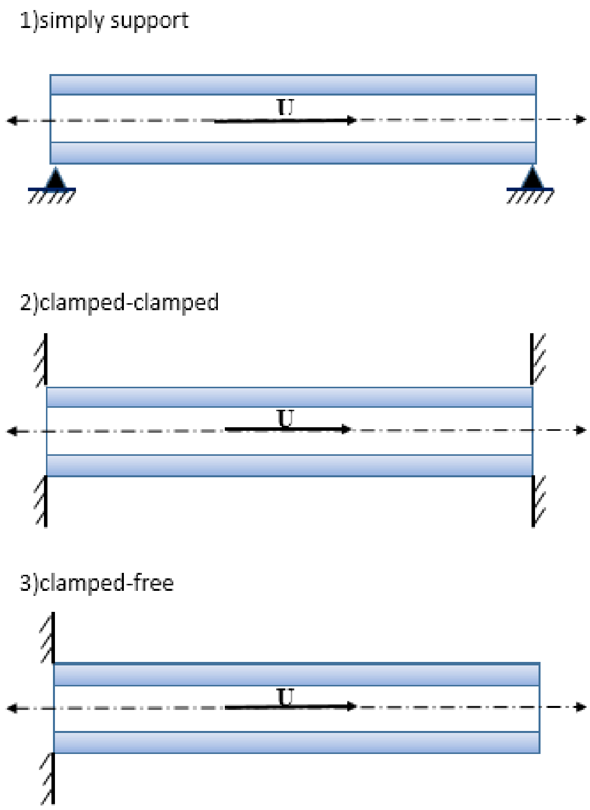


Figure 4. The boundary conditions for micro-pipe.

3. Solution method

The technique of differential transformation, which was first suggested by Zhou (1986) [7] considered as one of the numerical processes for solving ordinary and the partial differential equations with fast small arithmetic errors and convergence rate. It employed a polynomial model that is sufficiently differentiable in order to the convergence to the exact solution. The technique is based on the Taylor series expansion. The main variance between differential transformation (DT) method and Taylor series method is that the former requires calculations of higher order derivatives that are quite often tremendous and difficult, while the latter includes iterative procedures instead of that. Applying of differential transformation (DT) method in resolving vibration problems in general includes two transformations; they are differential transformation and the inverse differential transformation [9].

The differential transformation of a k^{th}derivative of function $y(x)$is showed as follows:

$$y(k) = \frac{1}{k!}\left[\frac{d^k}{d\xi^k}\, y(\xi)\right]_{\xi=0} \tag{24}$$

And differential inverse transformation for $y(k)$is showed as follows:

$$y(\xi) = \sum_{k=0}^{\infty} y_k(\xi - \xi_o)^k \tag{25}$$

Combining Eq. (24) and (25), gives

$$y(\xi) = \sum_{k=0}^{\infty} \frac{(\xi-\xi_o)^k}{k!}\left[\frac{d^k\, y(\xi)}{d\xi^k}\right]_{\xi=\xi_o} \tag{26}$$

Which is Taylor series for $y(\xi)$ at $\xi = \xi_o$. Equation (26) refers to that the idea of differential transformation (DT) is derived for the expansion of the Taylor series. In practical applications, function $y(\xi)$ is expressed through a finite string and an inverse differential transform is written as follows:

$$y(\xi) = \sum_{k=0}^{n} y_k(\xi - \xi_o)^k \tag{27}$$

Eq. (27) implies that $\sum_{k=n+1}^{\infty} y_k(\xi - \xi_o)^k$ is negligibly small. Theorems that are repeatedly used in the conversion of the equations of motion and boundary conditions and they are listed in Tables 2 and 3, respectively.

$$y(k) = \frac{(k+m)!}{k!} U(k+m) \tag{28}$$

3.1 Formulation of (DT) method

The solution of Eq. (19) may be expressed as [9]:

$$w(\xi, \tau) = y(\xi)e^{\omega\tau} \tag{29}$$

where ω is the vibration frequency, Substituting Eq. (29) into Eq. (19) yields:

$$(\gamma + \mu)\frac{\partial^4 y(\xi)}{\partial \xi^4} + (\alpha U^2)\frac{\partial^2 y(\xi)}{\partial \xi^2} + 2UMr^{0.5}\omega\frac{dy(\xi)}{d\xi} - y(\xi)\omega^2 = 0 \tag{30}$$

Applying the DTM into the vibration equation (30) resulted as:

$$(\gamma + \mu)(k+1)(k+2)(k+3)(k+4)y(k+4) + (\alpha U^2)(k+1)(k+2)y(k+2) + 2UMr^{0.5}\omega(k+1)y(k+1) - y(k)\omega^2 = 0 \tag{31}$$

After simplifying Eq. (31):

$$y(k+4)=\frac{-(\alpha U^2)}{(\gamma+\mu)(k+3)(k+4)}y(k+2)-\frac{2UMr^{0.5}\omega}{(\gamma+\mu)(k+2)(k+3)(k+4)}y(k+1)+\frac{\omega^2}{(\gamma+\mu)(k+1)(k+2)(k+3)(k+4)}y(k) \tag{32}$$

Table 2. Basic theorems of the DTM for the equation of motion.

Original function	Transformed function
$\mathbf{w(\xi)=y(\xi)\pm z(\xi)}$	$W(k)=Y(k)\pm Z(k)$
$\mathbf{w(\xi)=\lambda y(\xi)}$	$W(k)=\lambda Y(k)$
$\mathbf{w(\xi)=\frac{d^n y(\xi)}{dx^n}}$	$W(k)=(k+1)(k+2)\ldots(k+n)Y(k+n)$
$\mathbf{w(\xi)=y(\xi)z(\xi)}$	$W(k)=\sum_{l=0}^{k}Y(l)Z(k-1)$
$\mathbf{w(\xi)=x^m}$	$W(\xi)=\delta(k-m)=\begin{Bmatrix}1, & k=m\\ 2, & k\neq m\end{Bmatrix}$

Table 3. The DTM theorem for boundary conditions.

