INTERNATIONAL JOURNAL OF
ENERGY AND ENVIRONMENT

Volume 6, Issue 5, 2015 pp.425-436

Journal homepage: www.IJEE.IEEFoundation.org

Experimental study of non-rectangular piano key weir discharge coefficient

Saleh I. Khassaf[1], Mohamed B. Al-Baghdadi[2]

[1] Civil Engineering Department, College of Engineering, University of Basrah, Basrah, Iraq.
[2] Civil Engineering Department, Faculty of Engineering, University of Kufa, Najaf, Iraq.

Abstract
Experimental investigation has been performed to understand the hydraulic behaviour of non-rectangular piano key weir where either the side wall angle or the side wall inclination angle is greater than zero. Five physical models were prepared: one standard type-A rectangular model, and four non-rectangular models designed in similar dimensions to the rectangular one. Tests were conducted in a 15m long, 0.3m wide and 0.45 m deep rectangular glass-walled experimental flume. Effects of side wall angle and side wall inclination angle on discharge coefficient were investigated, so that the head-discharge relationship for each model is achieved. It was concluded that changing those angle to about 10° has negative effect on discharge capacity, while changing them around 5° can increase the capacity when appropriate change in the inlet and outlet keys widths ratio.

Keywords: Physical modeling; Piano key weir; Discharge coefficient; Non-rectangular; Side wall angle; Side wall inclination angle.

1. Introduction
Piano key weir (abbreviated PKW) is a particular type of labyrinth weirs which has been developed in the recent years as an alternative to the standard types. It combines the interest of labyrinth layout with the use of sloped floors and overhangs in order to develop an innovative geometry that helps to overcome the problems of traditional labyrinth weirs. Schleiss [1] and Lempérière *et al* [2] present historical reviews on the PKW development.
The main advantages of PKW over labyrinth weirs are [3]:
- The reduced footprint area making it suitable for installation on top of existing or new gravity dams as well as on earth dams.
- It is structurally simple, easy to build with local resources in all countries. Also, it requires less reinforcement than labyrinth weirs.

Many studies have been published in the literature about the hydraulic behaviour of PKW. Three main studies [4-6] obtained general design formulae that predict the discharge capacity of PKW according to the main geometric parameters such as the developed crest length to the width ratio (L/W), the inlet and outlet keys widths ratio (W_i/W_o), and the upstream-downstream length of PKW to the weir height ratio (B/P).
Most of researches are concerned with the standard rectangular configuration of PKW; however, Schleiss [1] reported that using non-rectangular configuration may be advantageous in terms of discharge

capacity. Non-rectangular achieved by using non-zero side wall angle or side wall inclination angle. Cicero *et al* [7] studied the effect of increasing the side wall angle on discharge coefficient.
This article is devoted to the study of the free flow hydraulic performance of non-rectangular PKW. Firstly, a classical rectangular model was prepared, then, four non-rectangular models were designed with similar dimensions as the rectangular one. Two of them were designed for the study of side wall angle effect, while the effect of side wall inclination angle is studied by the other two. Results of non-rectangular models are analysed and compared to the rectangular model behaviour. Also the results of Cicero *et al* [7] are discussed and compared with the present study.

2. Description of non-rectangular PKW geometry

In order to design a non-rectangular PKW, we must start with a rectangular configuration. Figure 1 illustrates a standard rectangular PKW. Nomenclature of this article is in agreement with the naming convention of Pralong *et al* [8]. The notations of Cicero *et al* [7] for non-rectangular PKW are also adopted. Notations of the side wall angle and side wall inclination angle are α and β respectively. Pralong *et al* [8] have set the notation of α, but β has not been discussed in their article.
Parameters of rectangular PKW are defined in Table 1. However, when we change the angles α and β, new parameters arise as the PKW layout becomes non-rectangular (see Figure 2). Definitions of these parameters are given in Table 2.

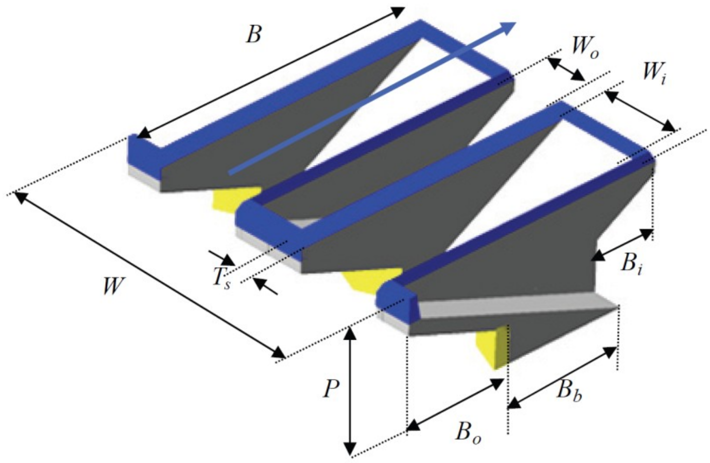

Figure 1. Sketch of standard rectangular PKW [8]

Table 1. Terminology of rectangular PKW geometric parameters [8]

Parameter symbol	Meaning
B	Upstream-downstream length of the PKW,B=B_b+B_i+B_o
B_o	Upstream (outlet key) overhang length
B_i	Downstream (inlet key) overhang length
B_b	Base length
B_h	Sidewall overflowing crest length measured from the outlet key crest axis to the inlet keycrest axis
P	Height of PKW measured from the crest(including possible parapet walls)
P_d	Dam height (or any platform under the PKW)
W	Total width of the PKW
W_i	Inlet key width (sidewall to sidewall)
W_o	Outlet key width (sidewall to sidewall)
T_s	Sidewall thickness
T_i	Horizontal crest thickness at inlet key extremity
T_o	Horizontal crest thickness at outlet key extremity
L	Total developed length along the overflowing crest axis

