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Analytical heat generation investigation for forced vibration beam with different crack characterizations influence

Diyaa H. J. Al-Zubaidi[1], Muhannad Al-Waily[2], Emad Q. Hussein[1]

[1] University of Kerbala, Faculty of Engineering, Mechanical Engineering Department, Iraq.
[2] University of Kufa, Faculty of Engineering, Mechanical Engineering Department, Iraq.

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Abstract
This paper investigates the effect of the depth and location of the crack on the heat generated in the beam, due to different variable load time with frequency values, in case of periodic load. The beam used made of carbon steel (1.5% C) which has main specifications of density (7750 kg/m3) and modulus of elasticity 200 Gpa. Two techniques are used, first, Theoretical technique is used in both analytical and numerical analysis. The analytical technique using the derived equation of motion for the beam and the crack model is included. Matlab (R 2017), computer program has been used to solve the derived equations to predict the amount of heat generated, and second, the numerical technique is employed using the finite element method adopting COMSOL program to verify the analytical results, it also gives a perception of the difference in temperature due to heat generated. In the first time, the heat generated was calculated as a result of force vibration at different locations and boundary conditions of the beam. It was observed that the heat generated as the frequency supplied is approaching to the natural frequency, and the generated heat is reduced as the frequency supplied goes away the natural frequency significantly. It has been observed that the effect of the crack for different types of supported beams affects the heat generation where the greater depth of the crack has increased the heat generation of the beam is increasing, as well as, the approaching the position of the crack than the moment higher of the beam, the heat generation of the beam is increased. In addition, analytical the results evaluated were comparison by numerical results evaluated by using COMSOL program, and found that the maximum percentage error is about (6.2%).

Keywords: Heat induce vibration; Heat generation crack structure; Crack beam vibration; Harmonic vibration; CFD heat vibration; Finite element vibration heat generation.

1. Introduction
The heat generation in the structure subjected to the high frequency due to mechanical viscoelastic losses in the materials. The thermo-elastic damping "represent between mechanical and thermal energy of the energy conversion" occur due to the slow temperature rise in a vibration structure. Due to internal damping of the materials, the loss of energy occur when s structure vibration under frequency effect. And, when the structure subjecting to high frequency and vibration under the high frequency, generation the heat and high displacement due to internal damping characteristic of materials. When a structure is exposed to vibrations of high frequency, an important amount of heat will be generated within the structure because of mechanical losses in the material due to the viscoelastic effects. A second mechanism contributing to the

slow temperature increase in a vibrating structure is called thermo-elastic damping, and represents the energy conversion between mechanical and thermal energy.

Vibration damping in solids is the phenomena whereby vibrational energy is converted to heat due to internal mechanisms within the solid. These mechanisms include internal friction between individual atoms, friction between different phases or precipitates, the movement of dislocations within a material, and others. Every material has different damping characteristics due to its unique structure.

The effect of crack on the mechanical behavior presented from many researchers with various parameters effect and different techniques used. Where, the Muhannad Al-Waily et. al, from 2012 to 2018, presented multi papers to shown the effect of crack on multi mechanical behaviors for various structure, as, at 2012, [1-3], and 2017, [4], investigated the effect of crack depth and location on the vibration behavior of different composite plate structure, by using various techniques. Then, also, at 2012, [5, 6], presented the effect of crack orientation angle on the vibration behavior for plate. Then, at 2012, [7], and 2013, [8, 9], investigation the vibration behavior of beam structure with effect of crack depth and location effect with theoretical and experimental techniques. After this, at 2015, [10], investigation the vibration of beam with various crack angle effect. Then, at 2016, [11], investigated the effect of crack on the buckling behavior of beam. Finally, at, 2017, [12], presented the effect of crack angle on the vibration behavior of pipe structure conveying fluid by using experimental and numerical techniques. in addition, other researches were investigation other effect of crack damage on the plate and beam structure with different parameters effect and various techniques for solution, [13, 14].

Therefore, since the crack damage is effect on the vibration behavior for structure, and since the heat generation dependent on the natural frequency and vibration behavior of structure, then, the crack damage is effect on the heat generation of beam. Then, the thermal induce vibration of structure are studied from many researchers with different technique as shown in same present review below,

In 1980, S. Pantellou et. al, [15], presented the analysis of heat generation for rotation shaft from materials damping, assuming both lumped mass and continuous system are considered. And, the forced torsional vibration is assumed. The elastic deformation rang and an elastoplastic materials, used hysteretic model for materials damping to yield the heat generation, assumed in the plastic range. From the heat conduction solved for cooling surface cylindrical shaft and the maximum temperatures and the maximum surface temperature obtained, shown that the develop of substantial temperature in shafts undergoing torsional vibration. At 1997, Michael I. F. et. al, [16], studied the active damping of thermal vibration smart structure, the simply supported baem made of aluminum materials is presented to studied the effect of thermal induce vibration beam. The control system design studied with effect of large temperature changes. The results of paper shown that the smart structure can be used to damping the vibration with thermal effect. In 2013, Jadwiga K. K., [17], presented the analysis of vibration cantilever beam with thermal effect. The periodically time varying heat source acts onto surface of beam are assumed in the paper. The variables of thermal moment effect are add to the Bernoulli-Euler equation to vibration beam. And, Green's function method used to solving the heat and vibration problem of beam. After this, at 2014, I. Jafarsadeghi P. et. Al, [18], studied the dynamic behaviors for cantilever beam made of graded materials and subjected to the harmonic thermal load. The metal and ceramic materials are used to product the graded materials of the beam used in this paper. The equation of coupled energy and equation of motion are derived in addition to Euler-Bernoulli beam theory is used. Also, the numerical method presented in the paper. Also, in 2014, Wenbo Zhang et. al, [19], presented the investigation of thermal effect on high frequency of beam vibration, and developed the energy flow analysis to predict frequency response beam. The methods used in the paper are EFA model and numerical simulations to study the effect of temperature effect onto thermal stress and materials properties changed. Finally, at 2018, Diyaa H.J. Al-Zubaidi, [20], investigated the heat generation for beam structure with various supported sue to vibration behavior for its beam. Where, the investigation included evaluated the effect for applied load frequency on the heat generation for beam by using analytical and numerical techniques, and then, comparison the results evaluate together.

In addition to other researchers studied the problem with different way to evaluating the heat generation and temperature distribution, but, not presented the analytical solution of problem as in this work. Where, the paper presented analytical solution evaluating the heat generation as a function of time and dependent onto natural frequency subjected onto beam and with different boundary condition of beam (cantilever beam, simply supported beam, and clamped beam) and various beam vibration modes. In addition, comparison the analytical results are evaluated with numerical results production by finite element method, with using CFD program. In addition, the investigation included evaluate the effect of crack depth and position on the heat generation for beam with various applied load frequency.

2. Analytical investigation

The analytical investigation is techniques used to give exact solution of problem with various parameters effect, [21-27], in addition to, the analytical solution is given agreement results comparison with other technique used, [28-35]. Thus, the analytical part included two section, first evaluated of natural frequency of beam, and then, in the section part derived the equation determine the generation heat due to supplied frequency onto beam and evaluated the equation of temperature as a function of time and temperature as a function of length for beam (distribution of temperature through length of beam).