ξ=0		ξ=1	
Original BC	Transformed BC	Original BC	Transformed BC
$\mathbf{w(0)=0}$	$W(0)=0$	$w(1)=0$	$\sum_{K=0}^{\infty}W(k)=0$
$\mathbf{\frac{dw(0)}{d\xi}=0}$	$W(1)=0$	$\frac{dw(1)}{d\xi}=0$	$\sum_{K=0}^{\infty}kW(k)=0$
$\mathbf{\frac{d^2w(0)}{d\xi^2}=0}$	$W(2)=0$	$\frac{d^2w(1)}{d\xi^2}=0$	$\sum_{K=0}^{\infty}k(k-1)W(k)=0$
$\mathbf{\frac{d^3w(0)}{d\xi^3}=0}$	$W(3)=0$	$\frac{d^3w(1)}{d\xi^3}=0$	$\sum_{K=0}^{\infty}k(k-1)(k-2)W(k)=0$

For the boundary conditions, Equations (21), (22), and (23) can be written in the differential transformation (DT) form as follows:
(1) Simply support:

$$W(0)=W(2)=0 \tag{33}$$

$$\textstyle\sum_{K=0}^{\infty}W(k)=0\,, \tag{34}$$

$$\textstyle\sum_{K=0}^{\infty}k(k-1)W(k)=0\,, \tag{35}$$

(2) clamped-clamped:

$$W(0)=W(1)=0 \tag{36}$$

$$\textstyle\sum_{K=0}^{\infty}W(k)=0\,, \tag{37}$$

$$\textstyle\sum_{K=0}^{\infty}kW(k)=0\,, \tag{38}$$

(3) Cantilevered pipe:

$$W(0)=W(1)=0 \tag{39}$$

$$\sum_{K=0}^{\infty} k(k-1)W(k) = 0 \tag{40}$$

$$\sum_{K=0}^{\infty} k(k-1)(k-2)W(k) = 0, \tag{41}$$

The differential transformation method is summarized in the flow chart shown in Figure 5.

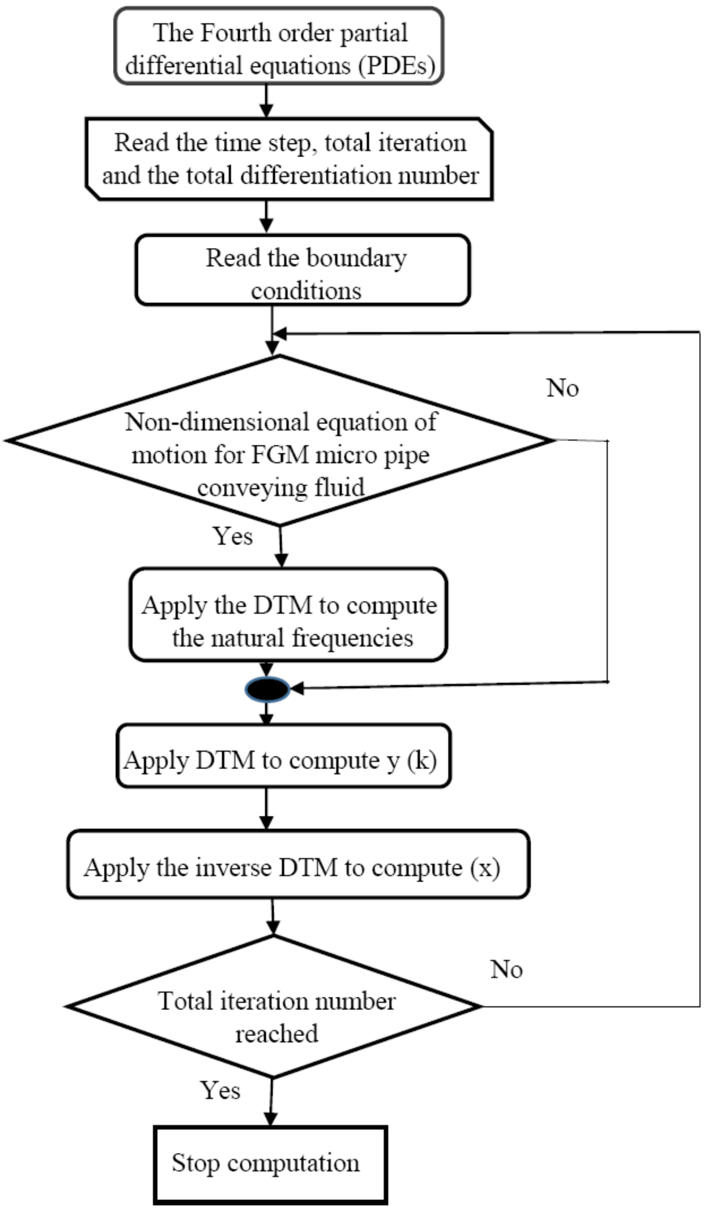


Figure 5. Flow chart for solving the partial differential equation of FGM micro pipe conveying fluid.

Considering Eq. (32) and boundary conditions for obtaining the solutions of the free vibration of fluid-conveying FGM micro-pipe in case of simply support. From Eqs. (32)-(35). W(2) and W(4) are anonymous parameters and known as $W(2) = c_1$,$W(4) = c_2$. With Eq. (30), W (k) can be computed by a repeated stride. After replacing W(k) into Eqs. (34) and (35), these equations can be written as in the following matrix:

$$\begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = 0 \tag{42}$$

where w_{ij} are connected with eigenvalues and another parameters of the system. To obtain the solution of Eq. (42), it is wanted that a determinant of the matrix equal to zero, namely

$$\begin{vmatrix} w_{11}^{(N)} & w_{12}^{(N)} \\ w_{21}^{(N)} & w_{22}^{(N)} \end{vmatrix} = 0 \tag{43}$$

Thus, an eigenvalue (w) can be calculated numerically from Eq. (43). Mostly, where w is complex number $w = a \pm ib$. The real part of w refers to the vibration frequency of system, and an imaginary part of w refers to the damping [28]. The accuracy of (DT) method counts on a number of terms N. The results of the (DT) method be nigh to exact solutions with an increase of N, and N is determined by

$$\left|w_i^N - w_i^{N-1}\right| < \delta \tag{44}$$

In the present study, the value of $\delta = 0.00001$ and it's showing the precision of calculations. With respect to a differential transformation method and the algorithm above, the MATLAB code has been developed in order to find the vibration characteristics of FGM micro-pipe conveying fluid.

4. Result and discussions

The parameters of geometrical for FGM micro-pipe were used as those utilized by Deng et al [27] as Di/Do=0.9, Do=20 µm, length l of FGM micro-pipe is assumed 17.6 µm, the density of fluid is $\rho_f = 1000\ kg/m^3$ and the mechanical properties for a constituent materials are shown in Table 1.