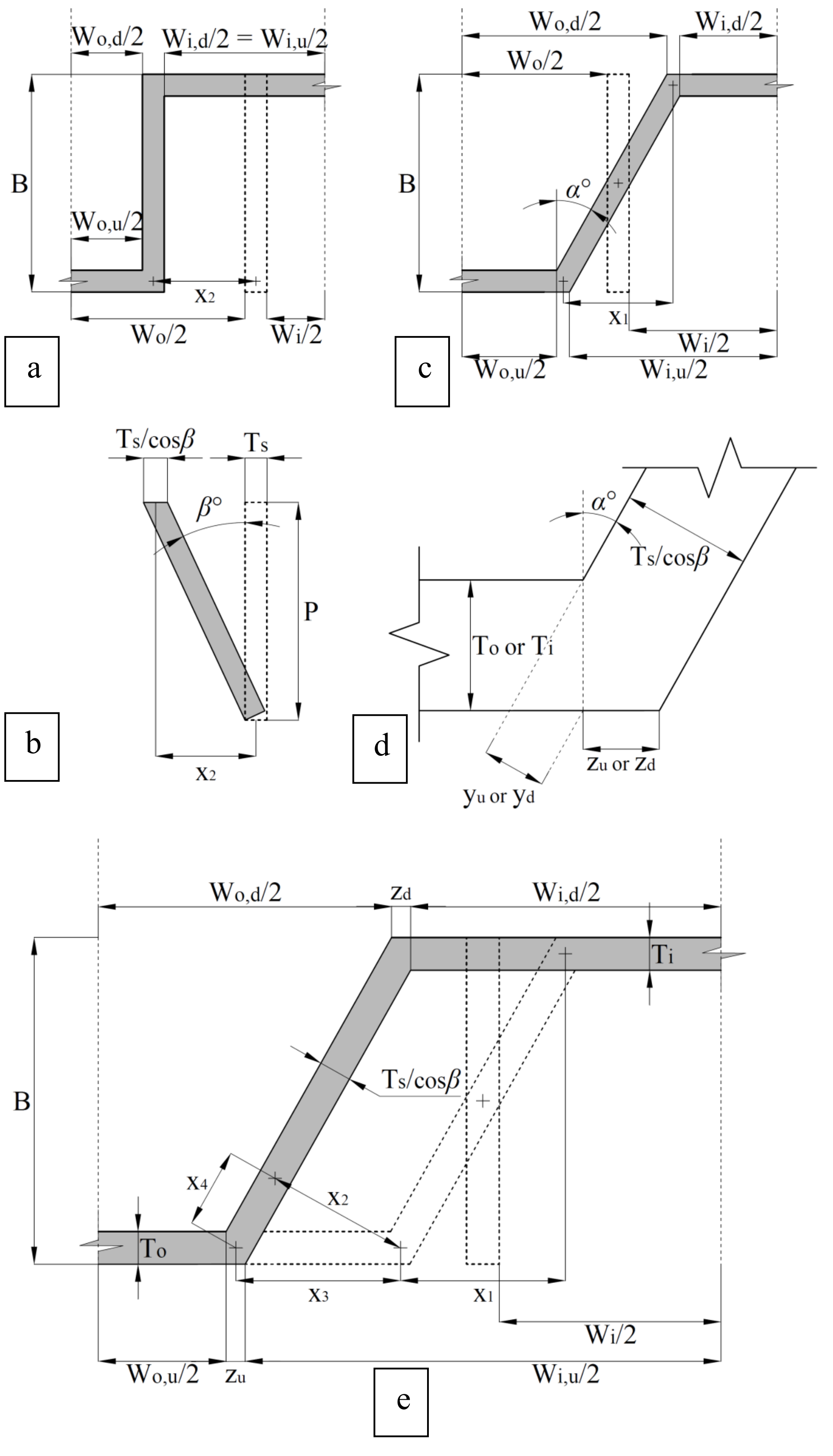

Figure 2. Half unit details of PKW with variations of angles α and β. (a) Top-view, β>0 and α=0, (b) Front-view, β>0 and α=0, (c) Top-view, α>0 and β=0, (d) Details of crest thickness at the transition between inlet (or outlet) key crest and side crest, and (e) Top-view, β>0 and α>0

ISSN 2076-2895 (Print), ISSN 2076-2909 (Online)

Table 2. New parameters that arise when using a non-rectangular PKW [7]

Parameter symbol	Meaning
$W_{i,u}$	Inlet key width at the upstream edge (sidewall to sidewall)
$W_{o,u}$	Outlet key width at the upstream edge (sidewall to sidewall)
$W_{i,d}$	Inlet key width at the downstream edge (sidewall to sidewall)
$W_{o,d}$	Outlet key width at the downstream edge (sidewall to sidewall)

Design calculations of non-rectangular PKW are given in equations 1 to 17. Note that when we substitute α=0 and β=0, the rectangular layout results in. Figure 2 presents details of non-rectangular PKW configuration with different cases of changing α, β, and both of them.
Following are the design calculation of non-rectangular PKW including some related dimensions which appear in Figure 2.