To evaluated the natural frequency of beam, solution of general equation of motion, [36-42], of free vibration uniform cross section and constant modulus beam, as, [43],

$$EI\frac{\partial^4 w(x,t)}{\partial x^4}+\rho A\frac{\partial^2 w(x,t)}{\partial t^2}=0 \tag{1}$$

Where, E, I, ρ, A are modulus of elasticity, moment of inertia, density, and cross section area of beam, respectively, x is beam direction through length of beam, t is time, and w is the deflection of beam through lateral direction of beam. Then, by using separation of variable as,

$$W(x,t)=W_n(x)\times W_n(t)$$

$$\omega_n=(\beta l)^2\sqrt{\frac{EI}{\rho A l^4}} \tag{2}$$

Where, l is the length of beam and n is the number of mode of beam. Then, by using Duhamel integral, for zero initial conditions (displacement and velocity), and substation the solution into Eq. 2, get, [43],

$$W(x,t)=W_n(x)\left(\frac{1}{\rho A b\omega_n}\int_0^t Q_n(\tau)\sin\omega_n(t-\tau)\ d\tau\right) \tag{3}$$

Then, for beam with crack effect, By using Euler-Bernoulli beam theory, the equation of motion for beam, assumed to have uniform cross section, is given by [44],

$$EI\frac{\partial^4 W(x.t)}{\partial x^4}+\rho A\frac{\partial^2 W(x.t)}{\partial t^2}=f(x,t) \tag{4}$$

By letting the forcing term, $f(x,t)$ to be zero. Then, using the separable solutions $W(x,t)=W(x)e^{j\omega t}$ in equation (4) leads to an associated eigenvalue problem, [45],

$$\frac{\partial^4 W(x)}{\partial x^4}-\lambda^4 W(x)=0 \tag{5}$$

Where, $\lambda^4=\frac{\rho A\omega^2}{EI}$ and ω_n natural frequency of beam.

The general solution of equation (5), for each segment, [45],

$$for\ x=0\longrightarrow l_1$$
$$W_1(x)=A_1\sin(\lambda x)+B_1\cos(\lambda x)+C_1\sinh(\lambda x)+D_1\cosh(\lambda x)$$

$$for\ x=l_1\longrightarrow l$$
$$W_2(x)=A_2\sin\big(\lambda(x-l_1)\big)+B_2\cos\big(\lambda(x-l_1)\big)+C_2\sinh\big(\lambda(x-l_1)\big)+D_2\cosh\big(\lambda(x-l_1)\big) \tag{6}$$

Now, applying the boundary conditions for three cases to evaluate the general equation for heat generation for harmonic vibration beam structure with crack effect.

2.1 General mode response

I. Cantilever beam

The equations for cantilever beam will be derived in the case of a crack, as shown in Figure 1, by subject boundary conditions of beam, as, [45],

$$W_1(0)=0,\ \bar{W}_1(0)=0,\ \bar{\bar{W}}_2(l)=0,\ \bar{\bar{\bar{W}}}_2(l)=0 \tag{7}$$

Applied boundary conditions, Eq. 7, on deflection beam equation, Eq. 6, get,

$$W_1(x) = A_1(sin(\lambda x) - sinh(\lambda x)) + B_1(cos(\lambda x) - cosh(\lambda x))$$
$$W_2(x) = \begin{pmatrix} A_2\big(sin\big(\lambda(x-l_1)\big) + \alpha_1 \, sinh\big(\lambda(x-l_1)\big) + \alpha_2 \, cosh\big(\lambda(x-l_1)\big)\big) + \\ B_2\big(cos\big(\lambda(x-l_1)\big) + \alpha_3 \, sinh\big(\lambda(x-l_1)\big) + \alpha_4 \, cosh\big(\lambda(x-l_1)\big)\big) \end{pmatrix} \quad (8)$$

Where,
$\alpha_1 = cos(\lambda l_2).cosh(\lambda l_2) - sin(\lambda l_2).sinh(\lambda l_2)$, $\alpha_2 = sin(\lambda l_2).cosh(\lambda l_2) - cos(\lambda l_2).sinh(\lambda l_2)$
$\alpha_3 = -cos(\lambda l_2).sinh(\lambda l_2) - sin(\lambda l_2).cosh(\lambda l_2)$, $\alpha_4 = cos(\lambda l_2).cosh(\lambda l_2) + sin(\lambda l_2).sinh(\lambda l_2)$

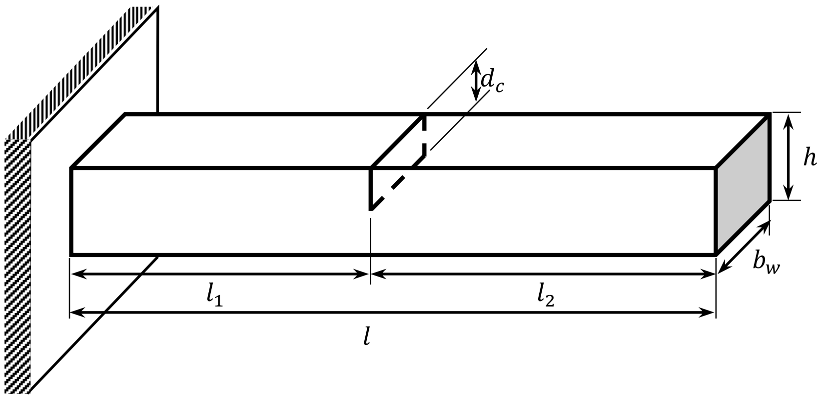


Figure 1. Cantilever beam with crack.

Then, for evaluating other constant in Eq. 8, using boundary condition at crack location, as, [45],

$$W_1(l_1) = W_2(l_1),\ \overline{\overline{W}}_1(l_1) = \overline{\overline{W}}_2(l_1),\ \overline{\overline{\overline{W}}}_1(l_1) = \overline{\overline{\overline{W}}}_2(l_1),\ \overline{W}_2(l_1) - \overline{W}_1(l_1) = \theta l \overline{\overline{W}}_2(l_1) \quad (9)$$

For, $\theta = 6\pi \begin{pmatrix} 0.6384\left(\frac{d_c}{h}\right)^2 - 1.035\left(\frac{d_c}{h}\right)^3 + 3.7201\left(\frac{d_c}{h}\right)^4 - 5.1773\left(\frac{d_c}{h}\right)^5 + \\ 7.553\left(\frac{d_c}{h}\right)^6 - 7.332\left(\frac{d_c}{h}\right)^7 + 2.4909\left(\frac{d_c}{h}\right)^8 \end{pmatrix}\left(\frac{h}{l}\right)$

There, substation boundary conditions in Eq. 9 into equation 8, get,

$$W_1(x) = A_1\big(sin(\lambda x) - sinh(\lambda x) + \alpha_7(cos(\lambda x) - cosh(\lambda x))\big)$$
$$W_2(x) = A_1\big(\alpha_8 \, sin\big(\lambda(x-l_1)\big) + \alpha_9 \, cos\big(\lambda(x-l_1)\big) + \alpha_{10} \, sinh\big(\lambda(x-l_1)\big) + \alpha_{11} \, cosh\big(\lambda(x-l_1)\big)\big) \quad (10)$$

Where,
$\alpha_5 = -\left(\frac{\alpha_4}{\alpha_2} sin(\lambda l_1) + \frac{1}{\alpha_2} sinh(\lambda l_1)\right)$, $\alpha_6 = -\left(\frac{\alpha_4}{\alpha_2} cos(\lambda l_1) + \frac{1}{\alpha_2} cosh(\lambda l_1)\right)$
$\alpha_7 = \frac{(cos(\lambda l_1) + cosh(\lambda l_1) - \alpha_5 + \alpha_5\alpha_1 + \alpha_3 \, sin(\lambda l_1))}{(sin(\lambda l_1) - sinh(\lambda l_1) + \alpha_6 - \alpha_6\alpha_1 - \alpha_3 \, cos(\lambda l_1))}$, $\alpha_8 = (\alpha_5 + \alpha_6\alpha_7)$, $\alpha_9 = (sin(\lambda l_1) + \alpha_7 \, cos(\lambda l_1))$
$\alpha_{10} = (\alpha_5\alpha_1 + \alpha_3 \, sin(\lambda l_1) + \alpha_6\alpha_1\alpha_7 + \alpha_3\alpha_7 \, cos(\lambda l_1))$
$\alpha_{11} = (\alpha_5\alpha_2 + \alpha_4 \, sin(\lambda l_1) + \alpha_6\alpha_2\alpha_7 + \alpha_4\alpha_7 \, cos(\lambda l_1))$
Then, with using of boundary conditions, $\overline{W}_2(l_1) - \overline{W}_1(l_1) = \theta l \overline{\overline{W}}_2(l_1)$, get the general characterizations equation to evaluate natural frequency of beam with crack effect, as,

$$\alpha_8 + \alpha_{10} - cos(\lambda l_1) + cosh(\lambda l_1) + \alpha_7 \, sin(\lambda l_1) + \alpha_7 \, sinh(\lambda l_1) = \theta l \lambda(\alpha_{11} - \alpha_9) \quad (11)$$

By solving equation (11), the value of λ can be evaluated, then the value of ω_n can be evaluated.