MATLAB package was developed to investigate the influence of different parameters on the free vibration of fluid-conveying FGM micro-pipe. The natural frequencies for flow conveying FGM micro-pipes are dependent on a fluid velocity (u). It should be referred to that the pipe will act as a straight beam when u = 0, and natural frequencies from which can be acquired analytically.

Table 4 presents the first four dimensionless vibration frequency with flow velocity u=0 and the graded material p=0, which represents a homogenous pipe conveying fluid. As presented in this table, the results of DTM become closer to the results of VIM solution reported in [6]. In addition, good agreement is found.

Table 4. Comparison of dimensionless natural frequency for homogenous pipe with different end condition for (u=0).

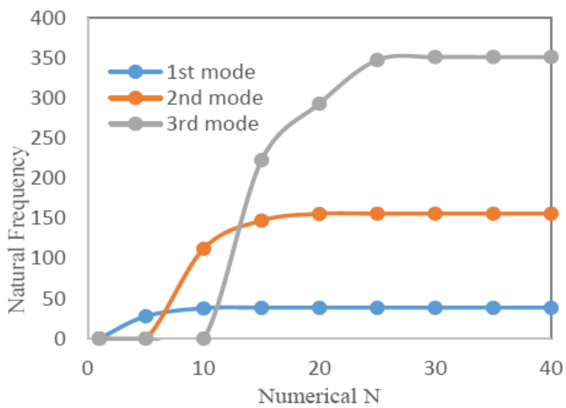
Type of support	Method	w_1	w_2	w_3	w_4
Pinned – pinned	DTM	9.8696	39.4784	88.8264	157.9099
	VIM [6]	9.8696	39.4784	88.8264	157.9137
Clamped - clamped	DTM	22.3733	61.6728	120.9034	199.8050
	VIM [6]	22.3733	61.6728	120.9034	199.8594
Clamped - free	DTM	3.5160	22.0345	61.6972	120.9018
	VIM [6]	3.5160	22.0345	61.6971	120.9019

Figures 6a, 6b, and 6c show the fundamental first three modes of fluid-conveying FGM micro-pipe with three end conditions as a function of the number of terms (N) with p=1 and u=2. The precision of DTM increases with increasing of N as shown. The number of terms N, for the DTM, was selected to be N = 35 in all tables and figures appeared in this paper. The first, second and third modes for simply support are ($w_1 = 38.6069, w_2 = 155.954, w_3 = 351.528$), for clamped-clamped are ($w_1 = 88.3909, w_2 = 244.042, w_3 = 478.746$), and for clamped-free are ($w_1 = 14.0585, w_2 = 87.0209,\ w_3 = 244.14$).

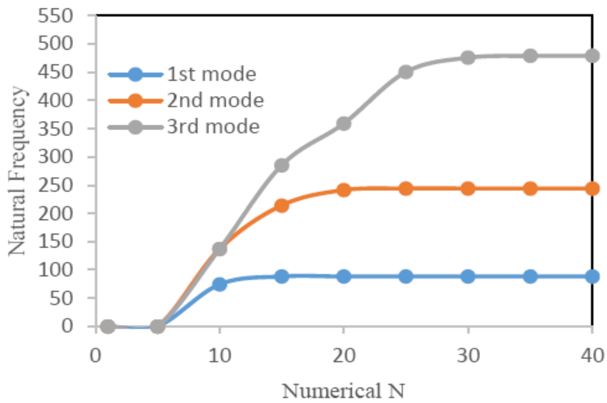
Table 5 shows the effect of volume fraction exponent (p) and flow velocity on the first natural frequency of FGM micro-pipe for different end conditions. This result indicates that the increase for the fluid velocity leads to a decrease in natural frequency values for each boundary conditions. While vibration frequency of FGM micro-pipe will increase with an increase in the exponent of volume fraction p. This is generally due to the fact that alumina content in FGM micro-pipe increases, whilst the aluminum content decrease with an increasing an exponent, and the alumina Young's modulus is frequently greater than that from aluminum.

The real components of the fundamental natural frequency for a simply supported, clamped-clamped and cantilever FGM micro-pipe conveying fluid with dimensionless fluid velocity at the various value of volume fraction exponent (p=0, 1, 5, 10, and 100) are presented in Figures 7a, 7b, and 7c, respectively. It is noted that the real part of vibration frequency and the critical of flow velocity will increase with an increase in a volume fraction index p. When p=10 the critical velocity and natural frequency are greater than that for p=0 and p=1 for three end conditions.

The dimensionless real components of eigenvalues for a simply supported, clamped-clamped and cantilever FGM pipe with dimensionless flow velocity for different dimensionless parameters of length scale Do/l are shown in Figures 8a, 8b, and 8c respectively. The results are presented for different Do/l, Do/l = 1, 2, 6, and 10. For numerical calculations, in this case, p=1. It can be noted that through increasing Do/l, each of the bending stiffness of the FGM pipe, the real eigenvalue and the critical flow velocity decreases.



(a)



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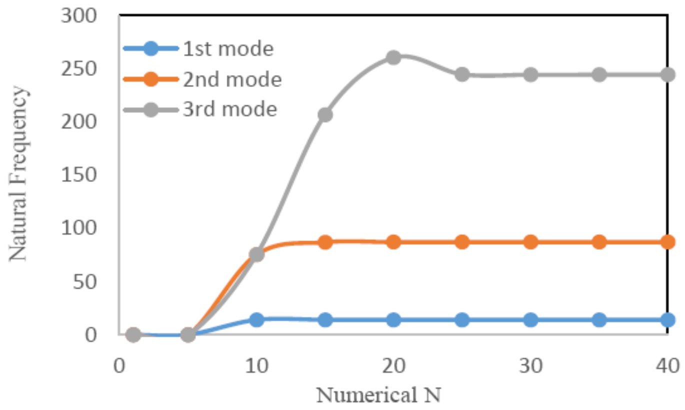


(c)

Figure 6. Natural frequency versus N for first, second and third modes with p=1 and u=2 m/s for (a) simply support, (b) clamped-clamped, and (c) clamped-free.

Table 5. The first natural frequency with various value of velocity and fraction index n.