$$W = W_u * N_u \quad (1)$$

$$L = L_u * N_u \quad (2)$$

where: W_u and L_u are the width and length of one unit of PKW respectively, while N_u is the number of units in the entire structure.

$$W_u = W_{i,u} + W_{o,u} + 2z_u = W_{i,d} + W_{o,d} + 2z_d \quad (3)$$

$$L_u = W_u + 2B_h(1 - sin\,\alpha) \quad (4)$$

$$B_h = \frac{2B - T_i - T_o}{2\,cos\,\alpha} \quad (5)$$

$$W_{i,u} = W_i + T_s + x_1 + 2x_3 - z_u \quad (6)$$

$$W_{i,d} = W_i + T_s - x_1 + 2x_3 - z_d \quad (7)$$

$$W_{o,u} = W_o + T_s - x_1 - 2x_3 - z_u \quad (8)$$

$$W_{o,d} = W_o + T_s + x_1 - 2x_3 - z_d \quad (9)$$

$$x_1 = B_h\,sin\,\alpha \quad (10)$$

$$x_2 = P\,tan\,\beta \quad (11)$$

$$x_3 = P\left(\frac{tan\,\beta}{cos\,\alpha}\right) \quad (12)$$

$$x_4 = P\,tan\,\alpha\,tan\,\beta \quad (13)$$

$$z_u = \frac{\left(\frac{T_s}{cos\,\beta}\right) - y_u}{cos\,\alpha} \quad (14)$$

$$z_d = \frac{\left(\frac{T_s}{cos\,\beta}\right) - y_d}{cos\,\alpha} \quad (15)$$

$$y_u = T_o\,sin\,\alpha \quad (16)$$

$$y_d = T_i\,sin\,\alpha \quad (17)$$

3. Experimental setup

Experimental tests were conducted in a 15 m long, glass-walled flume having a rectangular section of 0.3m wide by 0.45 m deep. The flume has a closed-loop water system. A main tank, of 4.5m^3 capacity, is located at the downstream end of the flume. Water is conveyed from the main tank to an inlet tank, of 0.5m^3 capacity, at the upstream end by means of a pump having maximum discharge of 36 litre/sec. Flume discharge is measured by means of a pre-calibrated sharp-crested rectangular weir. The flume is equipped with a rolling point gauge apparatus with accuracy of ±0.5mm.

Five physical models were prepared in this research. Firstly, a rectangular PKW model was made for purpose of comparison. According to the recommendation of Lempérière [9], a type-A PKW configuration has been selected with the following characteristics: ($L/W=5$, $W_i/W_o=1.25$, $B/P=2.4$, $B_i/B=0.25$, $B_o/B=0.25$). This model will be referred to as (M) in this article.

Two models were built to study the effect of angle α (i.e. having β=0), while other two were built to study the effect of β(with α=0). These models are given the following symbols with respect to their associated values of α and β: (α5), (α10), (β5), and (β10). Table 3 shows the values of α and β for each model. Note that model (α10) has α=10.25° as it is the maximum possible value within the available space (i.e. the model has a triangular layout).

Table 3. Values of α and β (degrees) for the models under study

Angle	(M)	(α5)	(α10)	(β5)	(β10)
α	0	5	10.25	0	0
β	0	0	0	5	10

All 2-units, flat-top crested, PKW models were manufactured of 2.5mm thick acrylic glass sheets cut with a CNC (computer numerical controlled) machine. Each model was fixed firmly to the flume bed by two screws. Then, enough quantity of silicon rubber was added to prevent movement and provide water tightness. Under each model, a platform was fitted so that the dam height ratio P_d/P=0.6. Free flow tests were executed at the mid-section of the flume to ensure that uniform flow is developed and to avoid the downstream effects.

Dimensions of each model are calculated by substituting the values of α, β, and other given design constraints in equations 1 to 17. The given ratios of model (M) should also be considered in calculations. Resulting dimensions are presented in Table 4.

Table 4. Calculated dimensions (centimetres)of the PKW models in this study

Model	B	P	B_i	B_o	$W_{i,u}$	$W_{i,d}$	$W_{o,u}$	$W_{o,d}$	P_d
(M)	30.3	12.6	7.6	7.6	8.06	8.06	6.44	6.44	7.6
(α5)	33.0	13.8	8.3	8.3	11.0	5.20	3.6	9.3	8.3
(α10)	36.2	15.1	9.1	9.1	14.6	1.60	0	13.0	9.1
(β5)	30.3	12.6	7.6	7.6	10.3	10.3	4.2	4.2	7.6
(β10)	30.3	12.6	7.6	7.6	12.5	12.5	2.0	2.0	7.6

Head-discharge relationship has been constructed for each model by recording the water head values associated with different discharges. There have been at least 12 readings for each model. Measurements of water head were taken at a distance of 32cm from the outlet key apex in the upstream direction. This is equal approximately to four times the maximum head over the PKW. Total head is obtained by adding the piezometric head to the velocity head corresponding to the average velocity of the cross-sectional area. Recordings were taken after the flow had been allowed to stabilize for 5 to 10 minutes.

Any reading of water head (above the crest level) that is below 3cm was avoided. This is because readings below this value are influenced by the scale effects (surface tension and viscosity effects) and would not reflect the behaviour of real prototypes [10].