II. Simply supported beam
The equations for simply supported beam will be derived in the case of a crack, as shown in Figure 2, by subject boundary conditions of beam, as,

$$W_1(0) = 0,\ \bar{\bar{W}}_1(0) = 0,\ W_2(l) = 0,\ \bar{\bar{W}}_2(l) = 0 \tag{12}$$

Therefore, by substitution boundary conditions for simply supported beam, Eq. 12, into general equation for mode beam, Eq. 6, get,

$$\begin{aligned} W_1(x) &= (A_1 \sin(\lambda x) + C_1 \sinh(\lambda x)) \\ W_2(x) &= A_2\big(\sin\big(\lambda(x - l_1)\big) + \alpha_1 \cos\big(\lambda(x - l_1)\big)\big) + C_2\big(\sinh\big(\lambda(x - l_1)\big) + \alpha_2 \cosh\big(\lambda(x - l_1)\big)\big) \end{aligned} \tag{13}$$

Where, $\alpha_1 = (-\tan(\lambda l_2))$, $\alpha_2 = (-\tanh(\lambda l_2))$
Then, by substitution boundary condition for crack location, Eq. 9, into Eq. 13, get,

$$\begin{aligned} W_1(x) &= A_1(\sin(\lambda x) + \alpha_5 \sinh(\lambda x)) \\ W_2(x) &= A_1\big(\alpha_4 \sin\big(\lambda(x - l_1)\big) + \alpha_6 \cos\big(\lambda(x - l_1)\big) + \alpha_7 \sinh\big(\lambda(x - l_1)\big) + \alpha_8 \cosh\big(\lambda(x - l_1)\big)\big) \end{aligned} \tag{14}$$

Where, $\alpha_3 = \frac{\sinh(\lambda l_1)}{\alpha_2}$, $\alpha_4 = \frac{\sin(\lambda l_1)}{\alpha_1}$, $\alpha_5 = \frac{[\cos(\lambda l_1) - \alpha_4]}{[\cosh(\lambda l_1) - \alpha_3]}$, $\alpha_6 = (\alpha_1\alpha_4)$, $\alpha_7 = (\alpha_3\alpha_5)$, $\alpha_8 = (\alpha_2\alpha_3\alpha_5)$

Also, by applying boundary condition, $\bar{W}_2(l_1) - \bar{W}_1(l_1) = \theta l \bar{\bar{W}}_2(l_1)$, get the general characterizations equation to evaluate the natural frequency for beam with various crack effect, as,

$$\alpha_4 + \alpha_7 - \cos(\lambda l_1) - \alpha_5 \cosh(\lambda l_1) = \theta l \lambda(\alpha_8 - \alpha_6) \tag{15}$$

By solving equation (15), the value of λ can be evaluated, then the value of ω_n can be evaluated.

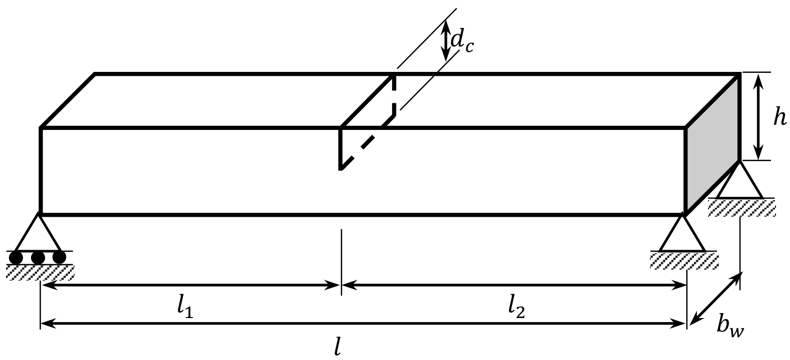


Figure 2. Simply supported Beam with crack.

III. Clamped supported beam
The equations for clamped supported beam will be derived in the case of a crack, as shown in Figure 3, by subject boundary conditions of beam, as,

$$W_1(0) = 0,\ \bar{W}_1(0) = 0,\ W_2(l) = 0,\ \bar{W}_2(l) = 0 \tag{16}$$

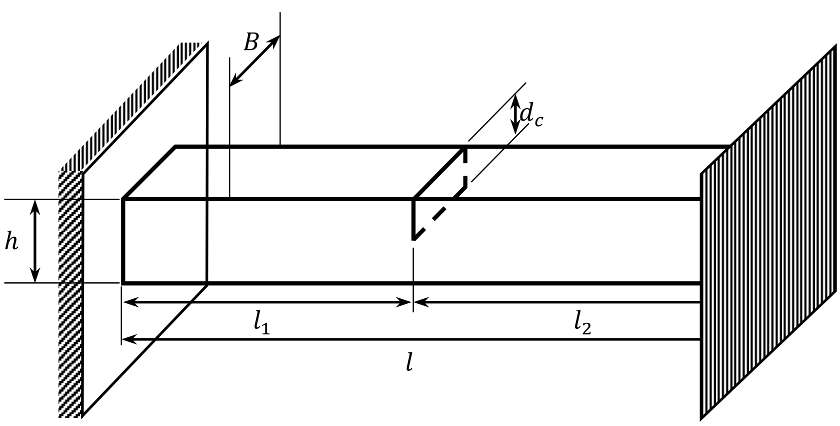


Figure 3. Clamped beam geometry with crack.

Then, by subject Eq. 16 into Eq. 6, get,

$$W_1(x) = (A_1(\sin(\lambda x) - \sinh(\lambda x)) + B_1(\cos(\lambda x) - \cosh(\lambda x)))$$

$$W_2(x) = \begin{pmatrix} A_2\big(\sin\big(\lambda(x-l_1)\big) + \alpha_1 \sinh\big(\lambda(x-l_1)\big) + \alpha_2 \cosh\big(\lambda(x-l_1)\big)\big) + \\ B_2\big(\cos\big(\lambda(x-l_1)\big) + \alpha_3 \sinh\big(\lambda(x-l_1)\big) + \alpha_4 \cosh\big(\lambda(x-l_1)\big)\big) \end{pmatrix} \tag{17}$$

Where,

$\alpha_1 = (\sin(\lambda l_2).\sinh(\lambda l_2) - \cos(\lambda l_2).\cosh(\lambda l_2))$, $\alpha_2 = (-\sin(\lambda l_2).\cosh(\lambda l_2) + \cos(\lambda l_2).\sinh(\lambda l_2))$
$\alpha_3 = (\cos(\lambda l_2).\sinh(\lambda l_2) + \sin(\lambda l_2).\cosh(\lambda l_2))$, $\alpha_4 = -(\cos(\lambda l_2).\cosh(\lambda l_2) + \sin(\lambda l_2).\sinh(\lambda l_2))$

Then, by subjecting boundary condition for crack defect, Eq. 9, into Eq. 17, get,

$$W_1(x) = A_1\big((\sin(\lambda x) - \sinh(\lambda x)) + \alpha_7(\cos(\lambda x) - \cosh(\lambda x))\big)$$

$$W_2(x) = A_1\big(\alpha_8 \sin\big(\lambda(x-l_1)\big) + \alpha_9 \cos\big(\lambda(x-l_1)\big) + \alpha_{10} \sinh\big(\lambda(x-l_1)\big) + \alpha_{11} \cosh\big(\lambda(x-l_1)\big)\big) \tag{18}$$