B.C	u	Volume fraction index (n)				
		p=0	p=1	p=3	p=10	p=100
Simply - Simply	0	21.8172	39.1148	45.3522	48.9098	50.6266
	2	20.8929	38.6069	44.9148	48.5045	50.2352
	4	17.8347	37.0413	43.5764	47.2679	49.0422
	6	10.9856	34.2733	41.2494	45.1316	46.9866
Clamped-clamped	0	49.4571	88.6689	102.8080	110.8730	114.7650
	2	48.9568	88.3909	102.5690	110.6510	114.5500
	4	47.4211	87.5512	101.8460	109.9810	113.9040
	6	44.7329	86.1316	100.6290	108.8560	112.8170
Clamped -free	0	7.56029	13.5544	15.7158	16.9486	17.5436
	2	6.97542	13.2608	15.4736	16.7301	17.3353
	4	4.96165	12.3525	14.7310	16.0625	16.7001
	6	0	10.7256	13.4325	14.9053	15.6027

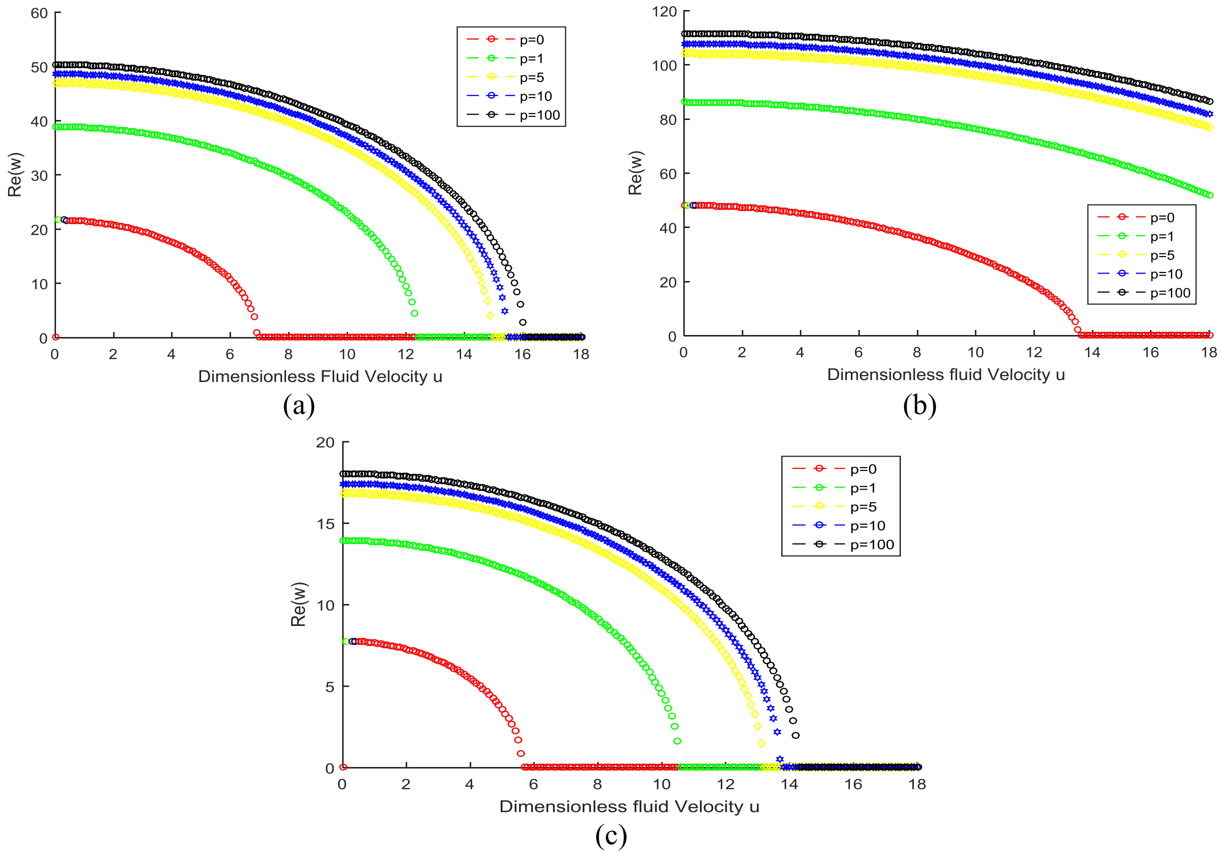


Figure 7. The natural frequency with velocity for various value exponent of volume fraction p: (a) for simply support, (b) for clamped-clamped and (c) for clamped free support.