4. Experimental results

Formulation of PKW discharge may be realised by using the formula of standard sharp-crested rectangular weir (equation 18), hence, the discharge coefficient may be calculated.

$$Q = C_{dW} \frac{2}{3}\sqrt{2g}WH_o^{\,1.5} \tag{18}$$

where: Q is the PKW discharge, C_{dW} is the PKW discharge coefficient, g is the gravitational acceleration, and H_o is the total head over the crest level.
Rating curve of each model as well as the plot of (C_{dW} vs. H_o/P) are presented in the following sections.

4.1 Effect of the side wall angle α

Two models were fabricated having the same initial value of W_i/W_o as the model (M) (i.e. W_i/W_o=1.25) with the value of α changing each time. The first model has α=5°. In the second model, the angle α was maximized within the available space so that the outlet key width at the upstream edge is zero, i.e. creating a triangular layout to the outlet keys. The value of α was found to be 10.25°.
Tests results of C_{dW} vs. H_o/P are shown in Figure 3. It is noticed that the model (α10) is less efficient than (M) relative to (α5) which is very similar to (M).

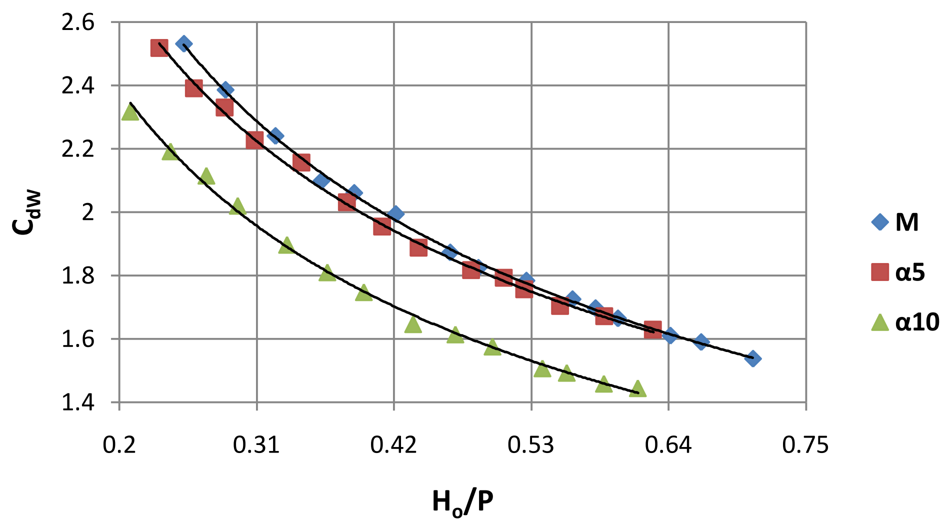

Figure 3. Variation of C_{dW}vs. H_o/P for three α values

In Figure 3, model (α5) is 3% less than (M) at low heads, but tend to be identical with (M) at high heads. Model (α10) is ranging from about 15% to 13% less than (M) at low and high H_o/P respectively. However, since the heights of these models are not equal, this chart does not represent how C_{dW} change with the increasing absolute total head H_o. Therefore, Figure 4 is prepared where the data of C_{dW} vs. H_o are plotted.
Contrary to Figure 3, data in Figure 4 show that the model (α5) performs slightly better than (M). At low heads, both models are similar, but (α5) becomes 4% larger than (M) at the maximum tested head. The model (α10) seems less efficient than (M). It ranges from about 8% to 5.5% less than (M) at low and high heads respectively.
Rating curves of these models are depicted in Figure 5 where (α5) seems slightly more effective than (M).
In Figure 6, the percentage change of C_{dW} is plotted against H_o. The percentage change of C_{dW} is calculated relative to the model (M) where:

$$\%\text{Change of } C_{dW} = \frac{\text{Tested model } C_{dW} - \text{(M)model } C_{dW}}{\text{(M)model } C_{dW}} \times 100\% \tag{19}$$