Where,

$\alpha_5 = -\left(\frac{\alpha_4}{\alpha_2}\sin(\lambda l_1) + \frac{1}{\alpha_2}\sinh(\lambda l_1)\right)$, $\alpha_6 = -\left(\frac{\alpha_4}{\alpha_2}\cos(\lambda l_1) + \frac{1}{\alpha_2}\cosh(\lambda l_1)\right)$
$\alpha_7 = \frac{(\cos(\lambda l_1)+\cosh(\lambda l_1)-\alpha_5+\alpha_5\alpha_1+\alpha_3\sin(\lambda l_1))}{(\sin(\lambda l_1)-\sinh(\lambda l_1)+\alpha_6-\alpha_6\alpha_1-\alpha_3\cos(\lambda l_1))}$, $\alpha_8 = (\alpha_5 + \alpha_6\alpha_7)$, $\alpha_9 = (\sin(\lambda l_1) + \alpha_7\cos(\lambda l_1))$
$\alpha_{10} = (\alpha_5\alpha_1 + \alpha_3\sin(\lambda l_1) + \alpha_6\alpha_1\alpha_7 + \alpha_3\alpha_7\cos(\lambda l_1))$
$\alpha_{11} = (\alpha_5\alpha_2 + \alpha_4\sin(\lambda l_1) + \alpha_6\alpha_2\alpha_7 + \alpha_4\alpha_7\cos(\lambda l_1))$

Also, by using boundary condition, $\bar{W}_2(l_1) - \bar{W}_1(l_1) = \theta l \bar{\bar{W}}_2(l_1)$, for Eq. 18, get the general characterization equation for evaluating the natural frequency of cantilever beam with various crack size and position effect, as,

$$\alpha_8 + \alpha_{10} - \cos(\lambda l_1) + \cosh(\lambda l_1) + \alpha_7\sin(\lambda l_1) + \alpha_7\sinh(\lambda l_1) = \theta l\lambda(\alpha_{11} - \alpha_9) \tag{19}$$

2.2 General heat generation

To applying the equation of deflection beam as a function of x-direction, Eq. 10, 14, 18 for cantilever, simply supported and clamped beam, respectively, in general equation of motion of beam with crack effect, must be transformation the equations of beam to continuous equation by using Fourier series formulation, and then subjecting the resulting equation into Eq. , as,

$$W(x) = A_o + \sum_{n=1}^{\infty} A_n \cos\frac{2n\pi x}{L} + \sum_{n=1}^{\infty} B_n \sin\frac{2n\pi x}{L} \tag{20}$$

Where,

$A_o = \frac{1}{l}\int_0^l W(x)dx = \frac{1}{l}\left(\int_0^{l_1} W_1(x)dx + \int_{l_1}^l W_2(x)dx\right)$
$A_n = \frac{2}{l}\int_0^l W(x)\cos\frac{2n\pi x}{L}dx = \frac{2}{l}\left(\int_0^{l_1} W_1(x)\cos\frac{2n\pi x}{l}dx + \int_{l_1}^l W_2(x)\cos\frac{2n\pi x}{l}dx\right)$
$B_n = \frac{2}{l}\int_0^l W(x)\sin\frac{2n\pi x}{l}dx = \frac{2}{l}\left(\int_0^{l_1} W_1(x)\sin\frac{2n\pi x}{l}dx + \int_{l_1}^l W_2(x)\sin\frac{2n\pi x}{l}dx\right)$

Then, by substation Eq. 10, 14, or 18, into Eq. 20, according of beam supported, and integration the resulting equation, get the constant Fourier series, then subjecting Eq. 20 onto Eq. 4, get,

$$\begin{pmatrix} EI\left(\sum_{n=1}^{\infty} A_n\left(\frac{2n\pi}{L}\right)^4\cos\frac{2n\pi x}{L} + \sum_{n=1}^{\infty} B_n\left(\frac{2n\pi}{L}\right)^4\sin\frac{2n\pi x}{L}\right)w(t) + \\ \rho A\left(A_o + \sum_{n=1}^{\infty} A_n\cos\frac{2n\pi x}{L} + \sum_{n=1}^{\infty} B_n\sin\frac{2n\pi x}{L}\right)\frac{\partial^2 w(t)}{\partial t^2} \end{pmatrix} = f(x,t) \tag{21}$$

Then, by using orthogonally technique, with multiplying Eq. 21 by Eq. 20, and then, integrating from 0 to l, get,

$$\omega^2 W(t) + \frac{\partial^2 W(t)}{\partial t^2} = \frac{\int_0^l\left(A_o+\sum_{n=1}^{\infty} A_n\cos\frac{2n\pi x}{L}+\sum_{n=1}^{\infty} B_n\sin\frac{2n\pi x}{L}\right)f(x.t)\,dx}{\rho A\int_0^l\left(A_o+\sum_{n=1}^{\infty} A_n\cos\frac{2n\pi x}{L}+\sum_{n=1}^{\infty} B_n\sin\frac{2n\pi x}{L}\right)^2 dx} = \frac{\int_0^l W(x)f(x,t)dx}{\rho A\int_0^l W^2(x)dx} = \frac{1}{\rho Ab}Q_n(t) \tag{22}$$

Where, $\mathrm{Q_n(t)}$: is the forced generalized (N), and can be evaluating from,

$$Q_n(t) = \int_0^l W(x) f(x,t) dx \tag{23}$$

And, b: orthogonally constant, can be evaluating from,

$$b = \int_0^l W^2(x) dx \tag{24}$$

Then, by using general solution form force d vibration single degree of freedom system and using Eq. 3, can be evaluating the general force vibration response for beam with various crack effect. Then, by substation Eq. 3, and the solution for its equation Eq. 22, into following equation for heat generation, can be calculating the heat generation for beam structure with vibration effect for various influence of crack depth and position, as, [20],

$$q = \frac{\pi}{48} b_w E h^3 \left(\frac{d^2 W(x,t)}{dx^2}\right)^2 f_n * \eta \tag{25}$$

Where, f_n: is the frequency [Hz], and η is the loss factor ($\eta = 0.0028$), [16].
Then, by substation Eq. 20 into Eq. 25, get,

$$\mathrm{q} = \frac{\pi}{48} \mathrm{b_w E h^3} \begin{pmatrix} \sum_{n=1}^{\infty} \mathrm{A_n} \left(\frac{2n\,\pi}{L}\right)^2 \cos\frac{2n\,\pi\,x}{L} + \\ \sum_{n=1}^{\infty} \mathrm{B_n} \left(\frac{2n\,\pi}{L}\right)^2 \sin\frac{2n\,\pi\,x}{L} \end{pmatrix}^2 \mathrm{f_n} * \eta * \left(\left(\frac{1}{\rho A b \omega_n} \int_0^t Q_n(\tau) \sin \omega_n (t-\tau)\; d\tau\right)\right)^2 \tag{26}$$

Therefore, by using Mathlab program can be evaluating the heat generation for beam with various crack effect of simply, clamped and cantilever beam, with time and beam length dependent function.

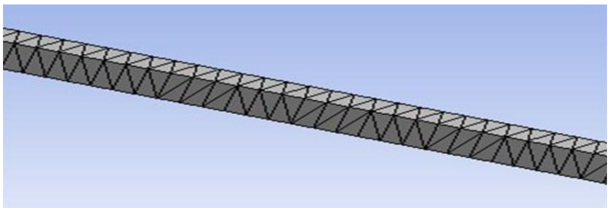
3. Numerical technique

The numerical examination is strategy utilized to assessment estimated answer for issue, in this way, the its method in more application utilized to investigation the troublesome structure its can't arrangement by scientific strategies. Likewise, the numerical procedure used to correlation the numerical outcomes by different outcomes are assessed with different methods, to given the understanding of results, [46, 47].
Therefore, the COMSOL program will be utilized, which will rely upon the finite element method (FEM) for outline and examination of the models, and give the results required by reproductions and simulations utilizing PC programming to achieve the yields and results of the fruition of the innovative work of the system of work, and contrasted with different results removed with the scientific arrangement, and given the percent error of its outcomes. This program is utilized to discover exact numerical arrangements and diverse outlines and structures and to discover the outcomes of complex scientific conditions that are hard to solve analytically, [48-50].
In this work, the COMSOL/CFD program was utilized to discover the heat generated values by the vibration of the beam presented to the harmonic force at various frequencies for various instances of supported with crack and to know the effect of its location and depth on the heat generated. Thus, to solution the problem by using its numerical technique, firstly, selecting the best number of element and node can be used, [51-54], then, different cases are studied for the computer mesh where the comparison is made for the design of the mesh using the COMSOL program in the absence of a crack in the beam as in Figure 4.a. Also, a computer mesh is designed in the case of a crack in the beam and what is the effect of the crack on the mesh as in Figure 4.b, and using the finite element method (FEM) where the model is divided into small parts and elements and simple geometric shapes that make it easy to distinguish between the two cases to give different results.