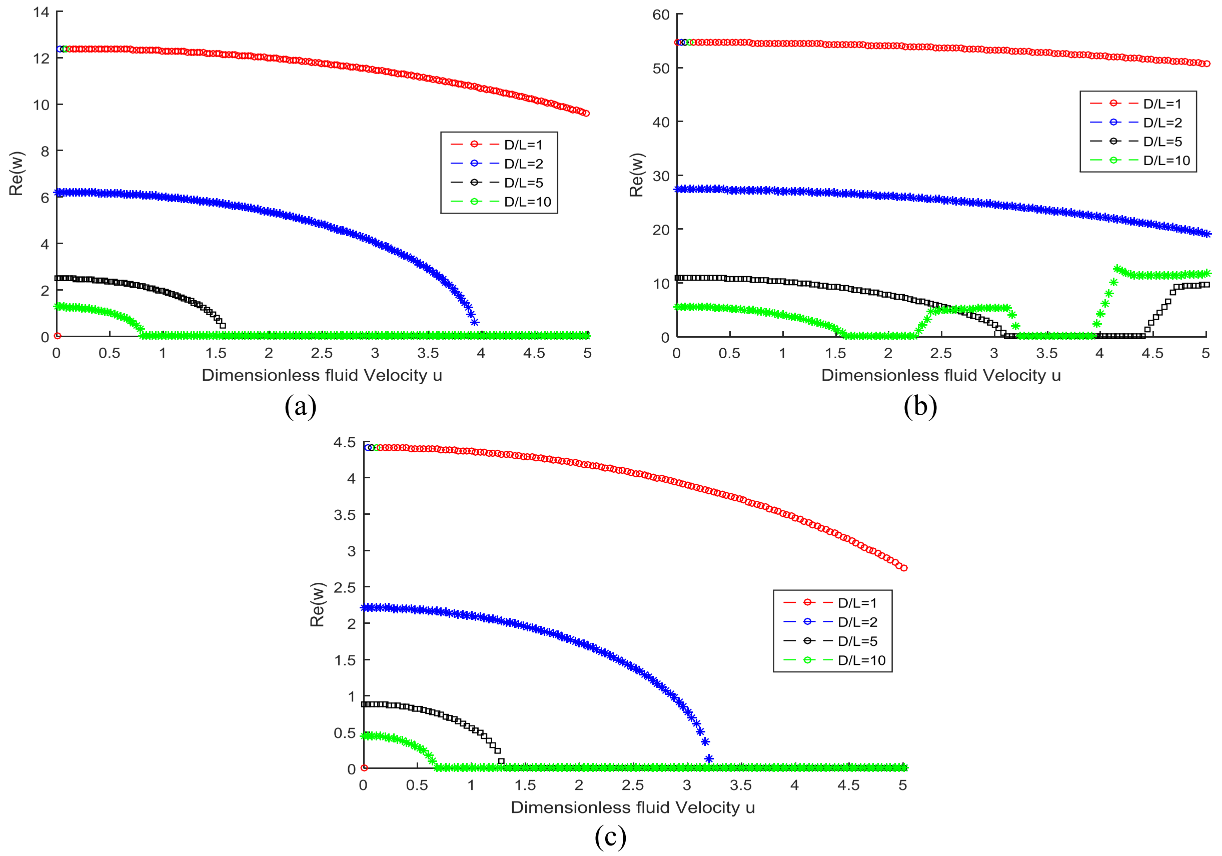


Figure 8. Natural frequency with velocity for different value of length scale parameter (D/L): (a) simply support (b) clamped-clamped (c) clamped –free.

It is concluded that the influence of length scale for material on a free vibration of FGM micro-pipe conveying fluid is very important, in addition, it makes a micropipe more stable, mostly when a diameter of micro-pipe is comparable to a parameter of length scale (Do/L) of material. This is due to that a parameter of length scale has an influence on increasing an equivalent bending hardness $[(EI_{eq}) + (GA_{eq}L^2)]$.

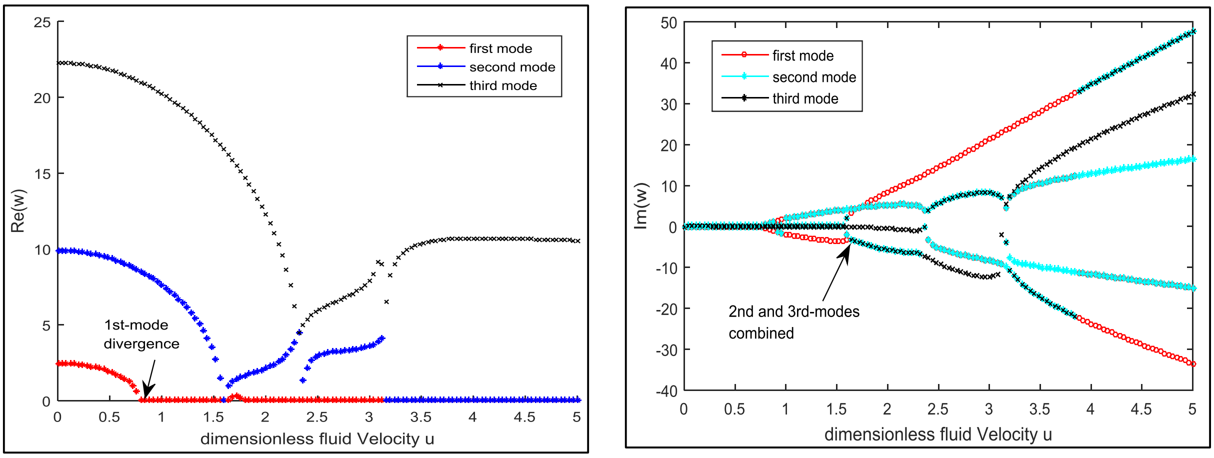
Figures 9a, 9b, and 9c show the effect of the exponent of volume fraction on free vibration of (FG) micro-pipes conveying fluid for the first three fundamental frequencies of pinned –pinned FGM micro-pipe and (Do/l = 10) as a function of dimensionless fluid velocity u. The results are presented for different p, p= 0, 1, and 10. From Figure 9a it can be seen that the FGM micro-pipe for simply support displays some more interesting and complex dynamic behaviors when an exponent p = 0, the divergence of the first mode happens at u =0.8, the divergence of a second mode happens at u =1.6, and the divergence happens in the third mode at u = 2.36. therefore, it is of important noted that when a fluid flow velocity u increases to 3.2, the combination of 2nd and 3rd modes appears, a real part is positive, and an imaginary part is negative. This means the simply support FGM micro-pipe loses the stability by a combined modes (flutter), and the conformable velocity is a flutter velocity.

Figures 10a, 10b, and 10c depicted the effect of volume fraction index on the real and imaginary eigenvalue for the first three modes of clamped FGM micropipe versus dimensionless flow velocity. When p=0, the divergence of the first mode happens at u =1.62, while the divergence of a second mode happens at u =2.24, and the divergence happens in the third mode at u = 3.34. Thereafter, it is of important noted that when fluid flow velocity (u) increase to 3.41, the combination form 2nd and 3rd modes happened as shown in Figure 10a. While, when p=1, a divergence of first mode is u=2.82, and, when p=10 the divergence of the first mode happens at u=3.56, the divergence of 2nd and 3rd modes does not happen for the range of fluid flow velocity u < 4 for p=10 as shown in Figures 10b and 10c respectively. Therefore, the stability for FGM micro-pipe increases with an increase in an exponent of the volume fraction. It is furthermore found that real parts and critical velocities will increase with an increase in the exponent of volume fraction. For the case of clamped-free support shown in Figures 11a, 11b, and 11c respectively, the divergence of a first mode happens at u =5.6 when p=0 while divergence of a second mode happened at u =11.8, it is of important noted that as a fluid flow velocity (u) increases to 12.7, the combination form 2nd and 3rd modes happens as shown in Figure 11a. And when p=1 the first mode divergence occurs at u=10.3 while when p=10 the first mode divergence at u=13.3 Figure (10c), doesn't occur divergence in a second and third mode in range u < 16 as shown in Figures 11b, and 11c respectively.