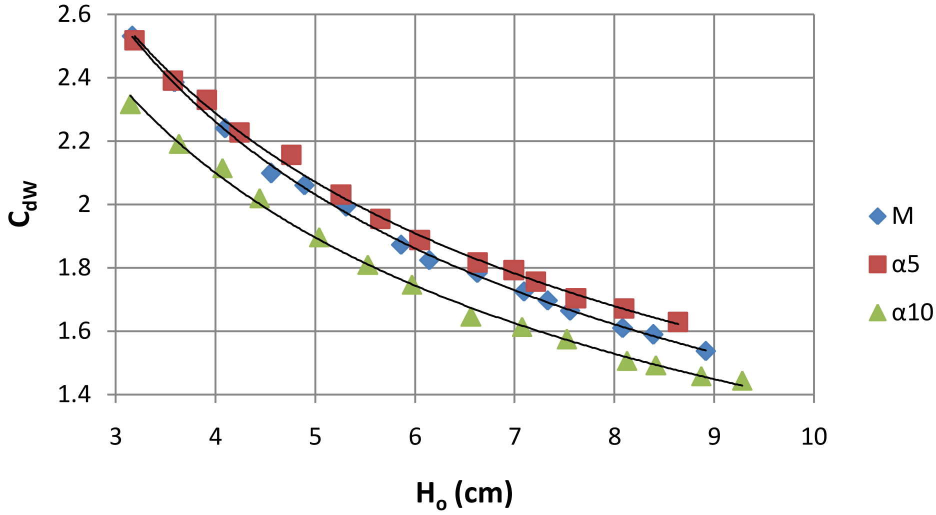

Figure 4. Variation of C_{dW}vs. H_o for three α values

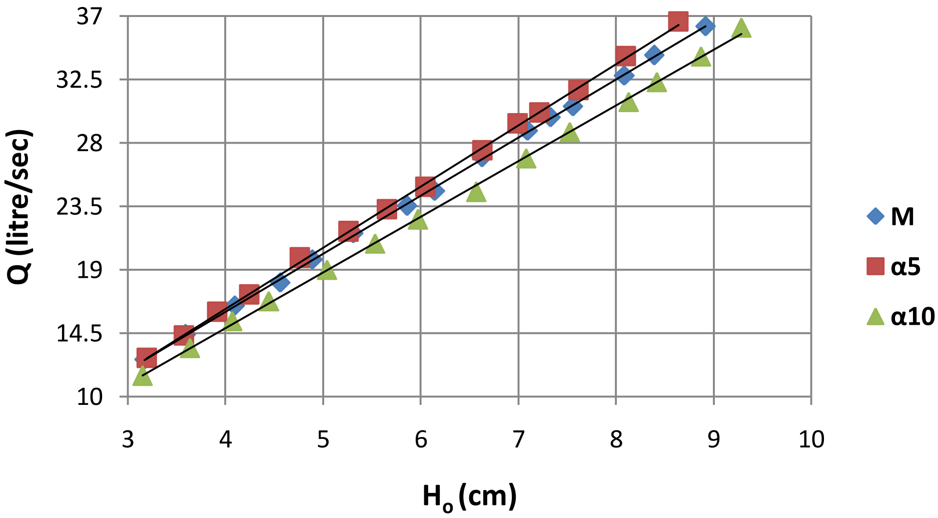

Figure 5. Experimental rating curves for models (M), (α5), and (α10)

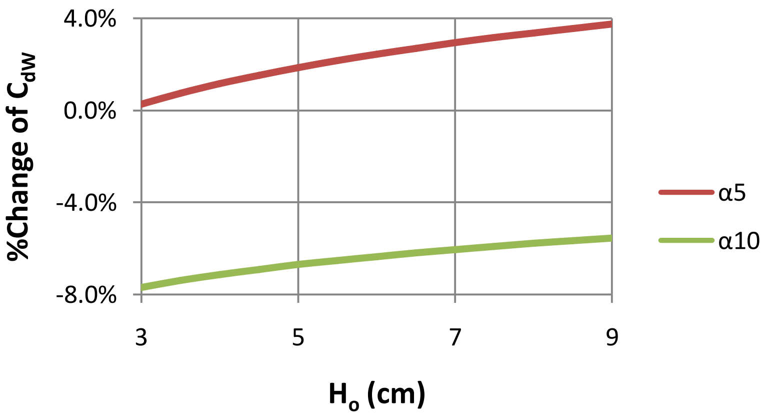

Figure 6. Percentage change of C_{dW} for the (α5) and (α10) relative to model (M) vs. H_o

It may be understood that adjusting α to 5° has a slight influence (may be neglected) on the discharge capacity, while increasing it up to 10° can reduce the capacity a little more intensely.

The negative effect of (α10) may be caused by the pronounced increase of local submergence in the upstream-side part of the outlet key making it inactive. This is due to the reduction of the outlet key cross-section resulted from angle α. Figure 7 shows the models (α5) and (α10) under operation.

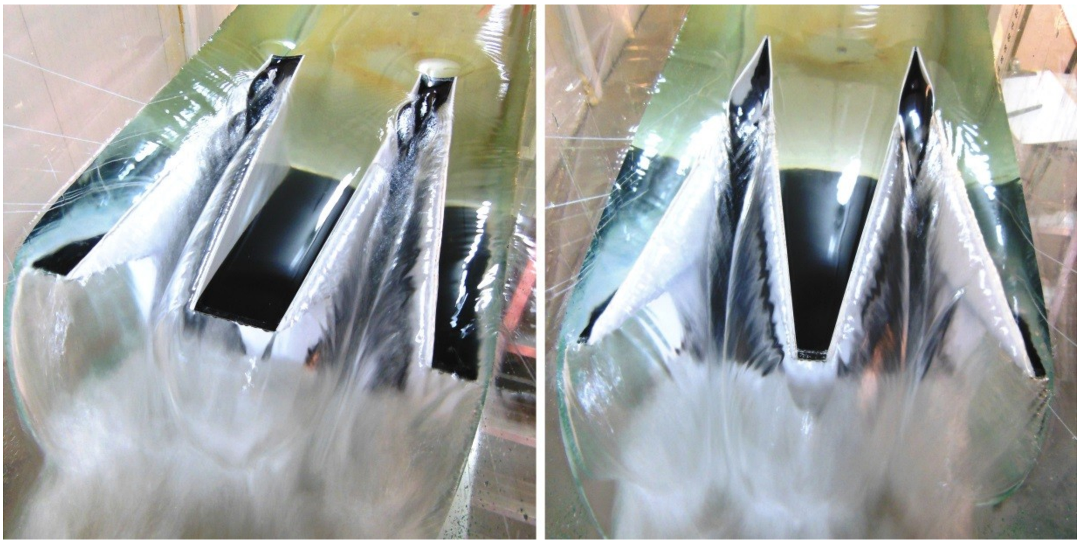

Figure 7. Views of PKW models (α5) (left), and (α10) (right)

4.2 Comparison of experimental results with those of Cicero et al [7]

Results presented in section 4.1 are dissimilar to those reported by Cicero *et al* [7] as they compared two trapezoidal models to a rectangular type-A model. Table 5 presents their properties. Note that the term W_i/W_o represents the initial rectangular condition of trapezoidal models prior to the application of α.