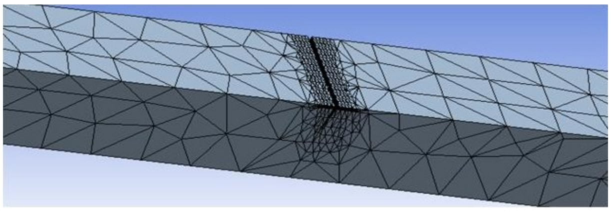
4. Results and discussion

The check and comparison of the outcomes and figures extricated from the analytical solution and the numerical solution with crack, for the heat generated by the vibration of the beam presented to the harmonic force with time. Since the analytical solution can compute the magnitude of heat generated in the cracking beam through the energy of elastic deformation, so the utilization of the COMSOL program to determine the value of heat generated and temperature changes in the beam due to vibration.

The numerical solution will be executed utilizing the COMSOL program by designing models like the beam specifications in terms of geometry, dimensions and properties of the material utilized the same forces applied during vibration of the cracking beam calculated by analytical results with compared between them. The value of heat generated is compared between the outcomes calculated in the analytical solution and the outcomes in the numerical solution for the same models with crack for multiple cases of supported.



(a) Without crack



(b) With crack

Figure 4. Beam mesh without crack is maximized.

The amount of heat generated in the assumed beam (length (L=0.84 m), width, height (w=h=2.5 cm)) is calculated analytically by using the Matlab program during the vibration of the beam with crack under the influence of harmonic force at constant frequency ($\omega = 0.9 \times \omega_n$) this frequency is chosen for the appearance of heat clearly approaching the resonance state. Sometimes the heat generated is calculated for the existence of the crack to different depths and locations on the beam for each location along the length of the beam on demonstrated constant time at other times it is calculated with time in a location for several cases of supported, as follows,

4.1 Cantilever beam

Figure 5 shows the amount of heat generated in the beam changes according to the time and length of the beam with crack ($L_1 = 0.5\ L = 0.42\ m;\ d_c = 0.5\ h = 1.25\ cm$).

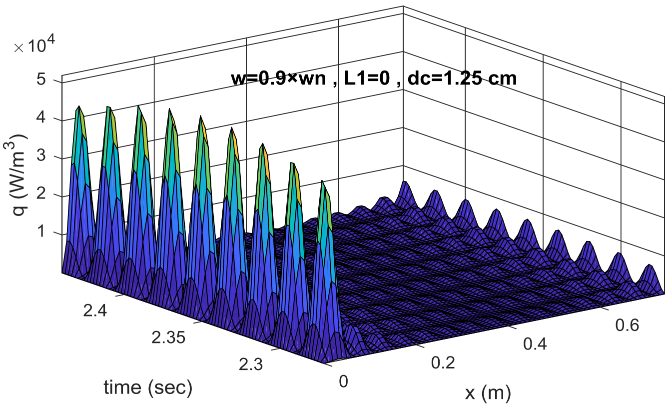


Figure 5. Heat generated with time and length of the beam with crack.

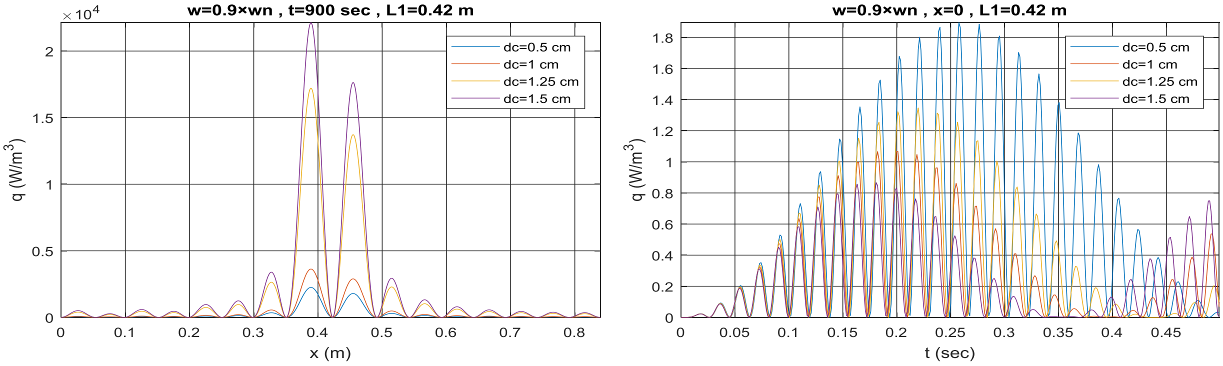
The heat generated is calculated according to the location and depth of the crack as shown,

I. The location of the crack (L_1 = 0.42 m), Figure 6, the relationship between the heat generated for different depths of the crack (dc = (0.5, 1, 1.25 and 1.5) cm) with the length of the beam at constant time as shown (a) or the time at the fixed end (x = 0) as shown (b).

II. The location of the crack (L_1 = 0.64 m), Figure 7, the relationship between the heat generated for different depths of the crack (d_c = (0.5, 1, 1.25 and 1.5) cm) with the length of the beam at constant time as shown (a) or the time at the fixed end (x = 0) as shown (b).

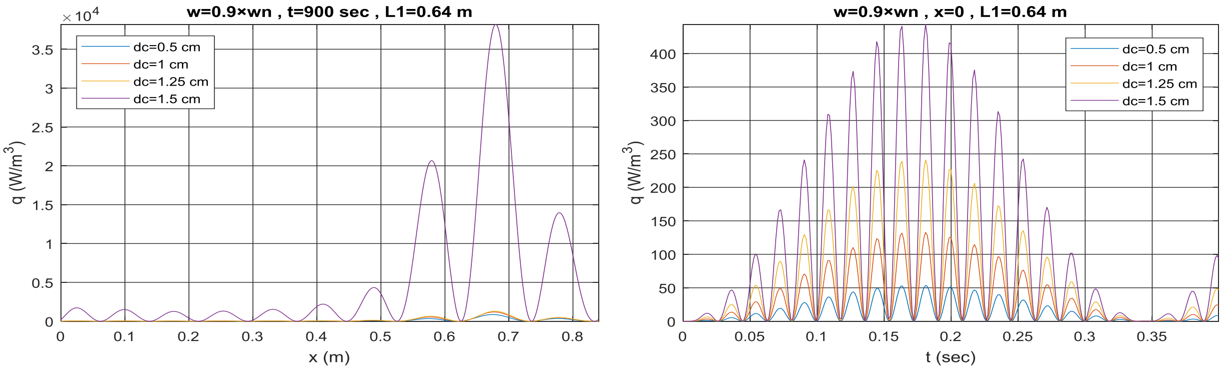
III. The location of the crack (d_c = 0.2h = 0.5 cm), Figure 8, the relationship between the heat generated for different locations of the crack (L_1 = (0.2, 0.42 and 0.64) m) with the length of the beam at constant time as shown (a) or the time at the fixed end (x = 0) as shown (b).

IV. The location of the crack (d_c = 0.5h = 1.25 cm), Figure 9, the relationship between the heat generated for different locations of the crack (L_1 = (0.2, 0.42 and 0.64) m) with the length of the beam at constant time as shown (a) or the time at the fixed end (x = 0) as shown (b).