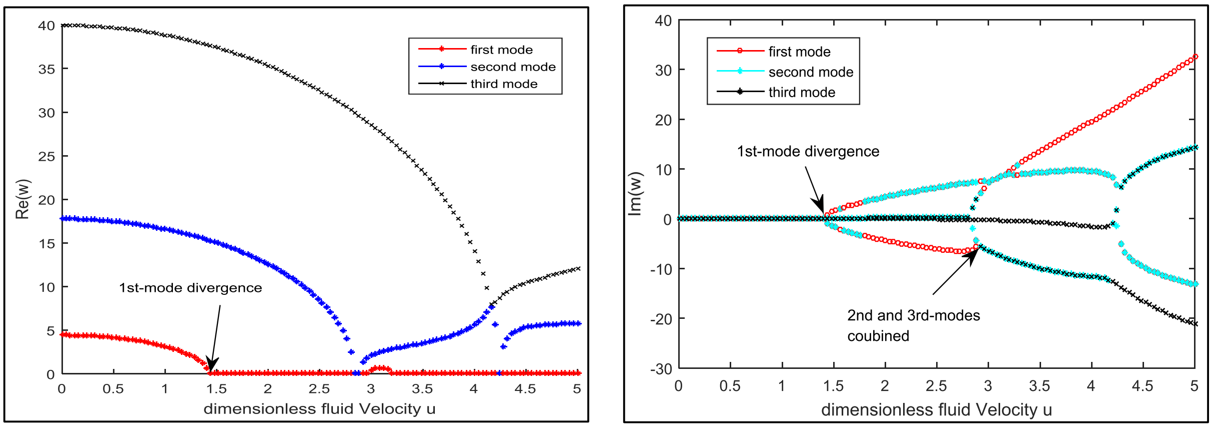
5. Conclusion

Free vibration and stability for a functionally graded (FG) material micro-pipe conveying fluid are investigated in this paper. Equations of motion are acquired by stratifying a modified couple stress (MCS) theory and the Hamilton's principle. Thereafter, the differential transformation (DT) method is progressed to find a complex eigenvalue. Some main inferences acquired from the results above are offered as follows:

1. The increase in a volume fraction exponent (p) leads to increase rapidly in the natural frequencies and can be modified by the distribution of natural frequencies readily by designing of the exponent of volume fraction (p).
2. The real natural frequency decreases with an increase in the parameter of length scale, a size effect is very important when the outer diameter comparable into the parameter of length scale for FGM micropipe and it makes a FGM micro pipe conveying the fluid more stable.
3. The critical velocities increase with an increase in an exponent of volume fraction (p), it can be concluded that a stability of FG micro-pipe increases with an increase in an exponent of volume fraction (p).
4. The present work has demonstrated that the differential transformation method has a computational efficiency and high degree of accuracy in vibration analysis for FGM micro pipes with flowing fluid.



(a)



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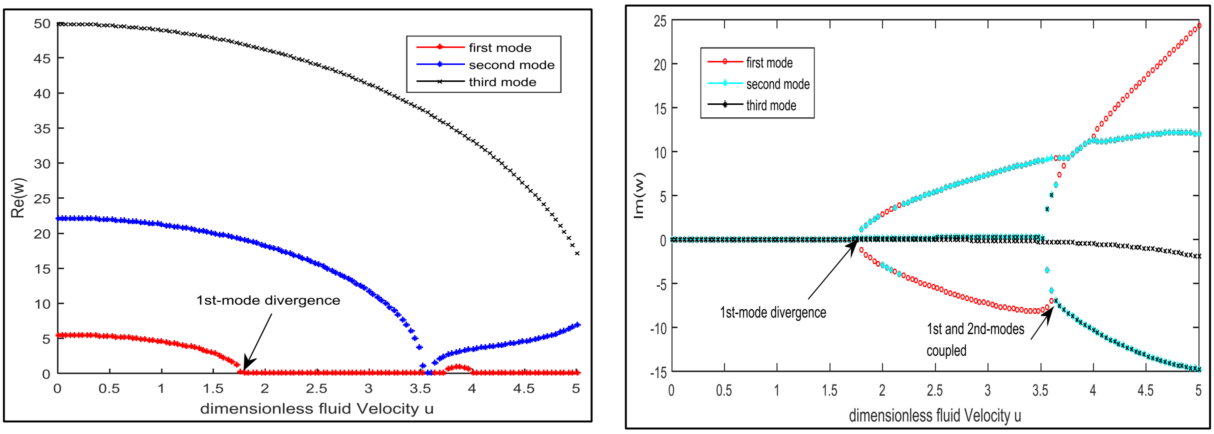
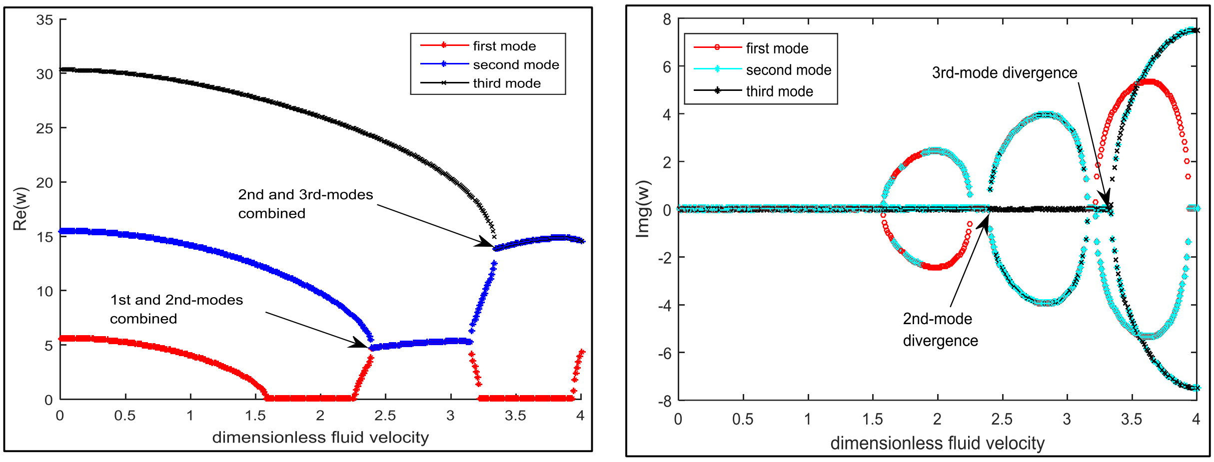
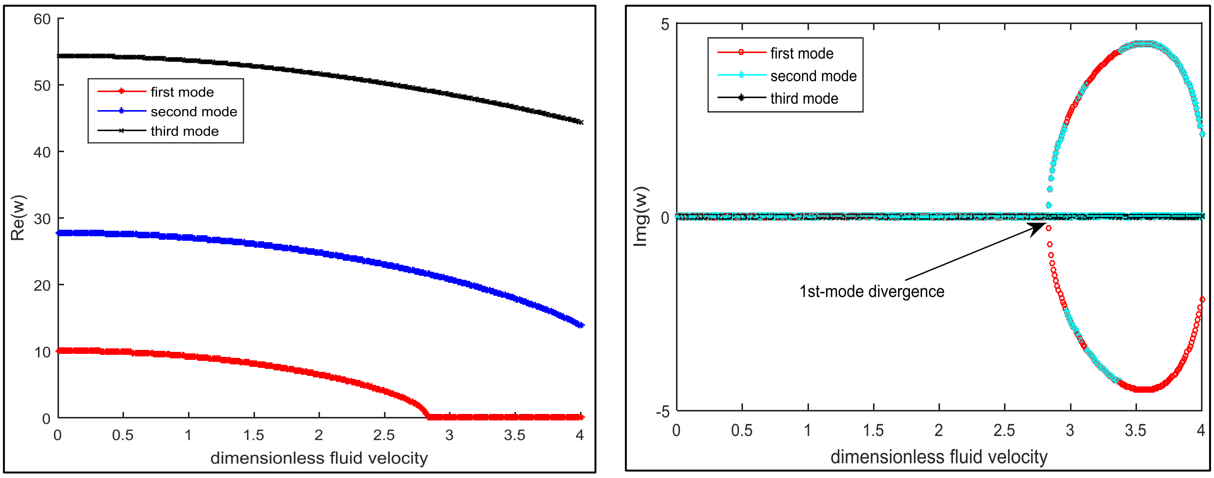