Table 5. Properties of the PKW models in the study of Cicero *et al* [7]

Model	L/W	B/P	W_i/W_o	B_i/B	B_o/B	P_d/P	α
Rectangular	4.61	2.58	1	0.27	0.27	1.63	0°
Trapezoidal 1	4.61	2.78	2.25	0.28	0.28	1.63	5°
Trapezoidal 2	4.35	2.58	2.1	0.27	0.27	1.63	5°

Selection of geometric parameters of *Trapezoidal 1* was such that the ratio L/W is the same as the model *Rectangular* as it has important effect on the discharge capacity. On the other hand, *Trapezoidal 2* was designed to maintain the same value of upstream-downstream length, B, as the *Rectangular* model because of its influence on the building cost of the PKW (i.e. the same ratio of B/P).

However, results showed that the model *Trapezoidal 1* is more efficient than *Rectangular* by approximately 20% in low heads (H_o/P=0.1), and about 5% in medium to high heads (H_o/P= from 0.3 to 0.7). *Trapezoidal 2* was about 2% less than *Trapezoidal 1* for all heads due to its reduced L/W.

In fact, this capacity improvement is probably due to the combined effect of the angle α and the increase in W_i/W_o as there is a considerable difference in W_i/W_o between *Rectangular* and trapezoidal models; (See Table 5).

In this study the separate investigation on the effect of the side wall angle α has proved that it has no positive effect on its own without being supplemented with an increase in W_i/W_o. Furthermore, when α is increased to about 10°, a decrease in capacity occurs. However, more detailed study should be made in future to explore how different angles of α associated with different values of W_i/W_o influence the discharge capacity of PKW.

4.3 Effect of the side wall inclination angle β

Two models were prepared to investigate the effect of the side wall inclination angle β, namely (β5) and (β10). Although no previous study was found in the literature about this parameter, it is expected to be similar to the side wall angle α to some degree since both α and β are aimed to widen the inlet key cross-section, hence, improving the discharge capacity.

Figure 8 presents the variation of C_{dW} vs. H_o/P for the three models (M), (β5), and (β10). It can be noticed that the model (β5) is very similar to (M) where the difference between them is around 2.5% at low heads (H_o/P=0.25), while the difference diminishes at high heads. The model (β10) is about 18% less than (M) at (H_o/P=0.25) but the decrease becomes only 9% at (H_o/P=0.7).

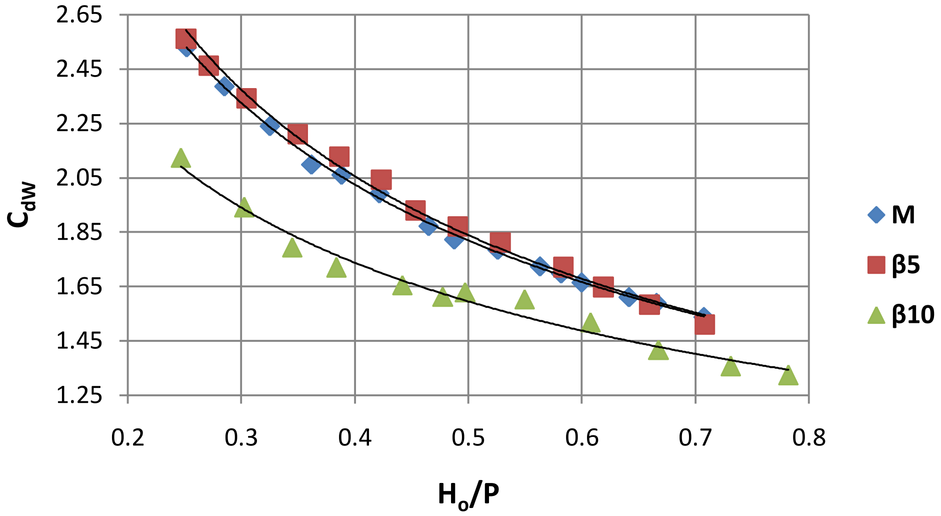

Figure 8. Variation of C_{dW}vs. H_o/P for three β values

The percentage changes relative to model (M) are illustrated in Figure 9. Rating curves of the three models are depicted in Figure 10.

It is clear how the models (M) and (β5) are almost identical. No advantage was gained by implementing an inclination angle β of 5°. On the other hand, model (β10) reveals a reduction in discharge capacity. This reduction (from 18% to 9%) is even more than the reduction of (α10) which is 8% to 5.5%.

Since the model (β10) has obviously reduced the discharge capacity relative to (M), it is not of interest. This decrease is probably to the reduction of the outlet key width at top, therefore, less quantity of water will be spilled over the side crest into the outlet key.