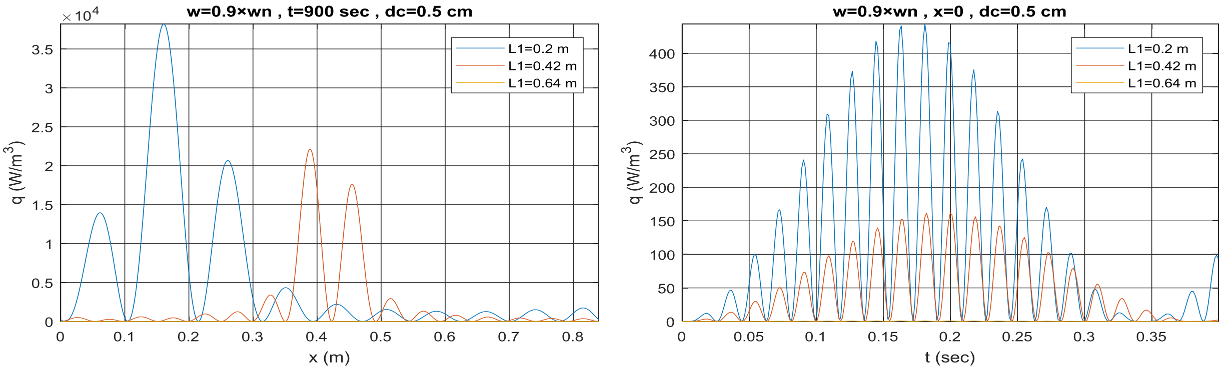
(a) (b)

Figure 6. The heat generated with crack (L_1=0.42 m).



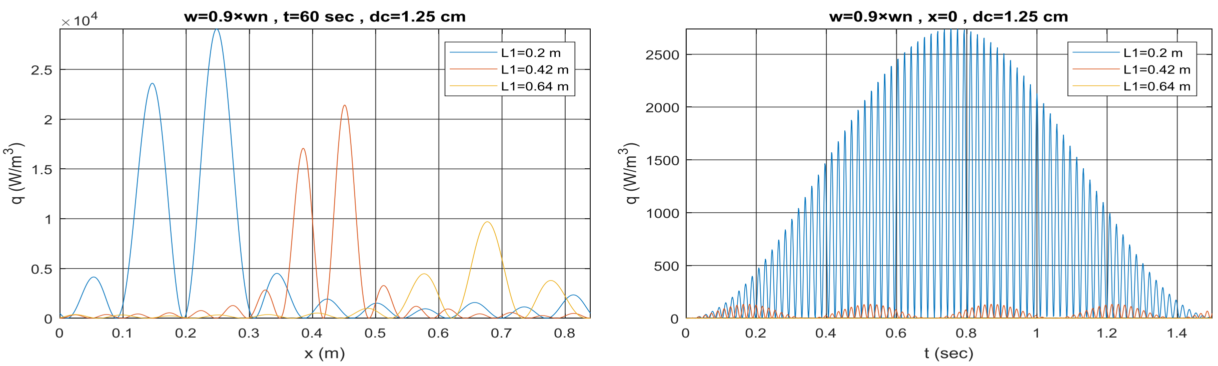
(a) (b)

Figure 7. The heat generated with crack (L1=0.64 m).



(a) (b)

Figure 8. The heat generated with crack (dc=0.5 cm).



(a) (b)

Figure 9. The heat generated with crack (dc=1.25 cm).

4.2 Simply supported beam

Figure 10, shows the amount of heat generated in the beam changes according to the time and length of the beam with crack ($L_1 = 0.5\ L = 0.42\ m;\ d_c = 0.5\ h = 1.25\ cm$).

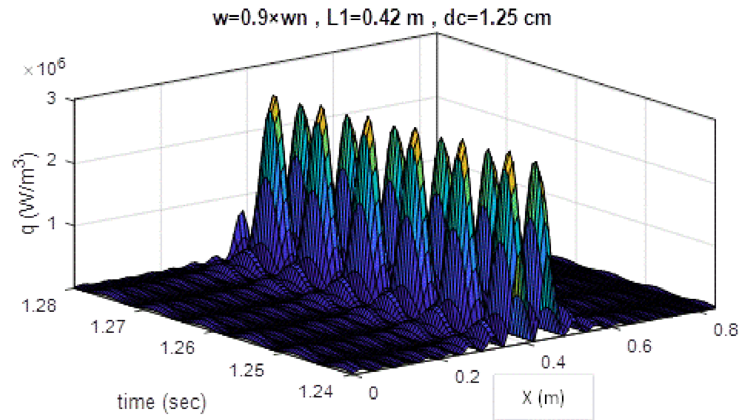
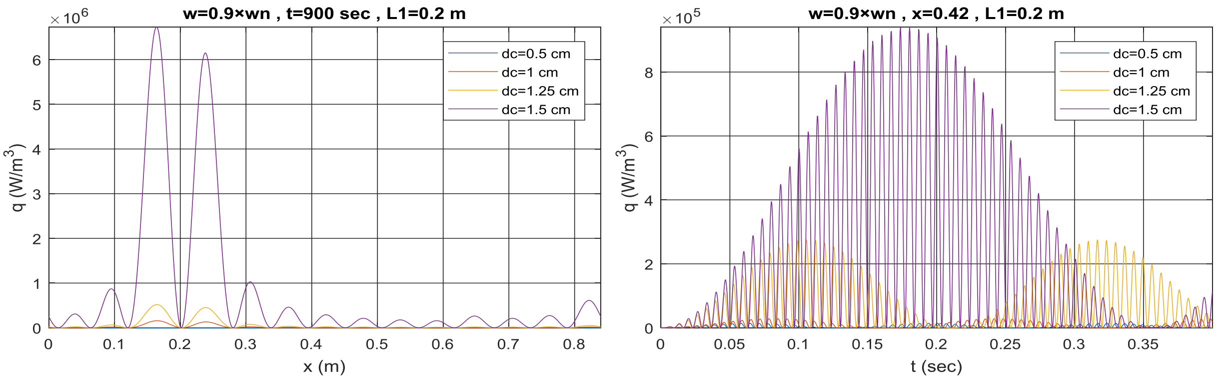


Figure 10. Heat generated with time and length of the beam with crack. (Three dimension).

The heat generated is calculated according to the location and depth of the crack as shown,

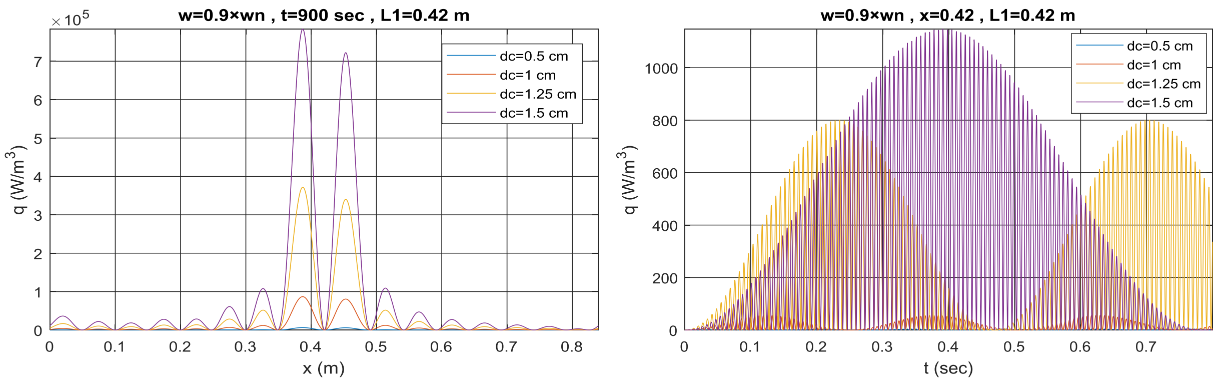
I. The location of the crack (L_1 = 0.2 m), Figure 11, the relationship between the heat generated for different depths of the crack (d_c = (0.5, 1, 1.25 and 1.5) cm) with the length of the beam at constant time as shown (a) or the time at the middle of the beam (x = 0.42 m) as shown (b).

II. The location of the crack (L_1 = 0.42 m), Figure 12, the relationship between the heat generated for different depths of the crack (d_c = (0.5, 1, 1.25 and 1.5) cm) with the length of the beam at constant time as shown (a) or the time at the middle of the beam (x = 0.42 m) as shown (b).