(c)

Figure 9. First three natural frquencies of FGM micro pipe with dimensionless fluid vilocity for simply support at: (a) p=0, (b) p=1 and (c) p=10.



(a)



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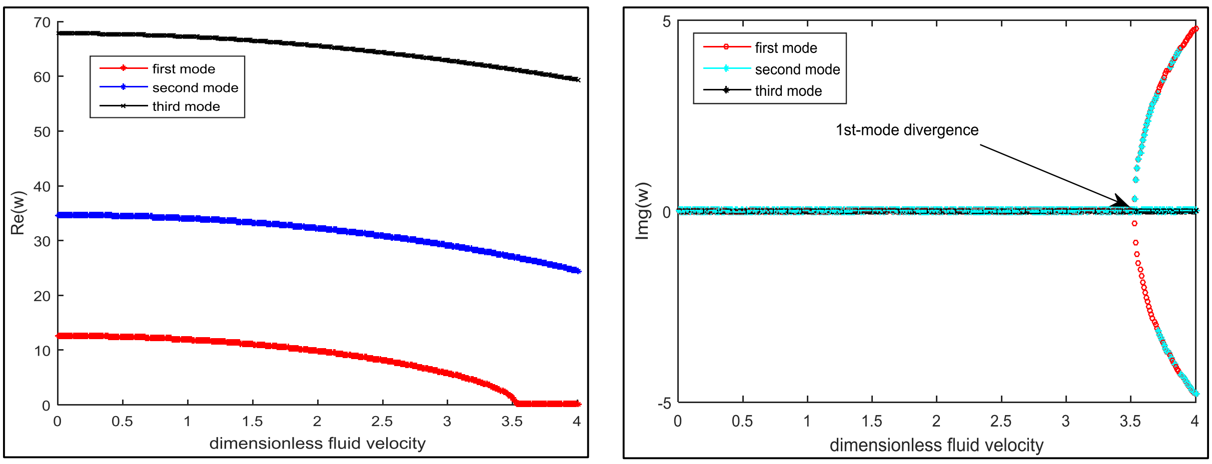
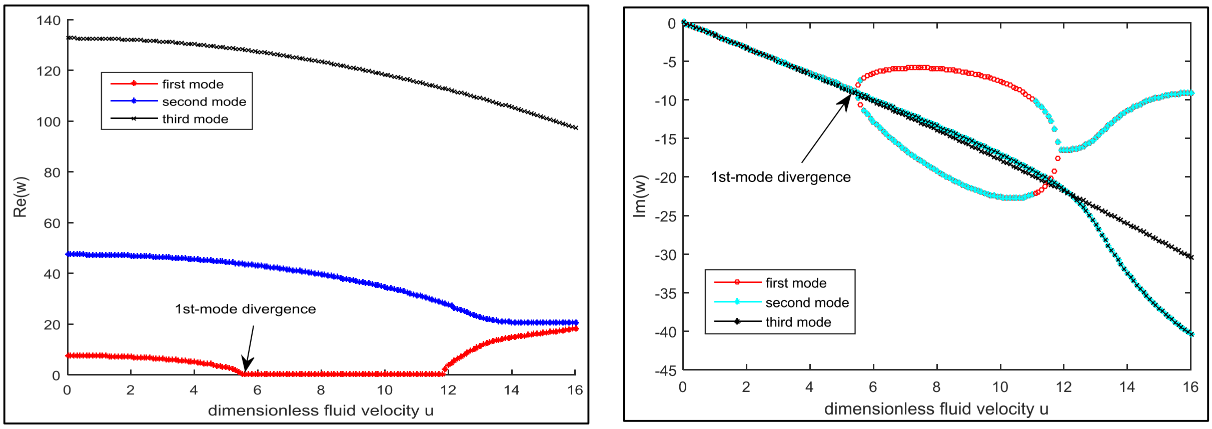
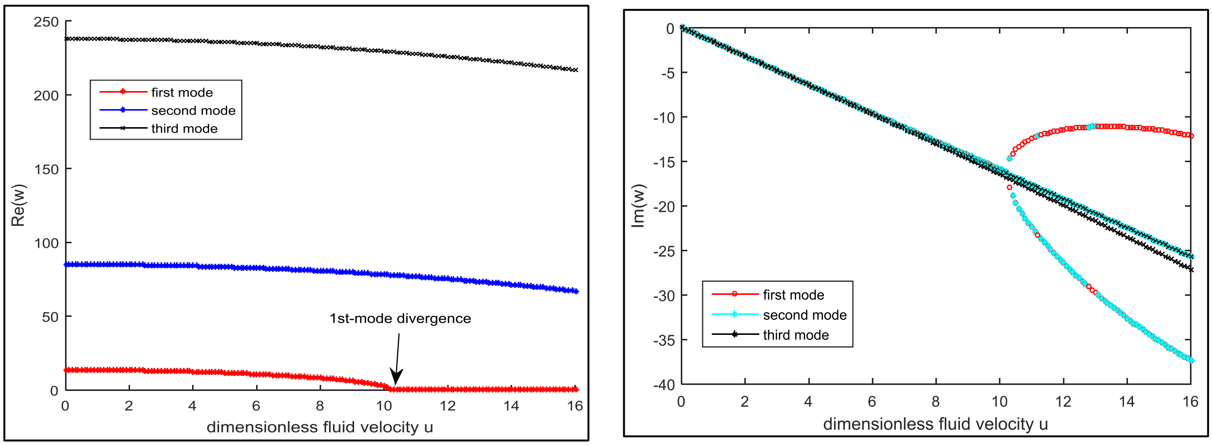