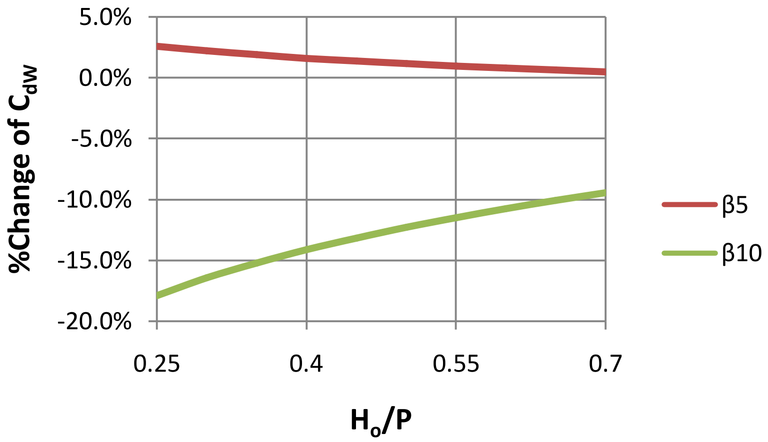

Figure 9. Percentage change of C_{dW} for the β models relative to model (M) vs. H_o/P

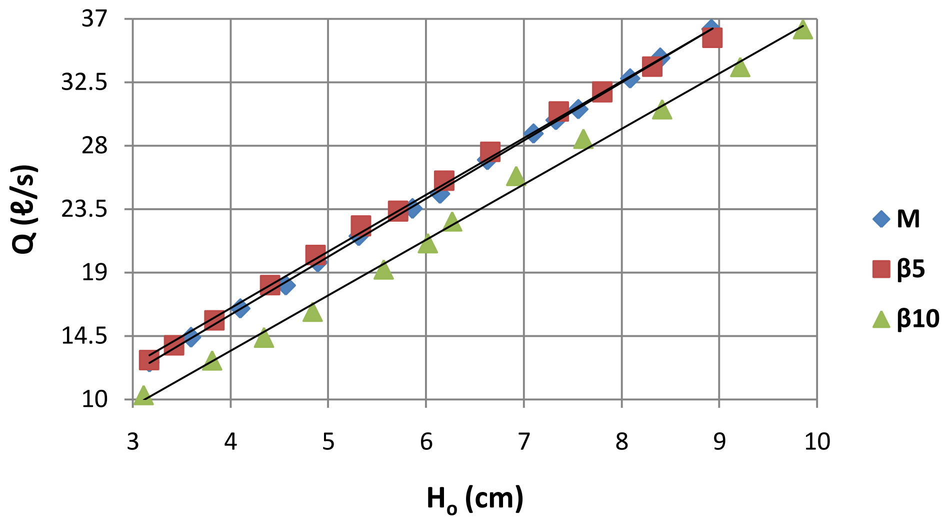

Figure 10. Experimental rating curves for models (M), (β5), and (β10)

It may be said that the as β increase, submergence occurs within the entire outlet key while in case of α increase, only the upstream half of the outlet key is submerged with the downstream half being widened and able to evacuate the flow freely. Thus, the negative effect of increasing α too much is less serious than that of increasing β. Photographs of models (β5) and (β10) are shown in Figure 11.

It seems that the model (β5) have somewhat similar effect to (α5) as both of them are close to (M) in their performance. Again, it is not possible according to the present results to determine how much the utilization of the inclination angle β combined with modifications in W_i/W_o can be helpful in capacity improvement. More detailed studies should be made about this aspect. Despite of that, it can be stated generally that future studies should concentrate on values around 5° for both α and β since increasing them up to 10° may cause a reduction in discharge capacity due to the outlet key inactivity resulted by its submergence. More interest should be given especially to the angle α since its effect of reducing C_{dW} is less in tense. In fact the parameter β could be of bad impact on the PKW cost since the construction of inclined walls is unfavourable option. However, it may be of interest in small structures manufactured from steel plates.

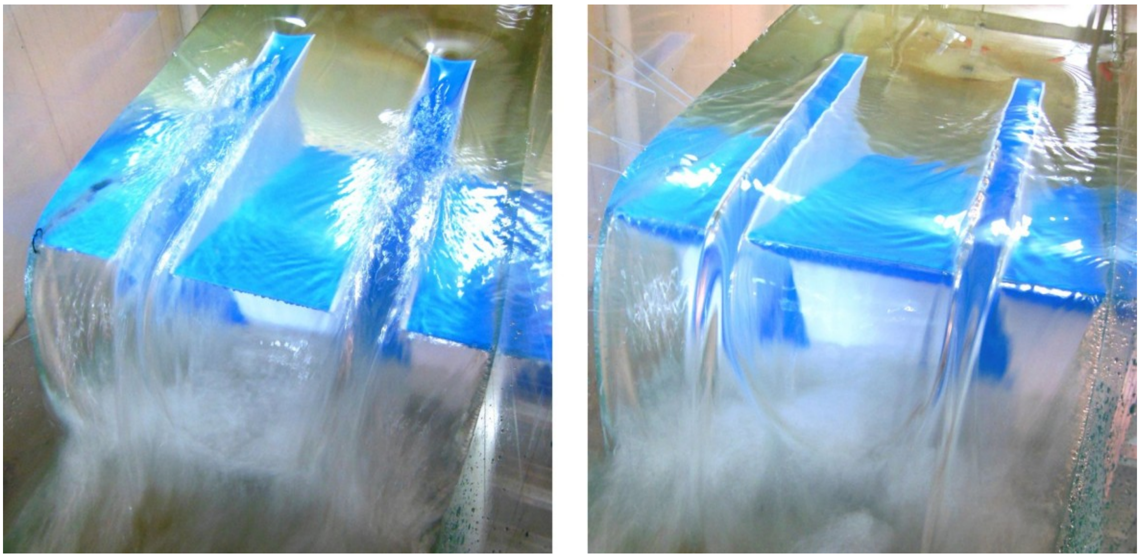

Figure 11. Views of PKW models (β5) (left), and (β10) (right)

4.4 Regression equations

In order to predict discharge capacity of non-rectangular configurations, a regression equation that determines C_{dW} of the tested models as a function of (H_o/P) is presented in the following form:

$$C_{dW} = a\left(\frac{H_o}{P}\right)^b \tag{20}$$

where a and b are coefficients which are given in Table 6 for each model. This power regression equation is valid within the given ranges of H_o/P. Refer to Figures 3 and 8 where this equation is graphically represented for each model as curve fitting.