(a) (b)

Figure 11. The heat generated with crack (L1=0.2 m).

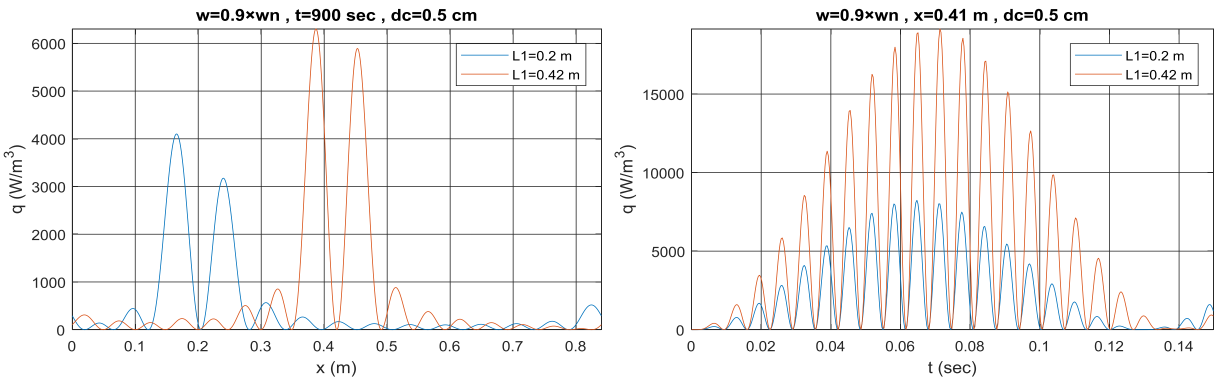


(a) (b)

Figure 12. The heat generated with crack (L1=0.42 m).

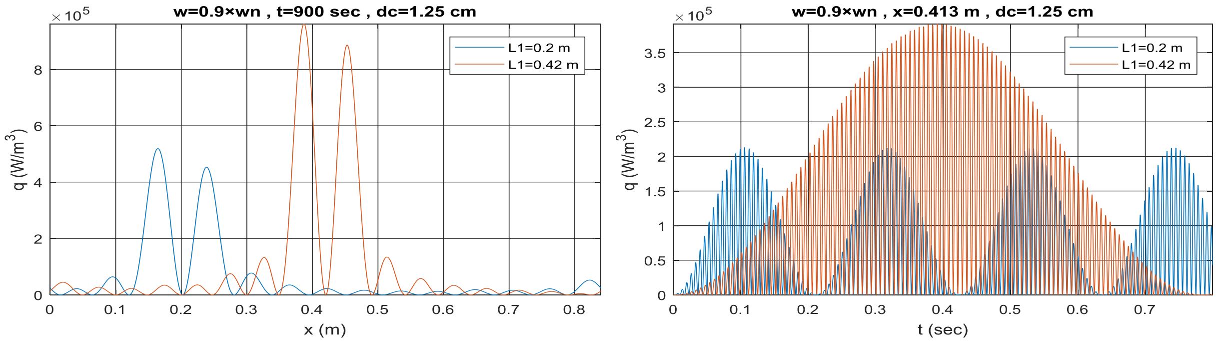
III. The location of the crack (d_c = 0.5 cm), Figure 13, the relationship between the heat generated for different depths of the crack (L_1 = (0.2 and 0.42) m) with the length of the beam at constant time as shown (a) or the time at the middle of the beam (x = 0.41 m) as shown (b).

IV. The location of the crack (d_c = 1.25 cm), Figure 14, the relationship between the heat generated for different depths of the crack (L_1 = (0.2 and 0.42) m) with the length of the beam at constant time as shown (a) or the time at the middle of the beam (x = 0.413 m) as shown (b).



(a) (b)

Figure 13. The heat generated with crack (dc=0.5 cm).



(a) (b)

Figure 14. The heat generated with crack (dc=1.25 cm).

4.3 Clamped beam

Figure 15, shows the amount of heat generated in the beam changes according to the time and length of the beam with crack (L_1 = 0.5 L = 0.42 m; d_c = 0.5 h = 1.25 cm).

The heat generated and the response deflection is calculated according to the location and depth of the crack as shown,

I. The location of the crack (L_1 = 0.2 m), Figure 16, the relationship between the heat generated for different depths of the crack (d_c= (0.5, 1, 1.25, 1.5) cm) with the length of the beam at constant time as shown (a) or the time at the fixed end (x=0) as shown (b).

II. The location of the crack (L_1 = 0.42 m), Figure 17, the relationship between the heat generated for different depths of the crack (d_c = (0.5, 1, 1.25 and 1.5) cm) with the length of the beam at constant time as shown (a) or the time at the fixed end (x = 0) as shown (b).

III. The location of the crack (d_c = 0.5 cm), Figure 18, the relationship between the heat generated for different depths of the crack (L_1 = (0.2 and 0.42) m) with the length of the beam at constant time as shown (a) or the time at the fixed end (x = 0) as shown (b).

IV. The location of the crack (L_1 = 1.25 cm), Figure 19, the relationship between the heat generated for different depths of the crack (L_1 = (0.2 and 0.42) m) with the length of the beam at constant time as shown (a) or the time at the fixed end (x = 0) as shown (b).

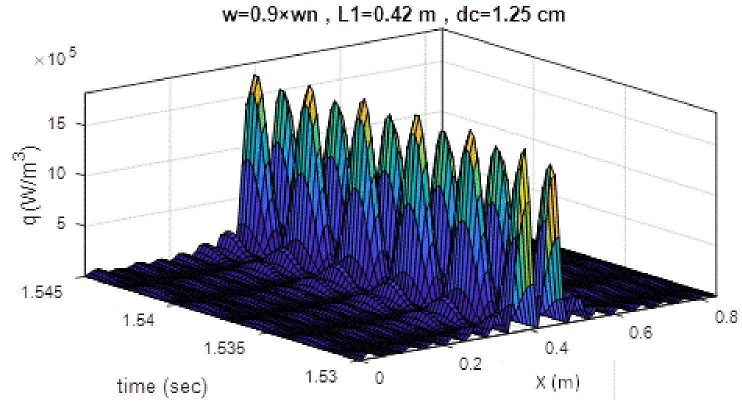


Figure 15. Heat generated with time and length of the beam with crack. (Three dimension).

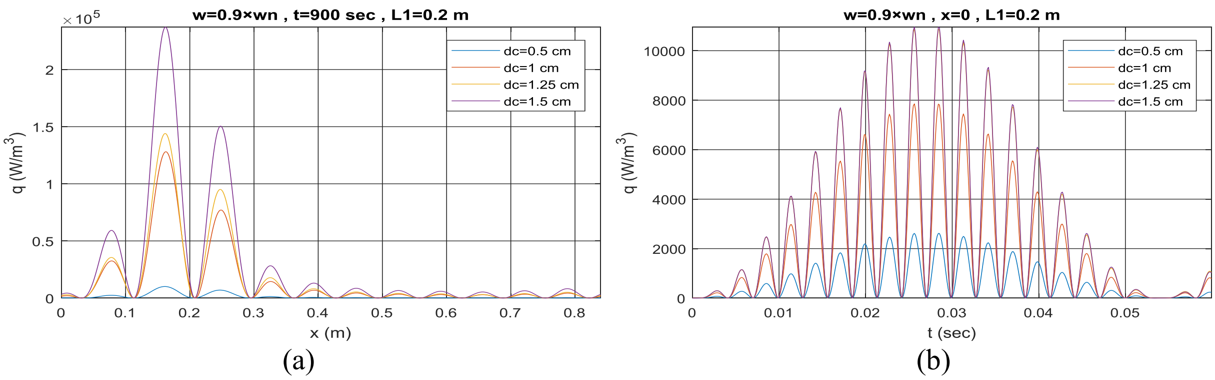


Figure 16. The heat generated with crack (L1=0.2 m).