(c)

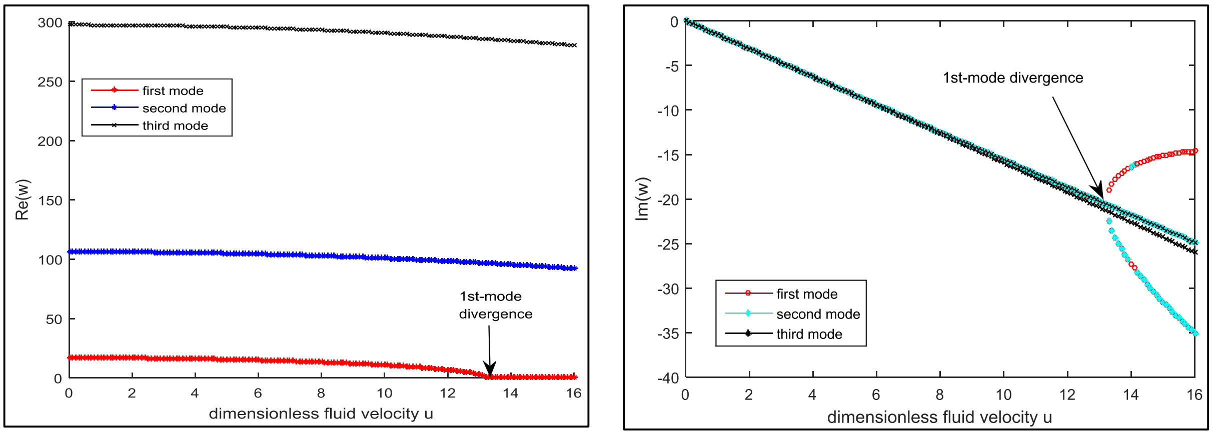
Figure 10. First three natural frquencies of FGM micro pipe with dimensionless fluid vilocity for clamped-clamped support at: (a) p=0, (b) p=1 and (c) p=10.



(a)



(b)



(c)

Figure 11. First three natural frquencies of FGM micro pipe with dimensionless fluid vilocity for clamped-free support at: a) p=0, b) p=1 and c) p=1.

List of symbols

E	Young's modulus	z	the distance to the mid-plane of the FG micro pipe
ρ_p	the density of FGM micro pipe	V_c	volume fraction at inner
ρ_f	the density of fluid	V_m	volume fraction at outer
ν	Poison ratio	m_f	The mass of the fluid
Ri	the inner radii of FGM micro pipe	m_p	The mass of the micro pipe
Ro	The outer radii of FGM micro pipe	A_f	Flow cross sectional area
h	the thickness direction	T	Kinetic energy
L	Length of micro pipe	α	the effect of size for micro-flow
u	fluid velocity	U	Strain energy
p	volume fraction exponent	Mr	the mass ratio
G	the modulus of rigidity		

Appendix

$$\delta \int_{t1}^{t2} \left[\frac{1}{2}\int_0^L m_p \left(\frac{\partial w}{\partial t}\right)^2 dx + \frac{1}{2}\int_0^L m_f \left(\frac{\partial w}{\partial t} + u_f \frac{\partial w}{\partial x}\right)^2 dx - \left[\frac{1}{2}\int_0^L EI_{eq} \left(\frac{\partial^2 w}{\partial x^2}\right)^2 dx + \frac{1}{2}\int_0^L GA_{eq} l^2 \left(\frac{\partial^2 w}{\partial x^2}\right)^2 dx \right] \right] dt = 0 \tag{A1}$$

$$m_p \int_0^L \int_{t1}^{t2} \left(\frac{\partial w}{\partial t}\right) \delta \left(\frac{\partial w}{\partial t}\right) dx\, dt + \frac{1}{2} m_f \int_{t1}^{t2} \int_0^L \left[2\left(\frac{\partial w}{\partial t}\right) \delta \left(\frac{\partial w}{\partial t}\right) + 2u_f \left(\left(\frac{\partial w}{\partial x}\right) \delta \left(\frac{\partial w}{\partial t}\right) + \left(\frac{\partial w}{\partial t}\right) \delta \left(\frac{\partial w}{\partial x}\right) \right) + 2u_f^2 \left(\frac{\partial w}{\partial x}\right) \delta \left(\frac{\partial w}{\partial t}\right) \right] dx\, dt + \frac{1}{2}\int_{t1}^{t2} \int_0^L \left(EI_{eq} + GA_{eq} l^2\right) \left(\frac{\partial^2 w}{\partial x^2}\right) \delta \left(\frac{\partial^2 w}{\partial x^2}\right) dx\, dt = 0 \tag{A2}$$

$$\because\ \delta w_{t1,t2} = 0 \tag{A3}$$

$$-m_p \int_{t1}^{t2} \int_0^L \frac{\partial^2 w}{\partial t^2}\, \delta w\, dx\, dt - m_f \int_{t1}^{t2} \int_0^L \frac{\partial^2 w}{\partial t^2}\, \delta w\, dx\, dt - m_f u_f \int_{t1}^{t2} \int_0^L \frac{\partial^2 w}{\partial x \partial t}\, \delta w\, dx\, dt - m_f u_f^2 \int_{t1}^{t2} \int_0^L \frac{\partial^2 w}{\partial x^2}\, \delta w\, dx\, dt - \int_{t1}^{t2} \int_0^L \left(EI_{eq} + GA_{eq} l^2\right) \frac{\partial^4 w}{\partial x^4}\, \delta w\, dx\, dt = 0 \tag{A4}$$

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