Table 6. Coefficients of regression equation $C_{dW}=f\ (H_o/P)$ for the tested models

Model	a	b	Limitation	R^2
(M)	1.3042	-0.479	$0.25 \le H_o/P \le 0.71$	0.9986
(α5)	1.3161	-0.448	$0.23 \le H_o/P \le 0.63$	0.9975
(α10)	1.1432	-0.458	$0.21 \le H_o/P \le 0.62$	0.9972
(β5)	1.3009	-0.499	$0.25 \le H_o/P \le 0.71$	0.9937
(β10)	1.2213	-0.384	$0.25 \le H_o/P \le 0.78$	0.9768

5. Conclusion

In this study, separate investigation of the side wall angle α and the side wall inclination angle β has been carried out on a standard rectangular PKW model. Each time one of the angles α or β is changed, all other geometric parameters are held constant. Values of W_i/W_o for non-rectangular models represent the initial rectangular configuration prior to application of α or β.

The side wall angle α is an interesting parameter that may be utilized to improve the PK weir discharge capacity. Increasing α to 5° has a minor effect of about 4% gain, while increasing α to 10.25° has a negative effect of 8% to 5.5% loss for a given upstream head H_o. More comprehensive studies should be made on this parameter in the range of (0° to 5°) along with changing W_i/W_o to enhance the PKW discharge capacity.

Inclination angle of the side wall β has somewhat similar effect to that of α. Increasing β to 5° does not influence the PK weir behaviour. Increasing it to 10° reduces the capacity by 18% to 9%. Again more studies should be made on β with the range of (0° to 5°) combined with variations in W_i/W_o to identify the effect of β on discharge capacity.

Acknowledgement

Experimental work has been conducted in the Laboratory of Hydraulics, Structures and Water Resources Engineering Department, Faculty of Engineering, University of Kufa, Iraq

References

[1] Schleiss, A. J. (2011). From Labyrinth to Piano Key Weirs – A historical review. Proceedings of the International Conference(Labyrinth and Piano Key Weirs – PKW 2011), Liège, Belgium, CRC Press, London, pp. (3-15).

[2] Lempérière, F., Vigny J.-P., and Ouamane, A. (2011). General comments on Labyrinths and Piano Key Weirs: The past and present. Proceedings of the International Conference(Labyrinth and Piano Key Weirs – PKW 2011), Liège, Belgium, CRC Press, London, pp. (17-24).

[3] Ouamane, A., and Lempérière, F. (2006). Design of a new economic shape of weir. <http://www.hydrocoop.org>.

[4] Leite Ribeiro, M., Pfister, M., Schleiss, A. J., and Boillat, J.-L. (2012). Hydraulic design of A-type Piano Key Weirs. Journal of Hydraulic Research(Taylor & Francis Group), Vol. (50), No. (4) (2012), pp. (400–408).

[5] Machiels, O. (2012). Experimental study of the hydraulic behaviour of Piano Key Weirs. Ph.D. Thesis, Faculty of applied science, University of Liège, Belgium.

[6] Kabiri-Samaini, A., and Javaheri, A. (2012). Discharge coefficients for free and submerged flow over Piano Key weirs. Journal of Hydraulic Research(Taylor & Francis Group), Vol. (50), No. (1) (2012), pp. (114–120).
[7] Cicero, G. M., Delisle, J. R., Lefebvre, V., and Vermeulen, J. (2014). Experimental and numerical study of the hydraulic performance of a trapezoidal Piano Key weir. Proceedings of the International Conference(Labyrinth and Piano Key Weirs II – PKW 2013), Chatou, Paris, France, CRC Press, London, pp. (265-272).
[8] Pralong, J., Vermeulen, J., Blancher, B., Laugier, F., Erpicum, S., Machiels, O., Pirotton, M., Boillat, J.-L., Leite Ribeiro, M., and Schleiss, A. J. (2011). A naming convention for the Piano Key Weirs geometrical parameters. Proceedings of the International Conference(Labyrinth and Piano Key Weirs – PKW 2011), Liège, Belgium, CRC Press, London, pp. (271-278).
[9] Lempérière, F. (2009). New Labyrinth weirs triple the spillways discharge – Data for an easy design of P.K. Weir. <http://www.hydrocoop.org>.
[10] Novak, P., Guinot, V., Jeffrey, A., and Reeve, D. E., (2010). Hydraulic modelling – an introduction. (1st edition), Spon Press, London.

Saleh Issa Khassaf received his degrees (BS 1986; MS 1991) in civil engineering from Baghdad University and (PhD 1999) in hydraulic structures engineering from Department of Building and Construction Engineering, Technology University, Baghdad, Iraq. His work includes topics like: scour phenomenon, sediment transport, seepage analysis. Prof Khassaf is the chair of editorial board of Basrah Journal for Engineering Sciences. He is the author or co-author of 45 research papers.
E-mail address: salehissakh@gmail.com

Mohamed Baqir Al-Baghdadi received his degrees (BS 2012) in structures and water resources engineering from Faculty of Engineering, University of Kufa, Iraq. He is a MSc student at the civil engineering department in the same university. His main research interest is hydraulics and hydraulic structures.
E-mail address: mohamedbn.en12p@uokufa.edu.iq