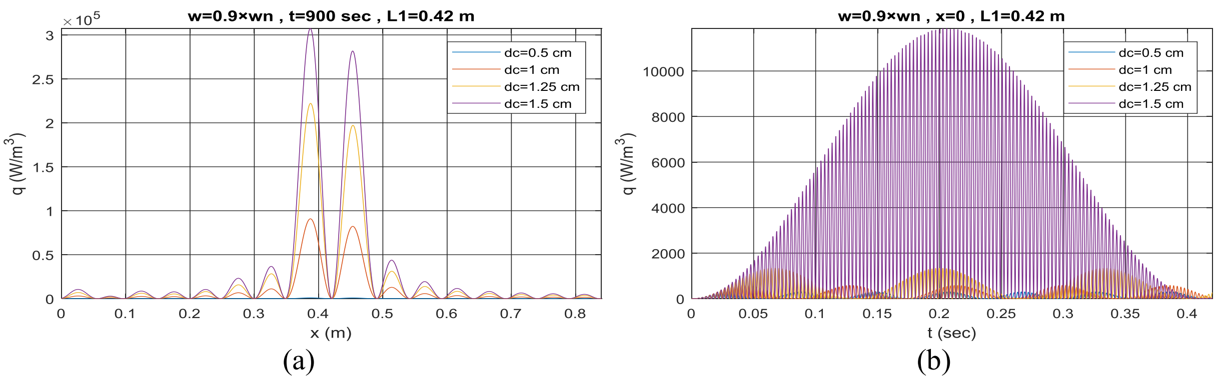


Figure 17. The heat generated with crack (L_1=0.42 m).

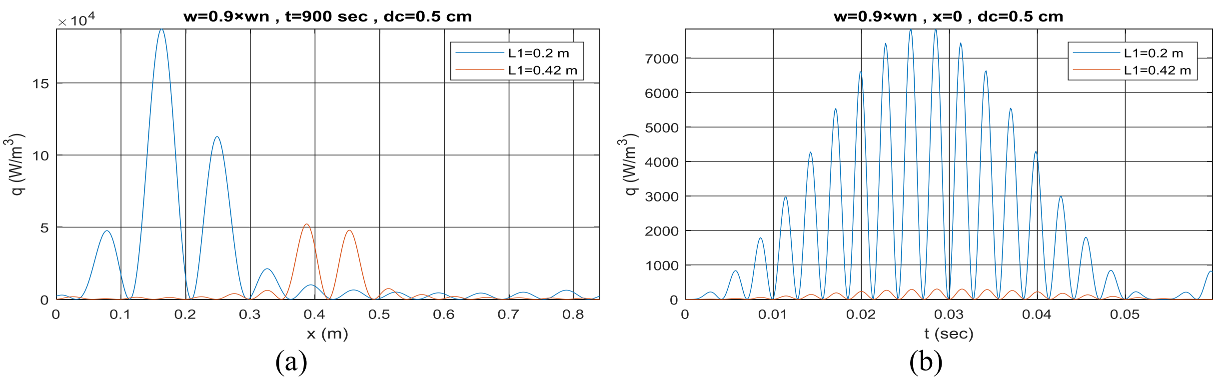
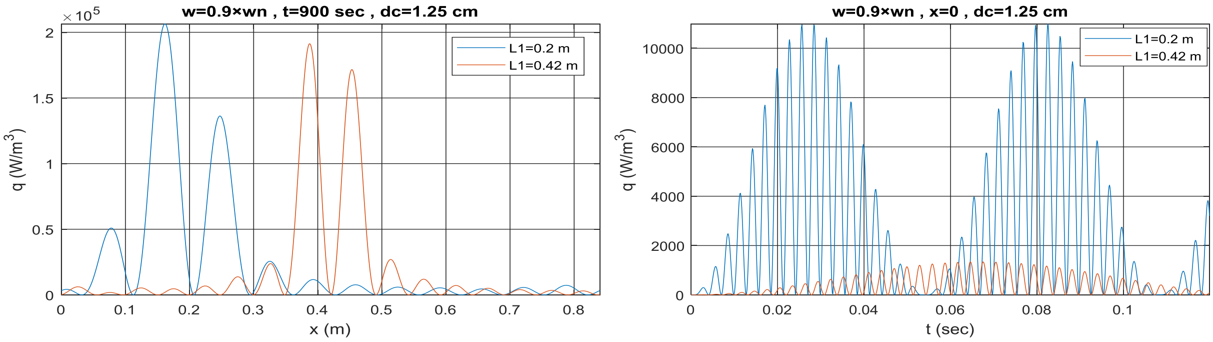


Figure 18. The heat generated with crack (d_c=0.5 cm).

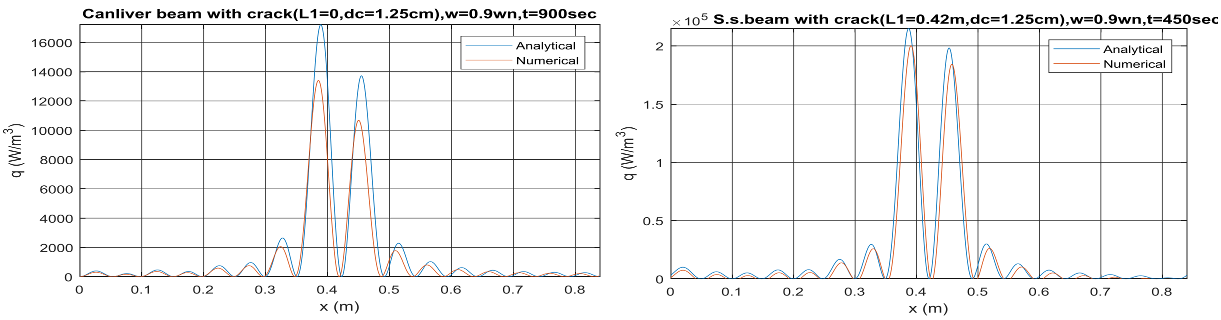


(a) (b)

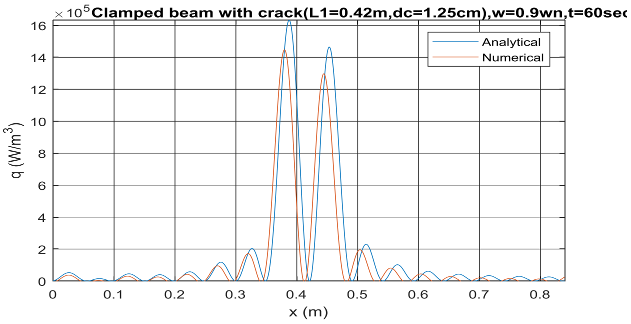
Figure 19. The heat generated with crack (L_l=1.25 cm).

4.4 The comparison the analytical solution and the numerical solution

The comparison heat generated for different supported beam, as follows, cantilever beam, Figure 20.a, the relationship between the heat generated with the length of the beam is explained for the analytical solution and the numerical solution at constant time and constant frequency with crack. Simply supported beam, Figure 20.b, the relationship between the heat generated with the length of the beam is explained for the analytical solution and the numerical solution at constant time and constant frequency with crack. Finally, Clamped beam, Figure 20.c, the relationship between the heat generated with the length of the beam is explained for the analytical solution and the numerical solution at constant time and constant frequency with crack.



(a) Cantilever beam. (b) Simply supported beam.



(c) Clamped beam.

Figure 20. Comparison for the heat generated (analytical & numerical).

5. Conclusions

From previous results, the following conclusions can be shown briefly as follows:

1. The theoretical method was followed by accurate and logical results to calculate the amount of heat generated in the cracking beam for the different types of supported according to the change of time or location in the beam and frequencies different according to the natural frequency. Also calculate the change in the natural frequency.
2. It has been observed that the effect of the crack for different types of support affects the natural frequency. The natural frequency decreased as the depth of the crack increased or the position of the crack in the beam approaches the highest moment in the beam.

3. Taking advantage of the above conclusions, early detection of the crack by observing the decrease of natural frequency or increase the heat generated or increase the response deflection before the crack grows more and thus leads to more losses.
4. When comparing the heat generated between the analytical results and the numerical results for the different cases did not exceed (6.2%).

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